Theory of Stellar Structure and Evolution

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Abstract

This paper presents a *computational* version of the theory of stellar structure and evolution developed by Dewey B. Larson in his books, especially *The Universe of Motion*. The theory is called the *Reciprocal System* because it is based on the postulated *reciprocal relationship* between space and time. In this paper, detailed equations are given for the structure of Main Sequence stars, Red Giants, and White Dwarfs / Pulsars. *Gravitational segregation* of elements, rather than mixing, occurs in all stars; density *decreases linearly with distance from the core* in Main Sequence stars and Red Giants, whereas it decreases *linearly with distance from the surface* in White Dwarfs and Pulsars. The paper presents the *step-by-step evolution* of Main Sequence stars, Red Giants, White Dwarfs, and Pulsars. Stars can go through a number of *cycles*, with a Type I Supernova terminating one cycle and beginning the next. They end their existence in a Type II Supernova, which results in Pulsars, and these may leave our material sector for the cosmic (inverse) sector. Main Sequence stars and White Dwarfs / Pulsars have a *surface*--this means that they are actually in the *condensed gas* or *liquid state*, *not* the ideal gas state; Red Giants are in the *gaseous state*, although as they contract and move toward the Main Sequence, they begin to *convert to the liquid state*. The thermal destructive limits of the elements in the stars and in supernovae are calculated, and detailed equations for the energy generation of stars and supernovae are given. Comparisons with the data of actual stars are presented to verify the theory. Plots are given for temperature, pressure, and density with distance from the core or surface for the major classes of stars.

keywords: Reciprocal System, stellar structure, stellar evolution, Main Sequence stars, Red Giants, White Dwarfs, Pulsars

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Introduction and Literature Review

The Reciprocal System represents a radical break with conventional physics and astrophysics. It is based on the concept that the *fundamental component* of the physical universe is *space-time*, or *motion* in the most general sense--not matter. Space and time are the *two reciprocal aspects* of space-time; this means that more space equals less time, and vice versa. It also means that space and time are *perfectly symmetrical* with each other. Based on two Fundamental Postulates (see Appendix A), the theory covers *all* of science and is thus a "Theory of Everything." This paper focuses on the astrophysical aspects of the theory, specifically the structure and evolution of every major class of star. Ref. [1], *The Universe of Motion*, Dewey B. Larson's major work in astronomy and astrophysics, is the starting point for the paper. Ref. [2] is Larson's earlier version of the theory, somewhat superceded by Ref. [1], but still worth reading.

A theoretical physicist must have a thorough knowledge of existing theories and equations. To that end, this author conducted a very extensive review of the astrophysics literature. Ref. [3], *Stellar Evolution*, edited by R. Stein and A. Cameron, has been most helpful in this regard. The first paper in that volume, by R. Stein, provides an excellent *linear density model* for stellar structure; this has formed the basis of the equations given in this paper--but there are major differences. For instance, Stein and his colleagues insist that there is *no gravitational segration of the elements* and so there is *hydrogen in the core*; this is not so in the Reciprocal System!

Ref. [4], *Theory and Problems of of Astronomy*, by Palen, is a simple exposition for beginners of basic astronomy with many solved problems.

Ref. [5], *Essential Astrophysics*, by Lang, is a simple and clear treatment of contemporary astrophysics, including both observations and theories.

Ref. [6], *Astrophysical Formulae*, also by Lang, is a comprehensive, high-level, mathematical treatment of current conventional theories.

Ref. [7], *The Physical Universe*, by Shu, is an older exposition of astronomy and astrophysics; it's still worth reading because of its clarity.

Ref. [8], *An Introduction to Modern Astrophysics*, 2nd ed., the famous work by Carroll and Ostlie, is the current favorite on college campuses. It has 1278 pages, not counting numerous appendices. It's this author's go-to-resource for current conventional theory.

Ref. [9], *Astrophysical Concepts*, by Harwit, has gone through numerous editions, but there has been relatively little change since it was first published.

Ref. [10], *Introductory Astronomy and Astrophysics*, by Smith and Jacobs, is this author's favorite astrophysical work (with the exception of Ref. [1], of course), simply because it is so concise and well-organized; it is also more focused on observations and proven equations, rather than on theory and unproven equations.

Ref. [11], *Allen's Astrophysics Quantities,* is the standard collection of astrophysical data and so is always helpful.

Ref. [12], *The Astronomy and Astrophysics Encyclopedia,* ed. by Maran, is a nice collection of articles.

Ref. [13], *The Tapestry of Modern Astrophysics*, by Shore, is a comprehensive, but difficult to follow, treatment of theoretical astrophysics.

Ref. [14], *Theoretical Astrophysics*, 3 volumes, by Padmanabhan, is a complete collection of the currently accepted equations of the establishment astrophysics community; its one weakness is the poor index to each volume.

Ref. [15], *Handbook of Space Astronomy and Astrophysics,* ed. by Zombeck, is somewhat similar to Ref. [11]; it is nicely organized and very helpful for the completeness of its data.

Ref. [16], *Stellar Structure and Evolution*, 2nd. ed, by Kippenhahn, Weigert, and Weiss, is the standard reference for the conventional theory of the subject matter of this paper.

Ref. [17], *Catalogue of the Universe*, by Murdin and Allen, is a nice collection of pictures and data on specific individual galaxies, clusters, stars, and the planets of our solar system.

Ref. [18], *Encyclopedia of Astronomy and Astrophysics*, 4 volumes, ed. by Murdin, is by far the most comprehensive treatment of the subject--but the writing is very wordy and there are relatively few equations.

Ref. [19], *The Canopus Encyclopedia of Astronomy,* ed. by Murdin and Penston, is a concise one volume version of Ref. [18]--it's actually much more helpful than Ref. [18].

Ref. [20], *Encyclopedia of Astronomy and Astrophysics*, ed. by Myers, is similar to Ref. [12].

Ref. [21], *Physics of the Sun*, by Mullan, is a nice monograph treating the data and theories of the Sun's operation; this work is unusual in that it is computational to some extent.

Ref. [22], *Stellar Interiors: Physical Principles, Structure, and Evolution*, by Hansen and Kawaler, is similar to Ref. [16].

Ref. [23], *Astrophysics Through Computation*, by Koberlein and Meisel, is quite rare in astrophysics because it provides actual computer code in *Mathematica* for stellar properties (according to the conventional equations such as those known as the Lane-Emden).

Ref. [24], *The New Cosmos*, 4th ed., by Unsold and Baschek, is nicely written and well organized; it is similar to Ref. [10].

Ref. [25], *The Cosmos: Astronomy in the New Millennium*, 4th ed., by Pasachoff and Filippenko, is a very contemporary account with numerous photographs and diagrams.

Ref. [26], *Patrick Moore's Data Book of Astronomy*, by Moore and Rees, contains a helpful collection of data tables.

Nomenclature

Av = Avogadro's Number

a0 = semi-major axis of first orbiting planet around a star, AU

adjFe = adjustment factor for element Fe to reduce it to a more normal amount

age_{Pulsar} = age of Pulsar, yr

age_{Pulsar_unit} = empirical coefficient for Pulsar age calculation, yr

an = calculated semi-major axis of planet from star, AU

c_{cgs} = speed of light, cm/sec

c_{SI} = speed of light, m/sec

$conv_{amu\_to\_u}$ = conversion factor from amu to u

$conv_{atmtodynescm2}$ = conversion factor from atm to dynes/cm^2

$conv_{dynescm2toatm}$ = conversion factor from dynes/cm^2 to atm

$conv_{MeV\_to\_erg}$ = conversion factor from Mev to erg

$conv_{MeV\_to\_J}$ = conversion factor from MeV to J

$conv_{u\_to\_kg}$ = conversion factor from u to kg

$conv_{u\_to\_Mev}$ = conversion factor from u to MeV

$E_{I\_el\_MeV}$ = total ionization energy of element *el*, MeV

$E_{S\_gen\_MeV}$ = energy generated by individual fission reaction in Sun, MeV

$E_{S\_gen\_erg}$ = energy generated by individual fission reaction in Sun, erg

$E_{star\_gen\_MeV}$ = energy generated by individual fission reaction in star, MeV

$E_{S\_gen\_conv\_MeV}$ = energy generated by indidual set of fusion reactions in Sun according to conventional theory, MeV

end_elem = element in last shell of star which is fissioning (the remaining mass of heavier elements is lumped-in)

F_S = energy flux or radiation emittance of Sun, erg/sec cm^2

F_{star} = energy flux or radiation emittance of star, erg/sec cm^2

G_{cgs} = gravitational constant, cgs units

G_{SI} = gravitational constant, SI units

G_{el} = number of neutrinos harbored by an atom of element *el*

I = neutrino ionization level

I_R = interregional ratio

k =empirical coefficient for planet spacing calculation

k_{B_J} = Boltzmann's constant, J/K

k_{B_MeV} = Boltzmann's constant, MeV/K

k_{s1} = solid geometric factor for the liquid molecule

k_{s2} = liquid geometric factor for the liquid molecule

k_{s3} = vapor/gas geometric factor for the liquid molecule

L_{S_cgs} = luminosity of the Sun, erg/sec

L_{star_cgs} = luminosity of star, erg/sec

L_{S_Mev} = luminosity of the Sun, MeV/sec

L_{S_SI} = luminosity of the Sun, J/sec

L_{star_SI} = luminosity of star, J/sec

L_{star_Mev} = luminosity of star, MeV/sec

M = general symbol for mass of star, g

M_{S_cgs} = current mass of the Sun, g

M_{S_SI} = current mass of the Sun, kg

$M_{S\_cgs\_el}$ = mass of the Sun at the time that element *el* is undergoing fission, g

$M_{S\_SI\_el}$ = mass of the Sun at the time that element *el* is undergoing fission, kg

M_{star_cgs} = mass of star, g

M_{star_SI} = mass of star, kg

$M_{star\_cgs\_el}$ = mass of star at the time that *el* is undergoing fission, g

$M_{star\_SI\_el}$ = mass of star at the time that el is undergoing fission, kg

metallicity = base 10 log of the ratio of the *mass* of "metal" atoms to that of hydrogen and helium in the star vs. that of the Sun

m_r = equivalent number of rotational electric time displacements of element (same as atomic number Z)

$m_{S\_el\_MeV}$ = mass of atom of fissioning element *el* in Sun, MeV

$m_{star\_el\_MeV}$ = mass of atom of fissioning element *el* in star, MeV

m_{u_g} = unit mass, g

$N_{S\_conv\_reac}$ = number of fusion reactions per second occurring in the Sun according to conventional theory

N_{S_reac} = number of fission reactions per second occurring in the Sun

N_{star_reac} = number of fission reactions per second occurring in a star

$N_{V\_frac\_S\_el}$ = fraction of atoms of element el reaching the Sun's core which have the destructive thermal velocity

$N_{V\_frac\_star\_el}$ = fraction of atoms of element el reaching the star's core which have the destructive thermal velocity

n = number of atomic magnetic rotational displacements relevant to the thermal destructive limit

n = speed distribution number of planet

n = speed distribution number for Pulsar

n_{s1} = number of close-packed groups per molecule in the solid state (as solution in liquid)

n_{s2} = number of close-packed groups per molecule in the liquid state

n_T = number of temperature units of the liquid molecule

n_V = number of volumetric groups of the liquid molecule

P = general symbol for pressure of star, dynes/cm^2

P = Pulsar pulse period, sec

P_{0_L} = initial pressure in liquid state, atm

P_c = pressure of star at the core or center, dynes/cm^2 (Main Sequence stars and Red Giants) (subscript may have star's name)

P_{c_atm} = pressure of star at the core or center, atm (Main Sequence stars and Red Giants) (subscript may have star's name)

P_{L_u} = unit pressure for liquid state, atm

P_s = pressure of star at surface, dynes/cm^2 (White Dwarfs / Pulsars) (subscript may have star's name)

P_{s_atm} = pressure of star at surface, atm (White Dwarfs / Pulsars) (subscript may have star's name)

R = general symbol for radius of star, cm (subscript may have star's name)

R_{gas} = gas constant, atm cm^3/mol K

R_{Pulsar} = radius of Pulsar, cm

R_{S_cgs} = current radius of the Sun, cm

R_{S_SI} = current radius of the Sun, m

r = general symbol for distance from stellar core to a specific element shell (Main Sequence stars and Red Giants); general symbo for distance from stellar surface to a specific element shell (White Dwarfs / Pulsars), cm

r_1 = shell outer radius (dynamically calculated for each element), cm

r_2 = shell inner radius (dynamically calculated for each element), cm

r_{mean} = mean value of r_1 and r_2, cm

s_{Pulsar_equiv} = space equivalent of the maximum Pulsar period of .62 seconds, cm

s_{pulse_width} = width of pulse for average Pulsar, cm

T = general symbol for temperature, K (for the shell computations, the *el* subscript is used)

T_c = temperature of star at core or center, K (Main Sequence stars and Red Giants) (subscript may have star's name)

$T_{destructive\_S\_el}$ = destructive temperature limit of Sun with fissioning element *el*, K

$T_{destructive\_star\_el}$ = destructive temperature limit of star with fissioning element *el*, K

T_{S_el} = temperature of atom of element *el* after it reaches the Sun's core or center, K

T_{star_el} = temperature of atom of element *el* after it reaches the star's core or center, K

T_{s_S} = temperature of Sun at surface, K; also known as effective temperature, T_{eff_S}

T_{s_star} = temperature of star at surface, K; also known as effective temperature, T_{eff_star}

T_{SL_u} = unit temperature of the solid and liquid state, K

T_{V_u} = unit temperature in the vapor state, K

U_{S_el} = factor in Maxwell's distribution to compute fraction of atoms in Sun's core with velocity >= destructive velocity

U_{star_el} = factor in Maxwell's distribution to compute fraction of atoms in star's core with velocity >= destructive velocity

V_{00} = total initial value of the liquid specific volume (at zero temperature and pressure), cm^3/g

V_{01} = initial value of the solid molecular component of the liquid specific volume, cm^3/g

V_{02} = initial value of the liquid molecular component of the liquid specific volume, cm^3/g

V_{03} = initial vlaue of the vapor/gaseous component of the liquid specific volume, cm^3/g

V_1 = solid molecular component of the liquid specific volume, cm^3/g

V_2 = liquid molecular component of the liquid specific volume, cm^3/g

V_3 = vapor/gaseous component of the of the liquid specific volume, cm^3/g

V_{el} = volume of element shell, cm^3

V_L = total specific volume of liquid, cm^3/g

V_{L_u} = unit specific volume for liquid state, cm^3/g

$v_{destructive\_S\_el\_SI}$ = velocity of atom of element *el* sufficient to neutralize one magnetic rotational displacement, in Sun, m/sec

$v_{destructive\_star\_el\_SI}$ = velocity of atom of element *el* sufficient to neutralize one magnetic rotational displacement, in star, m/sec

v_{Pulsar_equat} = equatoria speed of Pulsar, cm/sec

$v_{S\_el\_SI}$ = velocity of atom of element *el* when it reaches the core of the Sun, km/sec

$v_{star\_el\_SI}$ = velocity of atom of element *el* when it reaches the core of star, km/sec

w_{el} = atomic weight (average) of element *el*

X = mass fraction of H in a star

XpY = mass fraction of H plus He in a star (subscript may specify the Sun or star or before and after the metallicity is added)

xl = Excel table giving the mass fraction of each element for the Sun (based on the observed amounts in the photosphere)

xlelem2 = table of properties of Sun or star as computed

Y = mass fraction of He in a star

Z_{el} = atomic number of element $_{el}$

Z_m = mass fraction of elements with higher atomic number than He in a star ("metals")

ΔM_{S_el} = mass of element shell in Sun (determined for each shell), g

ΔM_{star_el} = mass of element shell in star (determined for each shell)

ε_S = radiation per unit mass of Sun, erg/sec g

ε_{star} = radiation per unit mass of star, erg/sec g

κ_S = overall opacity for Sun, cm^2/g

κ_{star} = overall opacity for star, cm^2/g

ρ = general symbol for density, g/cm^3

ρ_c = central or core density of Sun or star, g/cm^3 (Main Sequence stars and Red Giants) (subscript may have star's name)

ρ_s = surface density of star, g/cm3 (White Dwarfs) (subscript may have star's name)

σ = Stefan-Boltzmann constant, erg / cm^2-sec-K^4

A square in the upper right of an equation means that the equation is disabled from running in *Mathcad*. This is done because not all variables in the equation have, as yet, been given numerical values. Because of the nature of this subject, it is necessary to use a mixture of SI, cgs, and natural units in the paper, but each individual equation utilizes just one set of units. Star classes are capitalized rather than in lower case.

Unit Conversions and Physical Constants

$c_{cgs} := 2.997925 \cdot 10^{10}$ cm/sec $c_{SI} := 2.997925 \cdot 10^{8}$ m/sec

$l_R := \left[128 \cdot \left(1 + \frac{2}{9}\right)\right]$ $l_R = 156.444444$ $s_{u\_cgs} := 4.558816 \cdot 10^{-6}$ cm $s_{t\_u} := \frac{s_{u\_cgs}}{l_R}$ cm

$V_{L\_u} := s_{t\_u}^{3}$ $V_{L\_u} = 2.474433 \times 10^{-23}$ $t_u := 1.520655 \cdot 10^{-16}$ sec $T_{SL\_u} := 510.8$ K $T_{V\_u} := 3.5978 \cdot 10^{9}$ K

$conv_{amu\_to\_u} := .9996822$ $conv_{u\_to\_MeV} := 931.494061$ $conv_{u\_to\_kg} := 1.66 \cdot 10^{-27}$ $conv_{J\_to\_MeV} := 6.242 \cdot 10^{12}$

$conv_{MeV\_to\_erg} := 1.60218 \cdot 10^{-6}$ $m_{u\_g} := 1.65979 \cdot 10^{-24}$ g $G_{SI} := 6.67259 \cdot 10^{-11}$ $G_{cgs} := 6.67259 \cdot 10^{-8}$

$conv_{atmtodynescm2} := 1.013 \cdot 10^{6}$ $conv_{dynescm2toatm} := \frac{1}{conv_{atmtodynescm2}}$ $conv_{dynescm2toatm} = 9.871668 \times 10^{-7}$

$Av := 6.02486 \cdot 10^{23}$ $conv_{erg\_to\_MeV} := \frac{1}{conv_{MeV\_to\_erg}}$ $conv_{erg\_to\_MeV} = 6.241496 \times 10^{5}$

$conv_{MeV\_to\_J} := \frac{1}{conv_{J\_to\_MeV}}$ $conv_{MeV\_to\_J} = 1.602051 \times 10^{-13}$ $h := 4.14 \cdot 10^{-15}$ eV-sec

$conv_{erg\_to\_J} := 1 \cdot 10^{-7}$ $conv_{J\_to\_erg} := \dfrac{1}{conv_{erg\_to\_J}}$ $conv_{sec\_to\_yr} := 3.169 \cdot 10^{-8}$

$k_{B\_J} := 1.38065 \cdot 10^{-23}$ J/K $k_{B\_MeV} := 1.3806505 \cdot 10^{-23} \cdot conv_{J\_to\_MeV}$ $k_{B\_MeV} = 8.61802 \times 10^{-11}$

$M_{S\_cgs} := 1.9884 \cdot 10^{33}$ g $M_{S\_SI} := 1.9884 \cdot 10^{30}$ kg $R_{S\_cgs} := 6.9568 \cdot 10^{10}$ cm $R_{S\_SI} := 6.9568 \cdot 10^{8}$ m

$L_{S\_cgs} := 3.8416 \cdot 10^{33}$ erg/sec $L_{S\_SI} := L_{S\_cgs} \cdot conv_{erg\_to\_J}$ $L_{S\_SI} = 3.8416 \times 10^{26}$ J/sec

$L_{S\_MeV} := L_{S\_cgs} \cdot conv_{erg\_to\_MeV}$ $L_{S\_MeV} = 2.397733 \times 10^{39}$ MeV/sec $\sigma := 5.6704 \cdot 10^{-5}$ erg/(cm^2-sec-K^4)

ORIGIN := 1   so that matrix indices start at 1, rather than 0

I. Stellar Structure

A. Main Sequence Stars

1. the Sun as starting point for analysis of Main Sequence stars

We begin our analysis with the star we know best--the Sun. Because we are not able to peer below the Sun's photosphere, we will have to make a number of assumptions. These assumptions are *consistent* with the Postulates of the Reciprocal System, so we will label them Tentative Theorems.

Tentative Theorem 1: The mass fractions of the elements in the photosphere of the Sun are *roughly* the mass fractions of the elements in the Sun *as a whole*.

Tentative Theorem 2: For other stars, the mass fractions of the elements are that of the Sun but *modified* by the star's *metallicity* and *by the current main element undergoing fission.* Elements *heavier* than that undergoing fission are *lumped*-in with that element. (This is also done with the Sun itself.)

Ref. [27] gives the currently accepted mass fractions of the elements in the photosphere of the Sun. A few of the elements are listed as having zero mass fractions; we have replaced these with *extrapolated* values, simply because it would be extremely unlikely that these would *actually* be zero. Table I follows; it is an Acrobat PDF document based on an Excel document.

Table I. Mass Fraction of the Elements of the Sun's Photosphere

In studying Table I, one can see that the mass fraction of Fe is an *outlier*. It is stated as being 1.2735×10^{-3} which is way beyond that of the adjacent elements. Although this is apparently the observed mass fraction in the photosphere, it is *not* necessarily its mass fraction deeper in the Sun. In order to be compatible with Tentative Theorem 3 below, we will have to *reduce* the mass fraction of Fe by 29%. So

$adjFe := 0.71$ (for the Sun and most stars) (1)

Fe is the *only* element which has to be adjusted for the analysis. The mass fraction of the very heaviest elements are not known, but have been set to 1e-11, which is tiny, to avoid division by zero in what follows.

2. density ρ(r) from core to surface

We can now state

Tentative Theorem 3: *Gravitational segregation* of the elements occurs in all stars. In the Main Sequence stars and Red Giants, the heavier elements settle *nearer to the core* than the lighter elements. There is a *linear density gradient* from the core of the star to the surface. The elements concentrate into *individual spherical shells*.

This tentative theorem can be put into mathematical form as follows:

$$\rho(r) := \rho_c \cdot \left(1 - \frac{r}{R}\right) \quad \text{g/cm}^3 \qquad (2)$$

where $\rho(r)$ is the density of the star's contents at distance r from the center, ρ_c is the central density, and R is the radius of the star. As stated in the Introduction, Ref. [3] has been helpful in our formulation of the fundamental equations of stellar structure--but, unlike the Reciprocal System, it does *not* posit gravitational segregation of the elements. As the equation shows, the density is zero at the surface, and the density is ρ_c at the core.

Let r_1 = radius of the outer surface of a spherical shell and r_2 = radius of the inner surface of a spherical shell. The mean density of a particular spherical shell is calculated as follows (from the mean value theorem of calculus).

$$r_{mean} := \frac{r_1 + r_2}{2} \qquad (3)$$

$$\rho\left(r_{mean}\right) := \rho_c \cdot \left(1 - \frac{r_{mean}}{R}\right) \qquad (4)$$

3. mass M(r) inside radius *r*

By basic geometry, the *mass* of the star *inside radius r* is

$$M(r) := \int_0^r 4\cdot\pi\cdot r^2\cdot\rho(r)\,dr \qquad g \qquad (5a)$$

or

$$M(r) := \int_0^r 4\cdot\pi\cdot r^2\cdot\left[\rho_c\cdot\left(1-\frac{r}{R}\right)\right]dr \qquad g \qquad (5b)$$

Carrying out the integration:

$$M(r) := \frac{4}{3}\cdot\pi\cdot\rho_c\cdot r^3\cdot\left(1-.75\cdot\frac{r}{R}\right) \qquad g \qquad (5c)$$

The total mass of the star at r = R is, by inspection,

$$M := \frac{1}{3}\cdot\pi\cdot\rho_c\cdot R^3 \qquad g \qquad (6)$$

5. central density ρ_c

Usually we know the total mass and radius of the star, so with that knowledge we can compute the central density from Eq. (4):

$$\rho_c := \frac{3 \cdot M}{\pi \cdot R^3} \quad \text{g/cm}^3 \qquad (7)$$

6. pressure p(r) from from core to surface

As in conventional theory, we assume *hydrostatic equilibrium*, on *small* time scales, for Main Sequence stars. Thus

$$\frac{d}{dr}P(r) := \frac{-G_{cgs} \cdot M(r) \cdot \rho(r)}{r^2} \quad \text{dynes/cm}^2 \text{ / cm} \qquad (8)$$

Letting P_C be the pressure at the center, we have

$$P(r) := P_c - \int_0^r \frac{G_{cgs} \cdot M(r) \cdot \rho(r)}{r^2} \, dr \quad \text{dynes/cm}^2 \qquad (9a)$$

or

$$P(r) := P_c - \int_0^r \frac{G_{cgs}\cdot\left[\frac{4}{3}\cdot\pi\cdot\rho_c\cdot r^3\cdot\left(1 - .75\cdot\frac{r}{R}\right)\right]\cdot\left[\rho_c\cdot\left(1 - \frac{r}{R}\right)\right]}{r^2}\,dr$$ dynes/cm^2 (9b)

Carrying out the integration:

$$P(r) := P_c - \frac{2\cdot\pi}{3}\cdot G_{cgs}\cdot\rho_c^{\,2}\cdot r^2\cdot\left(1 - \frac{7}{6}\cdot\frac{r}{R} + \frac{3}{8}\cdot\frac{r^2}{R^2}\right)$$ dynes/cm^2 (9c)

Applying the boundary condition P(R) = 0:

$$P(r) := \frac{\pi}{36}\cdot G_{cgs}\cdot\rho_c^{\,2}\cdot R^2\cdot\left(5 - 24\cdot\frac{r^2}{R^2} + 28\cdot\frac{r^3}{R^3} - 9\cdot\frac{r^4}{R^4}\right)$$ dynes/cm^2 (9d)

The mean pressure of a spherical shell will be approximated by

$$P\left(r_{mean}\right) := \frac{\pi}{36}\cdot G_{cgs}\cdot\rho_c^{\,2}\cdot R^2\cdot\left(5 - 24\cdot\frac{r_{mean}^{\,2}}{R^2} + 28\cdot\frac{r_{mean}^{\,3}}{R^3} - 9\cdot\frac{r_{mean}^{\,4}}{R^4}\right)$$ (10)

7. central Pressure P_C

Setting r = 0 in Eq. (7d):

$$P_c := \frac{5\cdot\pi}{36}\cdot G_{cgs}\cdot\rho_c^{\,2}\cdot R^2 \qquad \text{dynes/cm}^2 \qquad (11)$$

8. equation of state for liquids and condensed gases

All of the conventional references state that the contents of a Main Sequence star are in the *ideal* or *perfect gas* state. But this is *not* so! The temperatures are very high--but so are the pressures. The atoms are within *unit distance*, s_u, from one another so that they are in the *condensed gas* or *liquid state*. A simple proof of this can be found by observing the Sun: our star has a *surface*, but an ideal or perfect gas would *not* form a surface!

Ref. [28] provides a detailed treatment of the Reciprocal System theory of liquids, vapors, and gases. We'll repeat here the relevant equations for the *liquid specific volume* calculations (which apply for both liquids and *condensed* gases).

According to the Reciprocal System, each *individual* molecule of a liquid aggregate may be in the solid, liquid, or vapor/gaseous state, regardless of the state of the *majority* of the molecules. Furthermore, if a particular *molecule* is in the liquid state, the *individual* atoms of which it is comprised may be in the *solid* state *relative* to each other. The specific volume (volume/mass) of a liquid, V_L, is the sum of the contributions of the solid (V_1), liquid (V_2), and vapor/gaseous (V_3) components:

$$V_L := V_1 + V_2 + V_3 \qquad cm^3/g \qquad (12)$$

The *initial* values of these three components are designated V_{01}, V_{02}, V_{03}. These differ only by a geometric factor (k_{s1}, k_{s2}, k_{s3}) applied to a *base initial value*, V_{00}, determined as follows.

Just as the volume of a gas is determined by the *number of molecules*, so the volume of a liquid is determined by the *number of volumetric groups* which it contains. In an organic compound, for instance, each of the common interior groups, such as CH_2, CH, or CO, constitutes one volumetric group. The CH_3 groups in the end positions of the aliphatic chains usually occupy two units each (although if the the H bonding to the CH_2 is very strong, it might be only one unit). So hexane, represented as $CH_3CH_2CH_2CH_2CH_2CH_3$, has 8 volumetric groups. Let n_V be the number of volumetric groups and recall that the factor .7071 expresses the geometric reduction obtained by the *close-packed* arrangement of the liquid groups because of their flexibility of movement. Then, in natural Reciprocal System units, the base initial volume is directly proportional to the number of volumetric groups, reduced by close-packing:

$$V_{00} := .7071 \cdot n_V \qquad \text{(natural volume/mass units)} \qquad (13)$$

In cgs units, this expression becomes

$$V_{00} := \frac{.7071 \cdot n_V \cdot V_{L\_u}}{w \cdot m_{u\_g}} \qquad \text{cm}^3/\text{g} \qquad (14a)$$

where w = atomic weight and m_{u_g} = unit atomic mass. Simplifying, using functional representation and the *Mathcad* global equality operator:

$$V_{00}(n_V, w) \equiv \frac{10.54004 \cdot n_V}{w} \qquad \text{cm}^3/\text{g} \qquad (14b)$$

For the critical (vapor or gaseous) specific volume increment, the geometric factor k_{s3} is always 1.000. For the solid specific volume increment, the geometric factor k_{s1} is the cube root of .7071, .8909, where close-packing in the solid state can be achieved. Where such packing cannot be achieved the geometric factor k_{s1} is 1.000. The same applies to the geometric factor k_{s2} for the liquid specific volume increment. Note: in situations involving *hydrogen (or another lower group element) bonding*, the effective value for k_{s1} and k_{s2} may be *lower* than .8909, like .7795 for acetic acid and .8713 for water. The initial values of the three initial specific volume components may now be expressed as

$$V_{01}(n_V, w, n_{s1}) := V_{00}(n_V, w) \cdot k_{s1}(n_{s1}, n_V) \qquad \text{cm}^3/\text{g} \qquad (15)$$

$$V_{02}(n_V, w, n_{s2}) := V_{00}(n_V, w) \cdot k_{s2}(n_{s2}, n_V) \qquad \text{cm}^3/\text{g} \qquad (16)$$

$$V_{03}(n_V, w, k_{s3}) := V_{00}(n_V, w) \cdot k_{s3} \qquad \text{cm}^3/\text{g} \qquad (17)$$

In a *multi-group* molecule, the values of the geometric factors k_{s1} and k_{s2} represent *averages*, because some groups may be at .8909 while others are at 1.000. The Reciprocal System Database allows the flexibility to designate the number of close-packed groups per molecule in the solid state, n_{s1}, and the number of close-packed groups per molecule in the liquid state, n_{s2}. Then

$$k_{s1}(n_{s1}, n_V) \equiv \left[\frac{n_{s1} \cdot .8909 + (n_V - n_{s1}) \cdot 1.000}{n_V}\right] \qquad (18)$$

$$k_{s2}(n_{s2}, n_V) \equiv \frac{\left[n_{s2} \cdot .8909 + (n_V - n_{s2}) \cdot 1.000\right]}{n_V} \qquad (19)$$

(Of course, if hydrogen (or another lower group element) bonding is involved, the coefficients could be lower.)

Now that we have the *initial* values as a function of *composition*, we can determine the values of the three components as a function of *temperature*. The solid specific volume increment not only includes the initial volume at 0 K but also a factor proportional to the number of *solid molecules* (and the difference in volume between solid and liquid) in the substance at any temperature, Δs, which can be determined by probability considerations.

$$V_1(n_V, w, n_{s1}, T, T_m, \Delta V) \equiv V_{01}(n_V, w, n_{s1}) + \Delta s(T, T_m, \Delta V) \qquad \text{cm}^3/\text{g} \qquad (20)$$

For the solid, liquid, and vapor states, the Reciprocal System usually uses the *normal probability distribution*; for the 3D gaseous state, the Reciprocal System, like conventional theory, uses the Maxwellian distribution.

To use the normal probability function or table we need to know the value of the applicable normal random variable, z_s. It should be proportional to the difference between the liquid temperature, T, and the melting point, T_m, both in degrees K, divided by the melting point. The coefficient and the intercept have--unfortunately--not yet been worked out theoretically, but are given empirically by Larson (Ref. [4]) as follows:

$$z_S(T, T_m) := \frac{4 \cdot (T - T_m)}{T_m} + .40 \qquad (21)$$

We want the *right tail* of the normal probability distribution, so we subtract the value of the normal function, $\Phi_m(z_s)$, from 1 and then multiply by the average difference in specific volume between solid and liquid molecules, ΔV:

$$\Phi_m(T, T_m) := \text{cnorm}(z_S(T, T_m)) \qquad (22)$$

(*Mathcad*'s cumulative normal probability function with mean = 0 and variance = 1)

$$\Delta s\left(T, T_m, \Delta V\right) := \left(1 - \Phi_m\left(T, T_m\right)\right)\cdot\Delta V \qquad \text{cm}^3/\text{g} \qquad (23)$$

Larson uses an average value of ΔV of .080 cm^3/g for paraffin hydrocarbons (C_{14} and below) and .084 cm^3/g for paraffins above C_{14} (rather than computing the individual values). A user of the Reciprocal System Database will be able to input the value as appropriate for each element or compound.

The thermal motion beyond the initial point of the liquid (extrapolating back to 0 K) is the *one-dimensional* equivalent of the thermal motion of a *gas*, and thus the volume generated is therefore *directly proportional to the temperature*, T. Let n_T be the number of *temperature units* or the temperature factor. Then

$$V_2\left(n_V, w, n_{s2}, T, n_T\right) := \frac{T}{n_T\cdot T_{SL\_u}}\cdot V_{02}\left(n_V, w, n_{s2}\right) \qquad \text{cm}^3/\text{g} \qquad (24)$$

For simple substances, $n_T = 1$. More complex or more electropositive substances have values of n_T of 2 up to 16. Hexane has a value of 1; water, 2; silver, 16. Compounds of electropositive and electronegative elements have intermediate values (some with half-integral values, which are averages), as would be expected.

The gaseous or vapor increment of specific volume depends on the proportion of *critical molecules* existing in the aggregate at each temperature, which can be computed from probability considerations. Larson uses two random variables for this computation, both a function of the critical temperature, T_c (not to be confused here with the central or core temperature of a star).

$$z_{c1}(T_c, T) := \frac{9 \cdot (T_c - T)}{T_c + \frac{T_{SL\_u}}{2}} \qquad (25)$$

$$z_{c2}(T_c, T) := \frac{27 \cdot (T_c - T)}{T_c + \frac{T_{SL\_u}}{2}} \qquad (26)$$

These random variables appear to be empirical, not theoretical, but it seems evident that 9 and 27 are dimensional factors. The $9 = 3^2$ factor is for two dimensions, and $27 = 3^3$ factor is for three dimensions; therefore we may conclude that the first is for the vapor component, the second is for the gaseous component. Note: these components are still "trapped" in the liquid and do not have sufficient energy to escape into the space above the liquid; thus the normal probability distribution applies, rather than the Maxwellian.

The specific volume increment due to critical molecules in the substance is then

$$\Phi_T(T_c, T) := \text{cnorm}(z_{c1}(T_c, T)) \qquad (27)$$

$$\Phi_{3T}(T_c, T) := \text{cnorm}(z_{c2}(T_c, T)) \qquad (28)$$

$$V_3(n_V, w, T_c, T, k_{s3}) := \left[2 - \left(\Phi_T(T_c, T) + \Phi_{3T}(T_c, T)\right)\right] \cdot V_{03}(n_V, w, k_{s3}) \quad \text{cm}^3/\text{g} \qquad (29)$$

Now we have V_1, V_2, and V_3 for Eq. (12):

$$V_L := V_1 + V_2 + V_3 \quad \text{cm3/g} \qquad (12)$$

Of course, the density is the inverse of the specific volume:

$$\rho_L := \frac{1}{V_L} \quad \text{g/cm}^3 \qquad (30)$$

Conditions in the Sun and stars are quite different from those on Earth, so we're going to make some *drastic simplifications* in the above equations. First we will neglect any contribution from *solid* or *critical gaseous/vapor* molecules.

$$V_1 := 0 \qquad V_3 := 0 \qquad (31)$$

So

Tentative Theorem 4: The specific volume or density of a Main Sequence star is calculated using only the liquid component of the liquid state equations.

$$V_L := V_2 \qquad cm^3/g \tag{32}$$

Second, we will assume that the atoms in a Main Sequence star are *not* associated together in molecules and are all "simple." So the temperature and volume parameters will be equal to one.

$$n_T := 1 \tag{33}$$

$$n_V := 1 \tag{34}$$

Third, we will assume that the number of close-packed groups is zero (because we're dealing with individual atoms, not molecules):

$$n_{s2} := 0 \tag{35}$$

With these simplifications we now have

$$V_L := \frac{T}{T_{SL\_u}} \cdot \frac{.7071 \cdot V_{L\_u}}{w \cdot m_{u\_g}} \qquad cm^3/g \tag{36a}$$

Knowing T_{SL_u}, V_{L_u}, and m_{u_g}, we can further simply Eq. (32):

$$V_L := \frac{T \cdot .020637}{w} \qquad cm^3/g \tag{36b}$$

Thus the specific volume of a substance comprised solely of *individual atoms in the liquid or condensed gas* state is a *linear* function of temperature and is inversely related to the atomic weight.

But we're not done yet--we must consider pressure. From Ref. [28], we have

$$V_L := \frac{T \cdot .020637}{w} \cdot \frac{P_{0\_L}}{P + P_{0\_L}} \qquad cm^3/g \qquad (37a)$$

where P = pressure, in atm. rather than dynes/cm^2, and P_{0_L} = initial pressure, also in atm, for convenience, and

$$P_{0\_L} := \frac{P_{L\_u} \cdot n_P}{\left(\frac{V_{00}}{1}\right)^{\frac{2}{3}}} \qquad atm \qquad (38a)$$

where P_{L_u} is the natural unit of pressure for the liquid state, and n_P is the number of pressure units. Just as we have set $n_T = 1$ and $n_V = 1$, we will set n_P to 1.

$$n_P := 1 \qquad (39)$$

Putting the known values of the parameters into Eq. (38a), we have

$$P_{0\_L} := w^{\frac{2}{3}} \cdot 88.037131 \qquad \text{atm} \qquad (38b)$$

Then Eq. (37a) becomes

$$V_L := \frac{T \cdot .020637}{w} \cdot \frac{w^{\frac{2}{3}} \cdot 88.037131}{P + w^{\frac{2}{3}} \cdot 88.037131} \qquad \text{cm}^3/\text{g} \qquad (37b)$$

Note, again, that P must be in atm, here, not dynes/cm^2. And, repeating Eq. (30):

$$\rho := \frac{1}{V_L} \qquad \text{g/cm}^3$$

We know $P(r_{mean})$ and $\rho(r_{mean})$ for each value of r_{mean}, from Eqs. (4) and (10), so we can solve Eq. (30) for V_L and then calculate T from Eq. (37b).

9. temperature T(r) from core to surface

Proceeding:

$$T(r) := \left(48.45665552 \cdot \frac{1}{\rho(r)} \cdot w + 0.55041157 \cdot P(r) \cdot conv_{dynescm2toatm} \cdot \frac{1}{\rho(r)} \cdot w^{\frac{1}{3}}\right) K \qquad (37c)$$

(where we have applied the factor to convert P(r) to atm). Note that the atomic weight, w, will be *different* for each *element shell*.

The mean temperature for a spherical shell will be approximated by

$$T(r_{mean}) := \left(48.45665552 \cdot \frac{1}{\rho(r_{mean})} \cdot w + 0.55041157 \cdot P(r_{mean}) \cdot conv_{dynescm2toatm} \cdot \frac{1}{\rho(r_{mean})} \cdot w^{\frac{1}{3}}\right) K$$

(37d)

At the surface, the temperature is not defined, because $\rho(0) = 0$, in our linear model. However, we are using the mean values of r, ρ, and P for each shell, including the top shell, and so, this is not an issue.

10. central temperature T_c

By inspection,

$$T_C := 48.45665552 \cdot \frac{1}{\rho_C} \cdot w + 0.55041157 \cdot P_C \cdot conv_{dynescm2toatm} \cdot \frac{1}{\rho_C} \cdot w^{\frac{1}{3}} \quad K \qquad (40)$$

where here w = atomic weight of the core element.

11. destructive temperature of the fissioning element

Table II provides a PDF of an Excel document for the energy calculations of all elements from Z = 26 (Fe) to Z = 117 (Uus).

Table II. Stellar Element Energy Generation Properties

Among others, the columns include the atomic number Z, the element symbol, the relevant magnetic rotational displacement, the stellar class, the atomic weight (assuming that the neutrino ionization is at level 1), the star radius (in solar units), the star mass (in solar units), the star luminosity (in solar units), the central density, the central pressure, the central temperature, and the element destructive temperature. Note: the values have been adjusted so that for class G2 V, the values are that of the Sun.

We will now show how these values are calculated for the Sun, after first stating the next tentative theorem.

Tentative Theorem 5: Because of the *very high temperatures* in a star, the appropriate natural temperature unit to use is that for the vapor or condensed gas state, T_{V_u}, and the appropriate probability distribution to use for the velocity of atoms of a star is the Maxwellian temperature distribution, rather than the Normal Probability distribution.

a. fissioning element Lu

The Sun is classified as a G2 V star. Looking at Table II, this implies that the core fissioning element is Lu (4-4-(15)). In this section of the paper we will use SI units, for convenience.

$n := 4 \quad Z := 71 \quad w_{Lu} := 174.967 \quad M_{S\_Lu} := M_{S\_SI}$ kg $\quad R_{S\_Lu} := R_{S\_SI}$ m

$m_{Lu\_MeV} := w_{Lu} \cdot conv_{u\_to\_MeV} \quad m_{Lu\_MeV} = 1.629807 \times 10^{5} \quad m_{Lu\_kg} := w_{Lu} \cdot conv_{u\_to\_kg}$

$$T_{destructive\_Lu} := \frac{T_{V\_u}}{1 - \frac{2 \cdot n^2}{Z}}$$ (ignoring ionization energy) (from Ref. [29]) $T_{destructive\_Lu} = 6.549841 \times 10^{9}$ K

(41)

The average velocity of an Lu atom reaching the core is (see Ref. [29] for the derivation):

$$v(M, m_{kg}, m_{MeV}, R) := \frac{\sqrt{G_{SI}} \cdot \sqrt{M} \cdot c_{SI} \cdot \sqrt{conv_{J\_to\_MeV}} \cdot \sqrt{m_{kg}} \cdot \sqrt{2 \cdot R \cdot m_{MeV} + G_{SI} \cdot M \cdot conv_{J\_to\_MeV} \cdot m_{kg}}}{R \cdot m_{MeV} + G_{SI} \cdot M \cdot conv_{J\_to\_MeV} \cdot m_{kg}}$$

(42)

$v_{Lu} := v(M_{S\_Lu}, m_{Lu\_kg}, m_{Lu\_MeV}, R_{S\_Lu})$

$v_{Lu} = 6.175261 \times 10^{5}$ m/sec $\qquad \frac{v_{Lu}}{c_{SI}} = 0.00206$

The average temperature of a Lu atom when it reaches the core is (again see Ref. [29] for the derivation):

$$T_{el}(m_{MeV}, v) := -\frac{2\cdot\left(m_{MeV} - \frac{m_{MeV}}{\sqrt{1-\frac{v^2}{c_{SI}^2}}}\right)}{3\cdot k_{B\_MeV}\cdot\sqrt{1-\frac{v^2}{c_{SI}^2}}} \qquad (43)$$

$$T_{Lu} := T_{el}(m_{Lu\_MeV}, v_{Lu}) \qquad T_{Lu} = 2.674723\times 10^{9}\ \mathrm{K}$$

The ratio of this temperature to that of the destructive thermal limit is:

$$\frac{T_{Lu}}{T_{destructive\_Lu}} = 0.408365$$

So the gravitational potential energy (at the Sun's surface) changing to kinetic energy (at the core) provides only about 41% of the energy necessary. But: the velocity calculated above is the RMS velocity. Some atoms will have a *higher velocity* and thus they *will* reach the "burning" temperature. We can calculate the fraction of atoms reaching the higher velocity as follows.

$$\frac{m_{Lu\_MeV}}{\sqrt{1-\frac{v_{destructive}^2}{c_{SI}^2}}} - m_{Lu\_MeV} = \frac{3}{2}\cdot k_{B\_MeV}\cdot T_{destructive\_Lu}\cdot\sqrt{1-\frac{v_{destructive}^2}{c_{SI}^2}}$$

Solving for the destructive velocity and putting into functional form:

$$v_{destructive\_el}(m_{MeV}, T_{destructive}) := \frac{c_{SI}\cdot\sqrt{4\cdot m_{MeV}^{2}\cdot\left(\frac{\sqrt{\frac{m_{MeV}+6\cdot T_{destructive}\cdot k_{B\_MeV}}{m_{MeV}}}}{2}+\frac{1}{2}\right)-4\cdot m_{MeV}^{2}+9\cdot T_{destructive}}}{3\cdot T_{destructive}\cdot k_{B\_MeV}}$$

(44)

$v_{destructive\_Lu} := v_{destructive\_el}(m_{Lu\_MeV}, T_{destructive\_Lu})$ $\quad v_{destructive\_Lu} = 1.092554\times 10^{6}$

$$\frac{v_{destructive\_Lu}}{c_{SI}} = 0.003644$$

Now we'll calculate T_c, the temperature exactly at the core (temporarily changing back to cgs units):

$\rho_{c\_S\_cgs} := \frac{3\cdot M_{S\_cgs}}{\pi\cdot(R_{S\_cgs})^{3}}$ $\quad \rho_{c\_S\_cgs} = 5.639578$ g/cm^3

$$P_{c\_S\_atm} := \frac{5\cdot\pi}{36}\cdot G_{cgs}\cdot\rho_{c\_S\_cgs}^{2}\cdot R_{S\_cgs}^{2}\cdot conv_{dynescm2toatm}$$

$P_{c\_S\_atm} = 4.423994\times 10^{9}$ atm

$$T_{c\_S} := 48.45665552 \cdot \frac{w_{Lu}}{\rho_{c\_S\_cgs}} + 0.55041157 \cdot \frac{P_{c\_S\_atm}}{\rho_{c\_S\_cgs}} \cdot w_{Lu}^{\frac{1}{3}} \quad \text{K} \quad \text{(based on Eq. (37))}$$

$T_{c\_S} = 2.414948 \times 10^{9}$ K $\qquad T_{c\_S\_Lu} := T_{c\_S}$ (for later use)

Note that T_{Lu} is a little higher than T_{c_S}; energy in the core will reduce it to the equilibrium value, on average. Now we can calculate the fraction of the atoms reaching the destructive thermal limit using equations from Ref. [29] and putting into functional form:.

$$U_{el}(v_{destructive}, m_{kg}, T_c) := v_{destructive} \cdot \sqrt{\frac{m_{kg}}{2 \cdot k_{B\_J} \cdot T_c}} \qquad (45)$$

$U_{Lu} := U_{el}(v_{destructive\_Lu}, m_{Lu\_kg}, T_{c\_S}) \qquad U_{Lu} = 2.280156$

$$N_{v\_frac}(U) := 1 + \frac{2}{\sqrt{\pi}} \cdot U \cdot e^{-U^2} - \text{erf}(U) \qquad (46)$$

$N_{v\_frac\_Lu} := N_{v\_frac}(U_{Lu}) \qquad N_{v\_frac\_Lu} = 0.015467$

In the algorithm below and in the Excel spread sheet we use the *mean* values of r, ρ, and P for the inner-most shell and all the other shells, so there may be some minor *numerical differences* between those and the calculations done directly above (which are done precisely at the core).

We conclude that 1.5% of the Lu atoms reaching the core will "burn"--thus creating the energy of the Sun (see below for that calculation).

b. fissioning element Yb

According to Part II of this paper, most stars in the Main Sequence move *up*--so, the Sun will become a type G1 V star. Looking at Table II, this implies that the core fissioning element will then be Yb (4-4-(16) or 4-3-(16)). Space displacement is favored at high temperatures, so we will use n = 4, rather than n = 3.5. From the table:

$n := 4$ $Z := 70$ $w_{Yb} := 173.04$ $M_{S\_Yb} := M_{S\_SI} \cdot 1.261$ kg $R_{S\_Yb} := R_{S\_SI} \cdot 1.044$ m

$m_{Yb\_MeV} := w_{Yb} \cdot conv_{u\_to\_MeV}$ $m_{Yb\_MeV} = 1.611857 \times 10^{5}$ $m_{Yb\_kg} := w_{Yb} \cdot conv_{u\_to\_kg}$

$$T_{destructive\_Yb} := \frac{T_{V\_u}}{1 - \frac{2 \cdot n^2}{Z}}$$ (ignoring ionization energy) $T_{destructive\_Yb} = 6.627526 \times 10^{9}$ K

The average velocity of a Yb atom reaching the core is:

$v_{Yb} := v\left(M_{S\_Yb}, m_{Yb\_kg}, m_{Yb\_MeV}, R_{S\_Yb}\right)$ $v_{Yb} = 6.786759 \times 10^{5}$ m/sec $\frac{v_{Yb}}{c_{SI}} = 0.002264$

The average temperature of a Yb atom when it reaches the core is:

$$T_{Yb} := T_{el}\left(m_{Yb\_MeV}, v_{Yb}\right) \qquad T_{Yb} = 3.195096 \times 10^{9} \quad K$$

The ratio of this temperature to that of the destructive thermal limit is:

$$\frac{T_{Yb}}{T_{destructive\_Yb}} = 0.482095$$

So the gravitational potential energy (at the Sun's surface) changing to kinetic energy (at the core) provides about 48% of the energy necessary. But: the velocity calculated above is the RMS velocity. Some atoms will have a *higher velocity* and thus they *will* reach the "burning" temperature. We can calculate the fraction of atoms reaching the higher velocity as follows.

$$v_{destructive\_Yb} := v_{destructive\_el}\left(m_{Yb\_MeV}, T_{destructive\_Yb}\right) \quad v_{destructive\_Yb} = 1.097599 \times 10^{6}$$

Now we'll calculate T_c, the temperature exactly at the core (temporarily changing back to cgs units):

$$\rho_{c\_S\_cgs} := \frac{3 \cdot M_{S\_cgs} \cdot 1.261}{\pi \cdot \left(R_{S\_cgs} \cdot 1.044\right)^{3}} \qquad \rho_{c\_S\_cgs} = 6.249714 \quad g/cm^{3}$$

$$P_{c\_S\_atm} := \frac{5 \cdot \pi}{36} \cdot G_{cgs} \cdot {\rho_{c\_S\_cgs}}^{2} \cdot \left(R_{S\_cgs} \cdot 1.044\right)^{2} \cdot conv_{dynescm2toatm}$$

$$P_{c\_S\_atm} = 5.92165 \times 10^{9} \qquad atm$$

$$T_{c\_S} := 48.45665552 \cdot \frac{w_{Yb}}{\rho_{c\_S\_cgs}} + 0.55041157 \cdot \frac{P_{c\_S\_atm}}{\rho_{c\_S\_cgs}} \cdot w_{Yb}^{\frac{1}{3}} \qquad K$$

$$T_{c\_S} = 2.906158 \times 10^{9} \qquad K \qquad\qquad T_{c\_S\_Yb} := T_{c\_S}$$

Note that T_{Yb} is a little higher than T_{c_S}; energy in the core will reduce it to the equilibrium value, on average. Now we can calculate the fraction of the atoms reaching the destructive thermal limit:

$$U_{Yb} := U_{el}\left(v_{destructive\_Yb}, m_{Yb\_kg}, T_{c\_S}\right) \qquad U_{Yb} = 2.076609$$

$$N_{v\_frac\_Yb} := N_{v\_frac}\left(U_{Yb}\right) \qquad N_{v\_frac\_Yb} = 0.034722$$

We conclude that 3.5% of the Yb atoms reaching the core will "burn"--thus creating the energy of the future Sun (see below for that calculation).

c. fissioning element Cu

Now we will skip ahead to the element before the trio of elements (Fe, Co, Ni) involved in or close to a Supernovae Type I: Cu. Here (using Table II):

$n := 3$ $Z := 29$ $w_{Cu} := 63.546$ $M_{S\_Cu} := M_{S\_SI} \cdot 31.89164$ kg $R_{S\_Cu} := R_{S\_SI} \cdot 13.421$ m

$m_{Cu\_MeV} := w_{Cu} \cdot conv_{u\_to\_MeV}$ $m_{Cu\_MeV} = 5.919272 \times 10^{4}$ $m_{Cu\_kg} := w_{Cu} \cdot conv_{u\_to\_kg}$

$T_{destructive\_Cu} := \dfrac{T_{V\_u}}{1 - \dfrac{2 \cdot n^{2}}{Z}}$ (ignoring ionization energy) $T_{destructive\_Cu} = 9.485109 \times 10^{9}$ K

The average velocity of a Cu atom reaching the core is:

$v_{Cu} := v\left(M_{S\_Cu}, m_{Cu\_kg}, m_{Cu\_MeV}, R_{S\_Cu}\right)$ $v_{Cu} = 9.519197 \times 10^{5}$ m/sec $\dfrac{v_{Cu}}{c_{SI}} = 0.003175$

The average temperature of a Cu atom when it reaches the core is:

$T_{Cu} := T_{el}(m_{Cu\_MeV}, v_{Cu})$ $T_{Cu} = 2.308363 \times 10^{9}$ K

The ratio of this temperature to that of the destructive thermal limit is:

$$\frac{T_{Cu}}{T_{destructive\_Cu}} = 0.243367$$

So the gravitational potential energy (at the Sun's surface) changing to kinetic energy (at the core) provides about 24% of the energy necessary. But: the velocity calculated above is the RMS velocity. Some atoms will have a *higher velocity* and thus they *will* reach the "burning" temperature. We can calculate the fraction of atoms reaching the higher velocity as follows.

$v_{destructive\_Cu} := v_{destructive\_el}(m_{Cu\_MeV}, T_{destructive\_Cu})$ $v_{destructive\_Cu} = 1.935134 \times 10^{6}$

Now we'll calculate T_c, the temperature exactly at the core (temporarily changing back to cgs units):

$$\rho_{c\_S\_cgs} := \frac{3 \cdot M_{S\_cgs} \cdot 31.89164}{\pi \cdot (R_{S\_cgs} \cdot 13.421)^{3}}$$ $\rho_{c\_S\_cgs} = 0.074399$ g/cm^3

$$P_{c\_S\_atm} := \frac{5 \cdot \pi}{36} \cdot G_{cgs} \cdot \rho_{c\_S\_cgs}{}^{2} \cdot (R_{S\_cgs} \cdot 13.421)^{2} \cdot conv_{dynescm2toatm}$$

$P_{c\_S\_atm} = 1.386848 \times 10^{8}$ atm

$$T_{c\_S} := 48.45665552 \cdot \frac{w_{Cu}}{\rho_{c\_S\_cgs}} + 0.55041157 \cdot \frac{P_{c\_S\_atm}}{\rho_{c\_S\_cgs}} \cdot w_{Cu}^{\frac{1}{3}} \qquad K$$

$$T_{c\_S} = 4.094316 \times 10^{9} \qquad K \qquad T_{c\_S\_Cu} := T_{c\_S}$$

Note that T_{Cu} is less than T_{c_S}; energy in the core will increase it to the equilibrium value, on average. Now we can calculate the fraction of the atoms reaching the destructive thermal limit:

$$U_{Cu} := U_{el}(v_{destructive\_Cu}, m_{Cu\_kg}, T_{c\_S}) \qquad U_{Cu} = 1.869226$$

$$N_{v\_frac\_Cu} := N_{v\_frac}(U_{Cu}) \qquad N_{v\_frac\_Cu} = 0.072281$$

We conclude that 7.2% of the Cu atoms reaching the core will "burn"--thus creating the energy of the future Sun (see below for that calculation of stellar energy generation).

12. stellar reaction rate; energy generation; luminosity; opacity

Before going on to discuss supernovae, let's return to the current situation in our Sun. The amount of energy so generated is quite astounding, compared with the supposed hydrogen-to-helium process. When Lu loses 32 electric rotational displacement units (the equivalent of the fourth magnetic rotational displacement which is destroyed, 2 x 4^2), it becomes Y, element number 39. Presumably, Y is fully ionized upon conversion (or shortly thereafter).

$E_{I\_Y\_MeV} := .020715837$ MeV (from Reciprocal System Database)

$E_{I\_Lu\_MeV} := .068658$ MeV (from Reciprocal System Database)

$$E_{S\_gen\_MeV} := 32 \cdot 2 \cdot conv_{amu\_to\_u} \cdot conv_{u\_to\_MeV} + \left(E_{I\_Lu\_MeV} - E_{I\_Y\_MeV}\right) \quad (47a)$$

$E_{S\_gen\_MeV} = 5.959672 \times 10^4$ MeV (no isotopes)

This is the situation where the magnetic ionization level is zero, i.e., I = 0, for both of the atoms. But suppose the atomic conversion takes place in an environment where the magnetic ionization level is 1 (as in our present day Solar System), so G is not equal to zero. The difference in G between the two atoms must also be converted to energy! In the Lu to Y case, we have

$G_{Lu} := 32$ $G_Y := 10$ So:

$$E_{S\_gen\_MeV} := \left[32 + .5 \cdot \left(G_{Lu} - G_Y\right)\right] \cdot 2 \cdot conv_{amu\_to\_u} \cdot conv_{u\_to\_MeV} + \left(E_{I\_Lu\_MeV} - E_{I\_Y\_MeV}\right) \quad (47b)$$

$E_{S\_gen\_MeV} = 8.008308 \times 10^4$ MeV (isotopes, mid-range, mag. ioniz. level = 1)

Even though heavy atoms are very rare, the amount of energy generated totally compensates.

According to Prof. Lang, Ref. [5], p. 236, the luminosity of the Sun is

$$L_{S\_MeV} := 3.828 \cdot 10^{26} \cdot conv_{J\_to\_MeV} \qquad L_{S\_MeV} = 2.389438 \times 10^{39} \qquad \text{MeV/sec}$$

His hydrogen-to-helium calculation yields, per reaction,

$$E_{S\_gen\_conv\_MeV} := 4.2 \cdot 10^{-12} \cdot conv_{J\_to\_MeV} \qquad E_{S\_gen\_conv\_MeV} = 26.2164 \qquad \text{MeV}$$

Therefore, the number of reactions per second for the conventional theory is

$$N_{conv\_reac} := \frac{L_{S\_MeV}}{E_{S\_gen\_conv\_MeV}} \qquad N_{conv\_reac} = 9.114286 \times 10^{37} \quad \text{reactions/sec} \qquad (48a)$$

For the Reciprocal System, using the Lu-Y calculation above:

$$N_{reac} := \frac{L_{S\_MeV}}{E_{S\_gen\_MeV}} \qquad N_{reac} = 2.983698 \times 10^{34} \quad \text{reactions/sec} \qquad (48b)$$

The ratio of the number of reacting atoms of Lu to that of H then needs to be at least

$$\frac{N_{reac}}{N_{conv\_reac}} = 3.27365 \times 10^{-4} \qquad (49)$$

Heavy elements may be rare, but obviously there is a sufficient number to produce the calculated stellar energy generation! Prof. Lang, Ref. [5], p. 164, says "all of the heavier elements comprise only .1%." But this is more than sufficient. The Reciprocal System calculations given above apply to all Main Sequence stars and all stars approaching the Main Sequence (of course substituting the appropriate stellar mass, radius, luminosity, and atomic species).

Note: the number of neutrinos generated in the Reciprocal System reaction (for the Sun) is not necessarily 22 x N_{reac}/sec (6.564137×10^{35}) (although it could be). The conventional calculation says there are two electron neutrinos created per reaction, yielding a flux of 1.822857×10^{38} neutrinos/sec from the Sun. The observed number in the Davis experiment is 2.6×10^{36} neutrinos/sec, so at least the Reciprocal System calculation is much closer! And: In the Kamiokande experiment in Japan, the detected results were 4.8×10^{35} neutrinos/sec, which is very close. Keep in mind that

$$\frac{E_{S\_gen\_MeV}}{E_{S\_gen\_conv\_MeV}} = 3054.693959 \qquad (50)$$

The number of assumed "yellow" photons created per second (equated in energy to the total energy of the initial gamma and x-ray photons) would be:

$$\nu_{yellow} := 4.1 \cdot 10^{14} \quad \text{cycles/sec}$$

$$\frac{E_{S\_gen\_MeV}}{h \cdot \nu_{yellow} \cdot 10^{-6}} = 4.717985 \times 10^{10} \quad \text{photons} \qquad (51)$$

So a conversion of just a few thousand of these to neutrinos is certainly possible; far less would be necessary if the observed neutrinos/sec in the Davis experiment is accurate. None at all would be needed if the Kamiokande experiment is accurate!

Similar calculations for other stars giving $E_{star\_gen\_MeV}$ and N_{star_reac} can be done. If necessary to convert to erg or J, the conversio factors $conv_{Mev\_to\_erg}$ and $conv_{Mev\_to\_J}$ can be used.

Conventional equations can be used to calculate a few other stellar properties. For example, to calculate κ_S, the overall opacity of the Sun:

From Eq. (15-14) of Smith and Jacobs (Ref. [10]):

$$\rho_{avg} := \frac{M_{S\_cgs}}{\frac{4}{3}\cdot\pi\cdot R_{S\_cgs}{}^{3}} \qquad \rho_{avg} = 1.409894 \quad \text{g/cm}^3 \qquad (52)$$

$$L_{S\_cgs} = -\frac{64\cdot\pi\cdot\sigma\cdot R_{S\_cgs}{}^{2}\cdot T_{c\_S\_Lu}{}^{3}}{3\cdot\kappa_S\cdot\rho_{avg}}\cdot\left(\frac{-T_{c\_S\_Lu}}{R_{S\_cgs}}\right)$$

Solving for κ_S

$$\kappa_S := \frac{64\cdot\pi\cdot R_{S\_cgs}\cdot T_{c\_S\_Lu}{}^{4}\cdot\sigma}{3\cdot L_{S\_cgs}\cdot\rho_{avg}} \qquad \kappa_S = 1.660216\times 10^{12} \quad \text{cm}^2\text{/g} \qquad (53)$$

This is considerably higher than the conventional value because of the much greater core temperature.

$$\varepsilon_S := \frac{L_{S\_cgs}}{M_{S\_cgs}} \qquad \varepsilon_S = 1.932006 \quad \text{erg/sec g} \quad \text{radiation per unit mass} \qquad (54)$$

Similar equations can be used to calculate opacity for the Sun at later stages or for other stars.

The effective temperature and energy flux of our current Sun are:

$$T_{eff} := \left(\frac{L_{S\_cgs}}{4\cdot\pi\cdot R_{S\_cgs}{}^{2}\cdot\sigma}\right)^{\frac{1}{4}} \qquad T_{eff} = 5777.199788 \qquad K \qquad (55)$$

$$F := \sigma\cdot T_{eff}{}^{4} \qquad F = 6.316598\times10^{10} \qquad \frac{erg}{sec\cdot cm^{2}}$$ radiation emittance or energy flux (56)

Again, similar equations can be used to calculate the effective temperature and energy flux of our Sun at later stages or for other stars.

Useful *observed* relationships for Main Sequence stars are the following (Ref. [22], p. 27):

$$\frac{R_{star}}{R_S} := \left(\frac{M_{star}}{M_S}\right)^{0.75} \qquad (57)$$

$$\frac{L_{star}}{L_S} := \left(\frac{M_{star}}{M_S}\right)^{3.5} \qquad (58)$$

13. Supernova Type I--thermal destructive limit of Fe

When the thermal destructive limit of Ni (two elements higher than Fe) is reached, the result is a pre-supernova star, a Wolf-Rayet (Ref. [1], pp. 49-50):

"These Wolf-Rayet stars are somewhat less massive than the stars of the O class, the highest on the main sequence, but they have about the same luminosity, and they are associated with the O stars in the disk of the Galaxy. Their principal distinguishing characteristic is a very disturbed condition in their surface layers, with ejection of material that forms an expanding shell around each star. These special conditions lead to the existence of a distinctive spectrum. It appears probable that the Wolf-Rayet star is the one whose central temperature has reached the destructive limit of nickel. We may interpret its observed characteristics as indicating that arrival at this temperature limit has resulted in an increase in the production of energy that is large enough to cause violent internal activity, and ejection of matter from the star, without being enough to initiate a full-scale explosion. On this basis, the star remains in the Wolf-Rayet condition until the greater part of the nickel is consumed. It then resumes accreting mass (probably picking up most of what was ejected) and reverts to the O status."

Reaching the Co limit results in the same as the above. But now, what about Fe? This element is far more abundant than Ni and Co and so when the thermal destructive limit of Fe is reached, the result is a huge explosion, a Type I Supernova!

Some iron atoms will have rotational displacements of 3-3-(10), others will have 3-2-8. However, space displacements dominate at high temperature, so the 3-3-(10) form is much more likely.

The value of n to use for Fe (3-3-(10)) is thus 3. The destructive temperature limit is then (using property data from Table II):

$n := 3$ $\quad Z := 26$ $\quad w_{Fe} := 55.847$ $\quad M_{S\_Fe} := M_{S\_SI} \cdot 31.89164 \ \text{kg}$ $\quad R_{S\_Fe} := R_{S\_SI} \cdot 13.421 \ \text{m}$

$m_{Fe\_MeV} := w_{Fe} \cdot conv_{u\_to\_MeV}$ $\quad m_{Fe\_MeV} = 5.202115 \times 10^{4}$ $\quad m_{Fe\_kg} := w_{Fe} \cdot conv_{u\_to\_kg}$

$$T_{destructive\_Fe} := \frac{T_{V\_u}}{1 - \frac{2 \cdot n^2}{Z}} \quad \text{(ignoring ionization energy)} \qquad T_{destructive\_Fe} = 1.169285 \times 10^{10} \ \text{K}$$

The average velocity of an Fe atom reaching the core is:

$$v_{Fe} := v\left(M_{S\_Fe}, m_{Fe\_kg}, m_{Fe\_MeV}, R_{S\_Fe}\right) \qquad v_{Fe} = 9.519197 \times 10^{5} \quad \text{m/sec} \qquad \frac{v_{Fe}}{c_{SI}} = 0.003175$$

The average temperature of an Fe atom when it reaches the core is:

$$T_{Fe} := T_{el}\left(m_{Fe\_MeV}, v_{Fe}\right) \qquad T_{Fe} = 2.02869 \times 10^{9} \quad \text{K}$$

The ratio of this temperature to that of the destructive thermal limit is:

$$\frac{T_{Fe}}{T_{destructive\_Fe}} = 0.173498$$

So the gravitational potential energy (at the star's surface) changing to kinetic energy (at the core) provides about 17% of the energy necessary. But: the velocity calculated above is the RMS velocity. Some atoms will have a *higher velocity* and thus they *will* reach the "burning" temperature. We can calculate the fraction of atoms reaching the higher velocity as follows.

$$v_{destructive\_Fe} := v_{destructive\_el}\left(m_{Fe\_MeV}, T_{destructive\_Fe}\right) \qquad v_{destructive\_Fe} = 2.281559 \times 10^{6} \quad \text{m/sec}$$

Now we'll calculate T_C, the temperature exactly at the core (temporarily changing back to cgs units):

$$\rho_{c\_S\_cgs} := \frac{3 \cdot M_{S\_cgs} \cdot 31.89164}{\pi \cdot \left(R_{S\_cgs} \cdot 13.421\right)^{3}} \qquad \rho_{c\_S\_cgs} = 0.074399 \quad \text{g/cm}^{3}$$

$$P_{c\_S\_atm} := \frac{5\cdot\pi}{36}\cdot G_{cgs}\cdot \rho_{c\_S\_cgs}{}^{2}\cdot\left(R_{S\_cgs}\cdot 13.421\right)^{2}\cdot conv_{dynescm2toatm}$$

$$P_{c\_S\_atm} = 1.386848\times 10^{8} \quad \text{atm}$$

$$T_{c\_S} := 48.45665552\cdot\frac{w_{Cu}}{\rho_{c\_S\_cgs}} + 0.55041157\cdot\frac{P_{c\_S\_atm}}{\rho_{c\_S\_cgs}}\cdot w_{Cu}{}^{\frac{1}{3}} \quad K$$

$$T_{c\_S} = 4.094316\times 10^{9} \quad K \qquad T_{c\_S\_Fe} := T_{c\_S}$$

Note that T_{Fe} is less than T_{c_S}; energy in the core will increase it to the equilibrium value, on average. Now we can calculate the fraction of the atoms reaching the destructive thermal limit:

$$U_{Fe} := U_{el}\left(v_{destructive\_Fe}, m_{Fe\_kg}, T_{c\_S}\right) \qquad U_{Fe} = 2.066038$$

$$N_{v\_frac\_Fe} := N_{v\_frac}\left(U_{Fe}\right) \qquad N_{v\_frac\_Fe} = 0.036124$$

We conclude that 3.6% of the Fe atoms reaching the core will "burn." But this is enough to create the supernova!

When Fe loses 18 (2×3^2) electric rotational displacement units, it becomes O. Presumably, O is fully ionized upon conversion (or shortly thereafter).

$$E_{I\_O} := 871.6479\cdot 10^{-6} \quad \text{MeV} \quad \text{(from Reciprocal System Database)}$$

$$E_{I\_Fe} := .0092777 \quad \text{MeV} \quad \text{(from Reciprocal System Database)}$$

$$E_{star\_gen\_MeV} := 18 \cdot 2 \cdot conv_{amu\_to\_u} \cdot conv_{u\_to\_MeV} + \left(E_{I\_Fe} - E_{I\_O}\right) \qquad (59a)$$

$E_{star\_gen\_MeV} = 3.352314 \times 10^{4}$ MeV (no isotopes)

This is the situation where the magnetic ionization level is zero, i.e., G = 0 for both of the atoms. But suppose the atomic conversion takes place in an environment where the magnetic ionization level is 1 (as in our present day Solar System), so G is not equal to zero. The difference in G between the two atoms must also be converted to energy! In the Fe to Cl case, we have

$G_{Fe} := 4$ $G_{O} := 0$ So:

$$E_{star\_gen\_MeV} := \left[18 + .5 \cdot \left(G_{Fe} - G_{O}\right)\right] \cdot 2 \cdot conv_{amu\_to\_u} \cdot conv_{u\_to\_MeV} + \left(E_{I\_Fe} - E_{I\_O}\right) \qquad (59b)$$

$E_{star\_gen\_MeV} = 3.724793 \times 10^{4}$ MeV (isotopes, mid-range, mag. ioniz. level = 1)

The observed peak luminosity for a typical Type I Supernova is

$L_{SN\_I\_gen\_MeV} := 3 \cdot 10^{50} \cdot conv_{erg\_to\_MeV}$ $L_{SN\_I\_gen\_MeV} = 1.872449 \times 10^{56}$ MeV/sec

For this, the average number of observed reactions per second would be

$$N_{reac\_SN\_I\_obs} := \frac{L_{SN\_I\_gen\_MeV}}{E_{star\_gen\_MeV}} \qquad N_{reac\_SN\_I\_obs} = 5.026988 \times 10^{51} \text{ reactions/sec} \qquad (60)$$

Based on the mass fraction of Fe in our current Sun multiplied by adjFe and the mass factor we have

$$.0012735 \cdot M_{S\_cgs} \cdot adjFe \cdot 31.89164 = 5.733739 \times 10^{31}$$ g of Fe right before the supernova

The number of moles of Fe is

$$\frac{5.733739 \cdot 10^{31}}{Z + \frac{G_{Fe}}{2}} = 2.047764 \times 10^{30}$$ number of moles of Fe (including the isotope factor)

So the number of Fe atoms is

$$Av \cdot 2.047764 \cdot 10^{30} = 1.233749 \times 10^{54}$$ atoms of Fe

If, as in the above calculation, 3.6% of these atoms reach the thermal destructive limit:

$$.036 \cdot 1.233749 \cdot 10^{54} = 4.441496 \times 10^{52}$$ number of atoms reaching thermal destructive limit

Then

$$4.441496 \cdot 10^{52} \cdot E_{star\_gen\_MeV} = 1.654365 \times 10^{57}$$ MeV total energy generated

$$1.654365 \cdot 10^{57} \cdot conv_{MeV\_to\_erg} = 2.650591 \times 10^{51} \quad erg$$

The observed *total* energy generated by a typical Type Ia Supernova is usually given as

$$1.5 \cdot 10^{44} \cdot conv_{J\_to\_erg} = 1.5 \times 10^{51} \quad erg$$ (Wikipedia, Ref. [30])

So this indicates that

$$\frac{1.5}{2.65} \cdot .036 = 0.020377$$ or typically *only* 2% of the Fe atoms actually "burn"

It's difficult to calculate how long the process lasts, and the number of reactions per second may not be constant. The observed time to peak luminosity is 19 days, and the time for the luminosity to decrease to 10% is 60 days (Ref. [30]).

The Fe atoms not destroyed (98%) are preserved in the matter *moving outward in time* which creates the White Dwarf star. The less massive atoms comprise the matter moving *outward in space*, the nebula, which we can observe. Part II will go into more detail about the results of Supernovae Type I. With this explosion, the star enters the next cycle.

14. Supernova Type II--age limit of stars

Supernovae Type I multiplies stars as follows (unless a solar system is created instead): 1 --> 2 --> 4 --> 8. It's doubtful that there would be more than four cycles before the star terminates in a *Supernova Type II explosion*. This Type II explosion creates a nebula, which is dispersed in space and eventually absorbed into other stars, and a Pulsar, which is simply a White Dwarf with an *added translational motion*. This motion normally carries it beyond the Galaxy and into the cosmic sector; in some cases, though, it comes back to the plane of the Galaxy; see below, Part II. The age limit of a star occurs when the *central element* reaches an atomic weight of 2 x 118 (the highest possible element, which is unstable even when I = 0) = 236 and there is *sufficient mass of that element*. Isotopes are determined by the neutrino magnetic ionization level. At level 1, the current level in our Solar System, U_{236} is at that level--but there's way too little of it. So, Supernovae Type II will occur only when I = 2, 3, 4, or 5.

To reach the 236 limit:

$$2 \cdot Z + \frac{I \cdot \left(Z^2\right)}{I_R} = 236 \qquad \text{(See Appendix 1 of Ref. [29])} \qquad (61a)$$

Solving for Z and putting into functional form:

$$Z(I) := \frac{I_R \cdot \left(\sqrt{\frac{236 \cdot I + I_R}{I_R}} - 1\right)}{I} \qquad (61b)$$

$I := 2$ $Z(I) = 78.555201$ Au has Z = 79

$I := 3$ $Z(I) = 70.434038$ Lu has Z = 71 (this is the same fissioning element as in our current Sun, but we're at I = 1!)

$I := 4$ $Z(I) = 64.618833$ Tb has Z = 65

$I := 5$ $Z(I) = 60.161493$ Pm has Z = 61

For these central elements, Table II gives the following data:

I	Element	Spectral Class	Core Mass, g	Atoms in Core	Maximum Total Energy for *Usual* Stars
2	Au	K0	$1.8583553 \cdot 10^{28}$	$3.1327567 \cdot 10^{49}$	$1.6705646 \cdot 10^{49}$
3	Lu	G2	$2.075514 \cdot 10^{28}$	$4.126302 \cdot 10^{49}$	$1.8658125 \cdot 10^{49}$
4	Tb	F6	$3.2828506 \cdot 10^{28}$	$7.4641923 \cdot 10^{49}$	$2.9511116 \cdot 10^{49}$
5	Pm	F2	$3.4002264 \cdot 10^{28}$	$8.5046748 \cdot 10^{49}$	$3.0566263 \cdot 10^{49}$

Table III. Supernovae Type II Central Element Properties

Higher elements have been lumped into the core mass. The maximum total energy assumes that the *entire core* is blown up; this is quite unlikely, but it does give a maximum bound. The core mass includes heavier elements with relatively low velocity and so haven't reached the thermal destructive velocity at the time the age limit is reached; but if Au reaches the age limit, for instance, then all the higher elements will, as well (but the calculation includes just the equivalent Au mass). SN1987A, supposedly a Type II Supernova but probably a Type I, had a total neutrino energy plus kinetic energy of 2.01 x 10^{53} erg. However, the mass in the core could have been much greater for this star. Type I have an average peak magnitude M_B = -19.7, whereas Type II have an average peak magnitude M_B = -18.0; these are in the visual range. However, much of the Type II output is in the radio, X-ray, and gamma ray region. Also, there are some unusually massive stars, which could increase the core mass considerably. On the other hand, Type I Supernovae *always occur in large, massive stars* (so they are relatively *homogeneous*), whereas Type II Supernovae can occur in a star of *any spectral class*. So, on average, Type II output will be *smaller*, although the *entire core mass* could potentially be converted into energy--with a considerable number of neutrinos. Also because of the radioactive nature of the Type II process, the light curve declines more slowly than that for Type I.

Astronomers have come up with other classes of supernovae, but it's evident by inspection that there are only two *major* classes, consistent with the Reciprocal System. The subclasses simply depend on which *element* reaches the thermal or age limit.

15. metallicity

Stars differ from the Sun in their metallicity--the proportion of elements higher in atomic number than H and He. Let X = mass fraction of H, Y = mass fraction of He, and XpY = sum of X and Y. Then

$XpY := X + Y$ — mass fraction of H plus He, which we will use because Y is often not known) (62)

$Z_m := 1 - XpY$ — mass fraction of "metals" (63)

The metallicity is defined here to be

$$metallicty = \log\left(\frac{ZtoXpY}{\frac{Z_{m\_S}}{XpY_S}}\right) \qquad (64a)$$

where the denominator represents the values for the Sun. Please note that we're using *mass fractions* here, not *numbers* of atoms. Again, we are lumping He in with H for convenience. Solving for ZtoXpY

$$ZtoXpY := \frac{10^{metallicty} \cdot Z_{m\_S}}{XpY_S} \qquad (64b)$$

From Table I, for the Sun:

$X_S := .7060480$ $Y_S := .2750293$ $XpY_S := X_S + Y_S$ $XpY_S = 0.981077$

$Z_{m\_S} := 1 - XpY_S$ $Z_{m\_S} = 0.018923$ $ZtoXpY_S := \frac{Z_{m\_S}}{XpY_S}$ $ZtoXpY_S = 0.019288$

Hence, for any star,

$$ZtoXpY := 10^{metallicity} \cdot ZtoXpY_S \quad \text{or} \quad ZtoXpY := 10^{metallicity} \cdot .019288 \qquad (64c)$$

$$Z_m := XpY \cdot 10^{metallicity} \cdot \frac{Z_{m\_S}}{XpY_S} \quad \text{or} \quad Z_m := XpY \cdot 10^{metallicity} \cdot .019288 \qquad (64d)$$

In using these expressions, we have to keep the mass of the star the same as that observed. So if we increase the metal content, we have to reduce the content of H and He, and vice versa. Let M be the mass of the star. Before we apply the metallicity factor, we have

$$M_{HplusHe\_1} := M - M_{metals\_1} \quad \text{or} \quad M_{metals\_1} := M - M_{HplusHe\_1} \qquad (65)$$

After we apply the metallicity factor:

$$M_{HplusHe\_2} := M - M_{metals\_2} \quad \text{or} \quad M_{metals\_2} := M - M_{HplusHe\_2} \qquad (66)$$

The mass M is, of course, the same for both Eq. (65) and Eq. (66).

From the above equations we therefore have

$$M_{metals\_1} + M_{HplusHe\_1} = M_{metals\_2} + M_{HplusHe\_2} \qquad (67)$$

$$M_{metals\_2} := XpY \cdot 10^{metallicity} \cdot .019288 \cdot M \qquad (68)$$

$$M_{HplusHe\_2} := XpY \cdot M \qquad (69)$$

$$M_{metals\_1} + M_{HplusHe\_1} = XpY \cdot \left(10^{metallicity} \cdot .019288 \cdot M + M\right) \qquad (70a)$$

Simplifying and solving for XpY:

$$1 = XpY \cdot \left(10^{metallicity} \cdot .019288 + 1\right)$$

$$XpY := \frac{1}{10^{metallicity} \cdot .019288 + 1} \qquad (70b)$$

worked example: Proxima Centauri, which has the observed properties

$$metallicity_{Proxima\_Centauri} := .21$$

$$M_{Proxima\_Centauri} := .123 \cdot M_{S\_cgs} \qquad M_{Proxima\_Centauri} = 2.445732 \times 10^{32} \text{ g}$$

$$XpY_{Proxima\_Centauri} := \frac{1}{10^{metallicity_{Proxima\_Centauri}} \cdot .019288 + 1} \qquad XpY_{Proxima\_Centauri} = 0.969667$$

$$Z_{m\_Proxima\_Centauri} := 1 - XpY_{Proxima\_Centauri} \qquad Z_{m\_Proxima\_Centauri} = 0.030333$$

$$M_{metals\_1} := Z_{m\_S} \cdot M_{Proxima\_Centauri} \qquad M_{metals\_1} = 4.627985 \times 10^{30} \text{ g}$$

$$M_{metals\_2} := Z_{m\_Proxima\_Centauri} \cdot M_{Proxima\_Centauri} \qquad M_{metals\_2} = 7.418547 \times 10^{30} \text{ g}$$

$$M_{HplusHe\_1} := XpY_S \cdot M_{Proxima\_Centauri} \qquad M_{HplusHe\_1} = 2.399452 \times 10^{32} \text{ g}$$

$$M_{HplusHe\_2} := XpY_{Proxima\_Centauri} \cdot M_{Proxima\_Centauri} \qquad M_{HplusHe\_2} = 2.371547 \times 10^{32} \text{ g}$$

Check:

$$M_{HplusHe\_1} + M_{metals\_1} = 2.445732 \times 10^{32} \text{ g}$$

$$M_{Proxima\_Centauri} = 2.445732 \times 10^{32} \text{ g}$$

$$M_{HplusHe\_2} + M_{metals\_2} = 2.445732 \times 10^{32} \text{ g}$$

By inspection, the ratio of $M_{HplusHe\_2}$ to $M_{HplusHe\_1}$ is

$$\frac{M_{HplusHe\_2}}{M_{HplusHe\_1}} = \frac{\dfrac{1}{10^{metallicity}\cdot .019288+1}}{.981077} \qquad (71)$$

For Proxima Centauri:

$$\frac{\dfrac{1}{10^{metallicity_{Proxima\_Centauri}}\cdot .019288+1}}{.981077} = 0.98837$$

Checks

$$\frac{M_{HplusHe\_2}}{M_{HplusHe\_1}} = 0.98837$$

Also, by inspection, for the metals:

$$\frac{M_{metals\_2}}{M_{metals\_1}} = \frac{\left(1 - \dfrac{1}{10^{metallicity}\cdot .019288 + 1}\right)}{.018923} \qquad (72)$$

For Proxima Centauri:

$$\frac{\left(1-\frac{1}{10^{metallicity_{Proxima\_Centauri}}\cdot .019288+1}\right)}{.018923}=1.60295$$

$$\frac{M_{metals\_2}}{M_{metals\_1}}=1.602975$$ close enough $$\frac{1.602975}{1.60295}=1.000016$$

These ratios (calculated for each star) will be utilized in the algorithm given in a later section to appropriately modify the mass fraction of each element.

16. ΔM for each spherical shell

Starting with Eq. (5c), one can see by inspection:

$$M(r) := \frac{4}{3}\cdot\pi\cdot\rho_c\cdot r^3\cdot\left(1-\frac{3}{4}\cdot\frac{r}{R}\right) \quad (5c)$$

$$M(r_1) := \frac{4}{3}\cdot\pi\cdot\rho_c\cdot r_1^3\cdot\left(1-\frac{3}{4}\cdot\frac{r_1}{R}\right) \quad (73)$$

$$M(r_2) := \frac{4}{3}\cdot\pi\cdot\rho_c\cdot r_2^3\cdot\left(1-\frac{3}{4}\cdot\frac{r_2}{R}\right) \quad (74)$$

Then, for each shell:

$$\Delta M\_el := \frac{4}{3}\cdot(\pi\cdot\rho_c)\cdot\left[r_1^3\cdot\left(1-\frac{3}{4}\cdot\frac{r_1}{R}\right)-r_2^3\cdot\left(1-\frac{3}{4}\cdot\frac{r_2}{R}\right)\right] \quad (75)$$

Table I gives the mass of each element in the Sun, for each consecutive shell. For a star of mass M, we have to multiply this value by the ratio of the star's mass to that of the Sun. So

$$\Delta M_{star\_el} := \Delta M_{S\_el}\cdot\frac{M}{M_S} \qquad \text{(for each shell)} \quad (76a)$$

And, of course, we have to multiply by the appropriate metallicity ratio:

$$\Delta M_{.star\_el} := \Delta M_{S\_el} \cdot \frac{M}{M_S} \cdot \frac{\dfrac{1}{10^{metallicity} \cdot .019288 + 1}}{.981077} \qquad \text{(for the H and He shells)} \qquad (76b)$$

$$\Delta M_{star\_el} := \Delta M_S \cdot \frac{M}{M_S} \cdot \frac{\left(1 - \dfrac{1}{10^{metallicity_{Proxima\_Centauri}} \cdot .019288 + 1}\right)}{.018923} \qquad \text{(for the "metals", each shell)} \qquad (76c)$$

For the Main Sequence stars, like our Sun or Proxima Centauri, we start at the surface, so

$$r_1 := R$$

Eq. (75) is then solved for r_2 and put into functional form:

$$r_{2\_calc}\left(R, r_1, \Delta M, \rho_c\right) := \frac{R}{3} + \frac{\sqrt{16 \cdot R \cdot r_1^{\,3} + 9 \cdot \left[\frac{\sqrt{3} \cdot \sqrt{\frac{512 \cdot R^4 \cdot \left(\frac{R^4}{27} - \frac{4 \cdot R \cdot r_1^{\,3}}{3} + r_1^{\,4} + \frac{R \cdot \Delta M}{\pi \cdot \rho_c} \right)^2}{9} + \frac{2048 \cdot R^{12}}{19683} - \frac{1280 \cdot R^8 \cdot \left(\frac{R^4}{27} - \cdots \right.}{\cdots}}}{18} \cdots \right.}}{\cdots}$$

(77)

After r_2 is calculated, then the volume (not specific volume) of the shell can be calculated.

$$V_{el} := \left(\frac{4}{3}\right) \cdot \pi \cdot \left(r_1^{\,3} - r_2^{\,3}\right)$$ (for any Main Sequence star, including the Sun) cm^3 (78)

The density of the shell is then

$$\rho_{el} := \frac{\Delta M_{el}}{V_{el}}$$ (for any Main Sequence star, including the Sun) (79)

Then, of course, the specific volume is

$$V_{L\_el} := \frac{1}{\rho_{el}} \tag{80}$$

Using the mean radius of the shell, we can then calculate the approximate mean pressure and mean temperature of the shell:

$$P_{mean\_el} := \frac{\pi}{36}\cdot G\cdot \rho_c^{\,2}\cdot R^2\cdot\left(5 - 24\cdot\frac{r_{mean}^{\,2}}{R^2} + 28\cdot\frac{r_{mean}^{\,3}}{R^3} - 9\cdot\frac{r_{mean}^{\,4}}{R^4}\right)\cdot conv_{dynescm2toatm} \tag{81}$$

$$T_{mean\_el} := 48.45665552\cdot V_{L\_el}\cdot w_{el} + 0.55041157\cdot P_{mean\_el}\cdot V_{L\_el}\cdot\left(w_{el}\right)^{\frac{1}{3}} \tag{82}$$

which is just Eq. (37d) instantiated for each shell. Of course, ρ_{el} could be used in Eq. (82), as well.

For the next shell, the next element is chosen. r_1 is set equal to the value of r_2 previously calculated. ΔM is then calculated and then the new value for r_2. With these values, the mean radius, volume, density, specific volume, pressure, and temperature of the shell are calculated.

17. the complete algorithm for the properties of Main Sequence stars; results and plots

First, the Sun's data are read in from Excel and put into a matrix called *xl*:

$xl :=$ ...\solar_properties_variation_from_center.xls

Second, we select those columns of this matrix we need for the calculations

$xl1 := xl^{\langle 1 \rangle}$ $xl2 := xl^{\langle 2 \rangle}$ $xl3 := xl^{\langle 3 \rangle}$ $xl15 := xl^{\langle 15 \rangle}$ $xl16 := xl^{\langle 16 \rangle}$ $xl17 := xl^{\langle 17 \rangle}$ $xl18 := xl^{\langle 18 \rangle}$ $xl19 := xl^{\langle 19 \rangle}$

$xl20 := xl^{\langle 20 \rangle}$ $xl21 := xl^{\langle 21 \rangle}$ $xl22 := xl^{\langle 22 \rangle}$ $xl23 := xl^{\langle 23 \rangle}$ $xl24 := xl^{\langle 24 \rangle}$ $xl25 := xl^{\langle 25 \rangle}$

Third, we combine these columns to form another matrix, called *xlelem*:

$xlelem := augment(xl1, xl2, xl3, xl15, xl16, xl17, xl18, xl19, xl20, xl21, xl22, xl23, xl24, xl25)$

Let *end_elem* = the atomic number Z of the fissioning element. Now, with the equations derived in the previous sections of this paper we can formulate an internal *Mathcad* program to calculate the star's properties and put the results into a third matrix, *xlelem2.* Note: the first element starts at row 2, not row 1.

$$r2\_calc\_loop(R, M, end\_elem, metallicity, adjFe, I) :=$$

$$xlelem2 \leftarrow xlelem$$

$$mass\_rel\_to\_Sun \leftarrow \frac{M}{M_{S\_cgs}}$$

$$mH \leftarrow \left(xlelem2_{2,4}\right) \cdot mass\_rel\_to\_Sun$$

$$mHe \leftarrow \left(xlelem2_{3,4}\right) \cdot mass\_rel\_to\_Sun$$

$$factormet \leftarrow \frac{\left(1 - \frac{1}{10^{metallicity} \cdot .019288 + 1}\right)}{.018923}$$

$$factorHplusHe \leftarrow \frac{\frac{1}{10^{metallicity} \cdot .019288+1}}{.981077}$$

$$\rho_c \leftarrow \frac{3 \cdot M}{\pi \cdot R^3}$$

$$G \leftarrow 6.67259 \cdot 10^{-8}$$

$$conv_{dynescm2toatm} \leftarrow 9.8716683 \cdot 10^{-7}$$

$$mass\_tot \leftarrow 0$$

$$\text{for } i \in 2 .. end\_elem$$

$$r_1 \leftarrow R \quad \text{if } i = 2$$

$$\Delta M\_actual \leftarrow xlelem2_{i,4}$$

$$\Delta M\_actual \leftarrow \Delta M\_actual \cdot mass\_rel\_to\_Sun \cdot factormet \quad \text{if } i > 3$$

$$\Delta M\_actual \leftarrow mH \cdot factorHplusHe \quad \text{if } i = 2$$

$$\Delta M\_actual \leftarrow mHe \cdot factorHplusHe \quad \text{if } i = 3$$

$$\Delta M\_actual \leftarrow adjFe \cdot \Delta M\_actual \quad \text{if } i = 27$$

$$r_2 \leftarrow r_{2\_calc}\left(R, r_1, \Delta M\_actual, \rho_c\right)$$

$$r_2 \leftarrow \sqrt{r_2^{\,2}}$$

$$r_{mean} \leftarrow .5 \cdot (r_1 + r_2)$$

$$\rho_{current} \leftarrow \rho_c \cdot \left(1 - \frac{r_{mean}}{R}\right)$$

$$P_{mean} \leftarrow \frac{\pi}{36} \cdot G \cdot \rho_c^{\,2} \cdot R^2 \cdot \left(5 - 24 \cdot \frac{r_{mean}^{\,2}}{R^2} + 28 \cdot \frac{r_{mean}^{\,3}}{R^3} - 9 \cdot \frac{r_{mean}^{\,4}}{R^4}\right) \cdot co$$

$$xlelem2_{i,5} \leftarrow \Delta M\_actual$$

$$xlelem2_{i,6} \leftarrow r_1$$

$$xlelem2_{i,7} \leftarrow r_2$$

$$xlelem2_{i,8} \leftarrow \Delta M\_actual$$

$$xlelem2_{i,9} \leftarrow \frac{\Delta M\_actual}{\rho_{current}}$$

$$xlelem2_{i,10} \leftarrow \rho_{current}$$

$$w \leftarrow 2 \cdot (i-1) \cdot conv_{amu\_to\_u} \quad \text{if} \quad l = 0$$

$$w \leftarrow xlelem2_{i,3} \quad \text{if} \quad l = 1$$

$$w \leftarrow \left[2 \cdot (i-1) + \frac{(i-1)^2}{I_R}\right] \cdot conv_{amu\_to\_u} \quad \text{if} \quad l > 1$$

$$T_{mean} \leftarrow 48.45665552 \cdot \frac{1}{xlelem2_{i,10}} \cdot w + 0.55041157 \cdot P_{mean} \cdot \frac{1}{xlelem2_{i,}}$$

$$xlelem2_{i,11} \leftarrow \frac{r_1}{R}$$

$$xlelem2_{i,12} \leftarrow \frac{r_2}{R}$$

$$xlelem2_{i,13} \leftarrow T_{mean}$$

$$xlelem2_{i,14} \leftarrow P_{mean}$$

$$r_1 \leftarrow r_2$$

$$\rho_H \leftarrow xlelem2_{i,10} \quad \text{if} \quad i = 2$$

$$\rho_{prev} \leftarrow xlelem2_{i,10}$$

$$mass\_tot \leftarrow mass\_tot + \Delta M\_actual$$

$$mass\_remaining \leftarrow M - mass\_tot$$

$$j \leftarrow end\_elem + 1$$

$$r_2 \leftarrow 0$$

$$xlelem2_{j,5} \leftarrow mass\_remaining$$

$$xlelem2_{j,6} \leftarrow r_1$$

$$xlelem2_{j,7} \leftarrow r_2$$

$$r_{mean} \leftarrow .5\cdot(r_1)$$

$$P_{mean} \leftarrow \frac{\pi}{36}\cdot G\cdot \rho_c^{\,2}\cdot R^2\cdot\left(5 - 24\cdot\frac{r_{mean}^{\,2}}{R^2} + 28\cdot\frac{r_{mean}^{\,3}}{R^3} - 9\cdot\frac{r_{mean}^{\,4}}{R^4}\right)\cdot conv_{d}$$

$$xlelem2_{j,8} \leftarrow xlelem2_{j,5}$$

$$xlelem2_{j,9} \leftarrow \frac{4}{3}\cdot\pi\cdot r_1^{\,3}$$

$$xlelem2_{j,10} \leftarrow \frac{mass\_remaining}{xlelem2_{j,9}}$$

$$w \leftarrow 2\cdot(i-1)\cdot conv_{amu\_to\_u} \quad \text{if} \quad l = 0$$

$$w \leftarrow xlelem2_{i,3} \quad \text{if} \quad l = 1$$

$$w \leftarrow \left[2\cdot(i-1) + \frac{(i-1)^2}{\ldots}\right]\cdot conv_{amu\_to\_u} \quad \text{if} \quad l > 1$$

$$w \leftarrow \left[2 \cdot (I - 1) + \frac{\ldots}{I_R} \right] \cdot conv_{amu\_to\_u} \quad \text{if } I > 1$$

$$T_{mean} \leftarrow 48.45665552 \cdot \frac{1}{xlelem2_{j,10}} \cdot w + 0.55041157 \cdot P_{mean} \cdot \frac{1}{xlelem2_{j,10}} \cdot ($$

$$xlelem2_{j,11} \leftarrow \frac{xlelem2_{j,6}}{R}$$

$$xlelem2_{j,12} \leftarrow 0$$

$$xlelem2_{j,13} \leftarrow T_{mean}$$

$$xlelem2_{j,14} \leftarrow P_{mean}$$

xlelem2

worked example: the Sun (spectral class G2, cycle 2B)

end_elem := 71 (Lu, current fissioning element) metallicity := 0 (by definition) I := 1

$M := M_{S\_cgs}$ $R := R_{S\_cgs}$

xlelem2 := r2_calc_loop(R, M, end_elem, metallicity, adjFe, I)

The results are below.

xlelem2 =

	1	2
1	"Z"	"Element"
2	1	"H"
3	2	"He"
4	3	"Li"
5	4	"Be"
6	5	"B"
7	6	"C"
8	7	"N"
9	8	"O"
10	9	"F"
11	10	"Ne"
12	11	"Na"
13	12	"Mg"
14	13	"Al"
15	14	"Si"
16	15	...

$ii := 2 .. end\_elem + 1$ index for plots

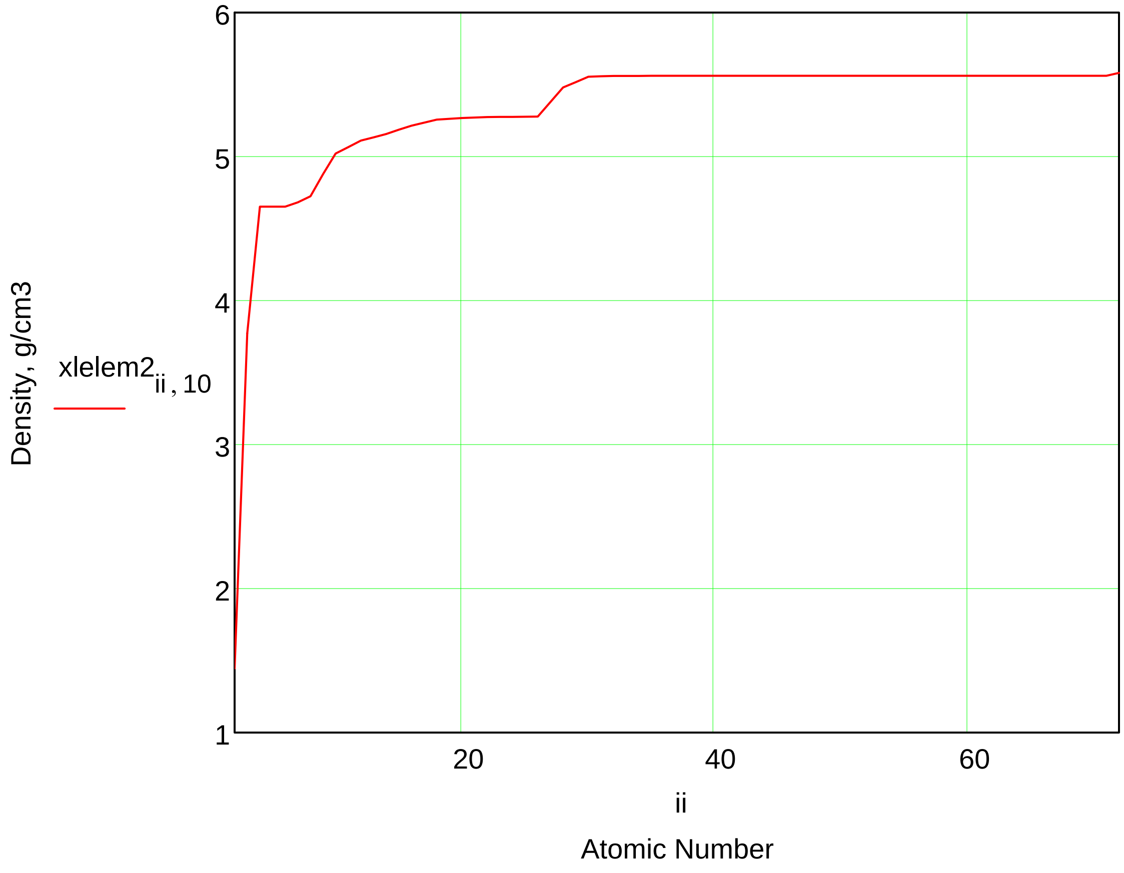


Figure 1. Density vs. Atomic Number for the Sun

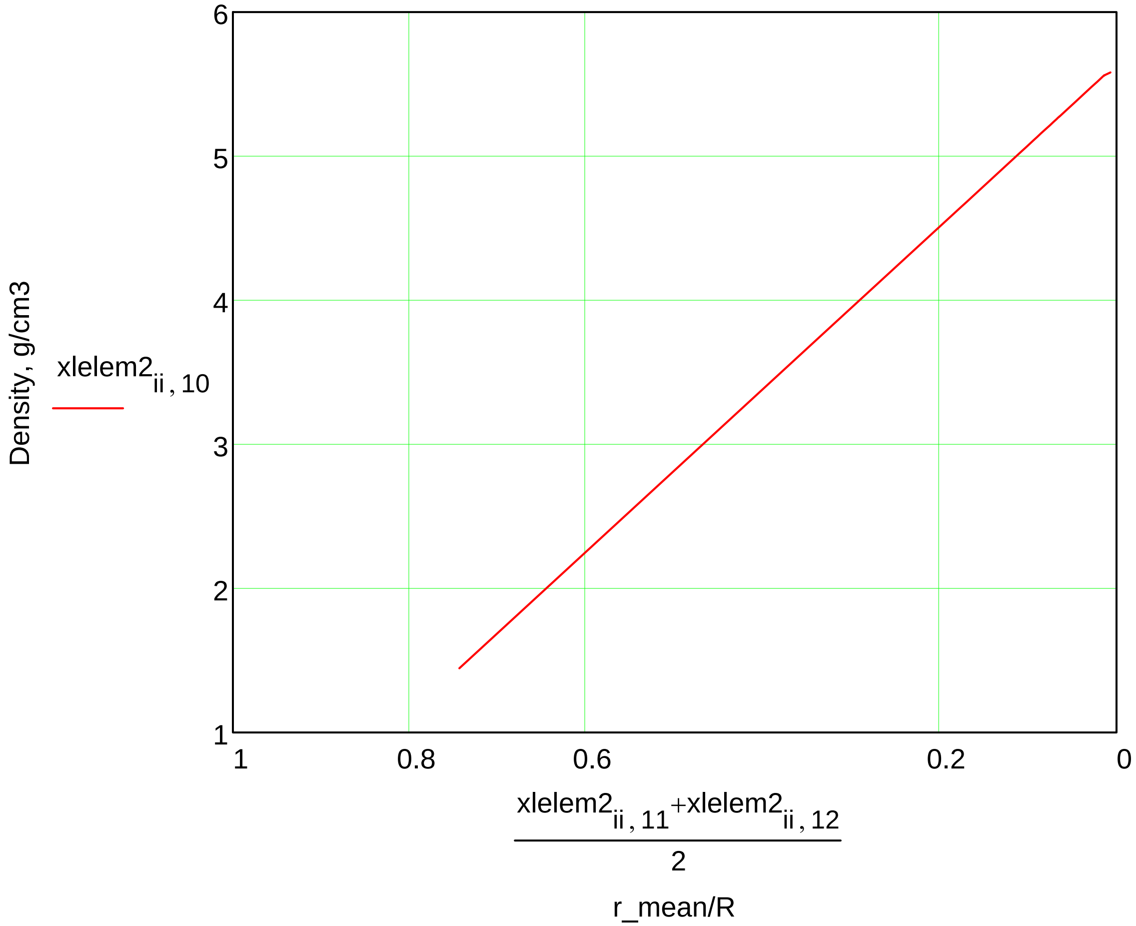


Figure 2. Density vs. r_{mean}/R for the Sun

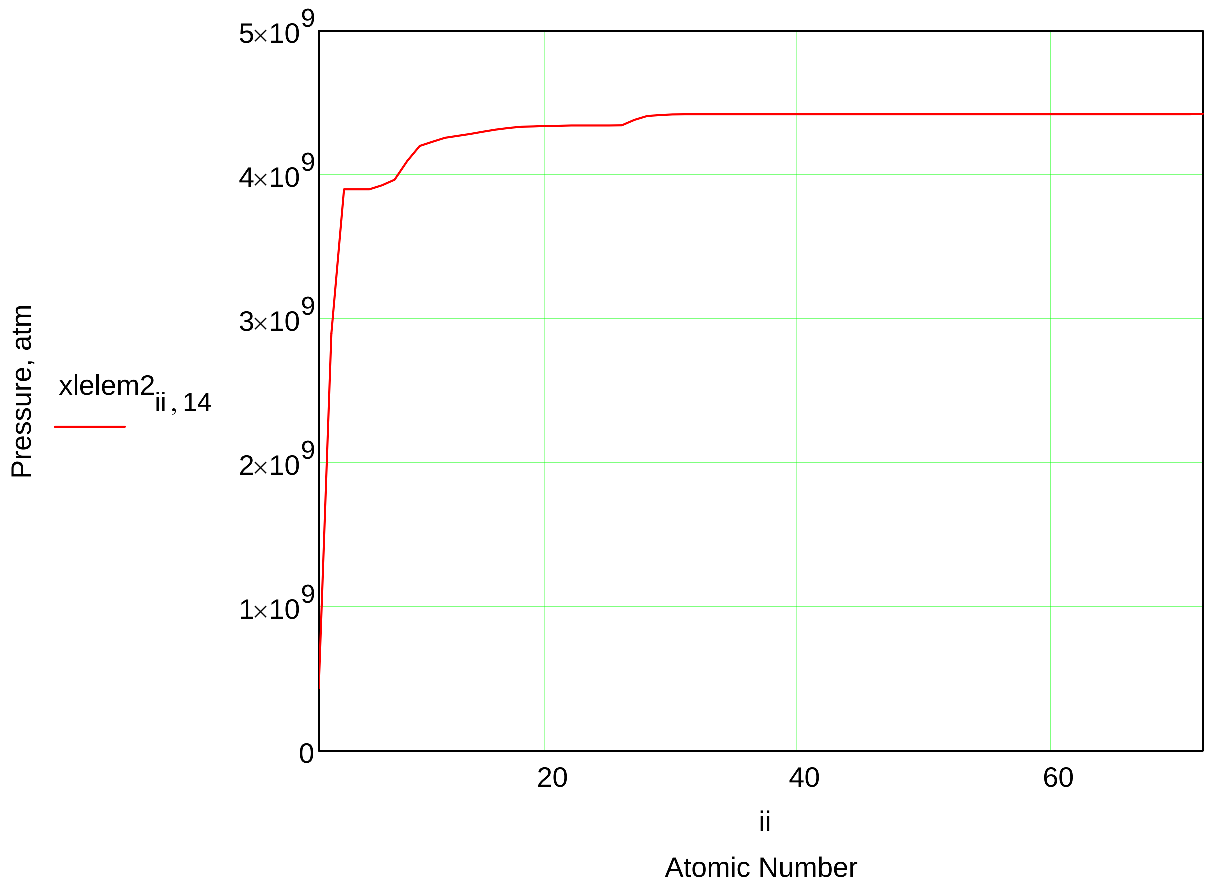


Figure 3. Pressure vs. Atomic Number for the Sun

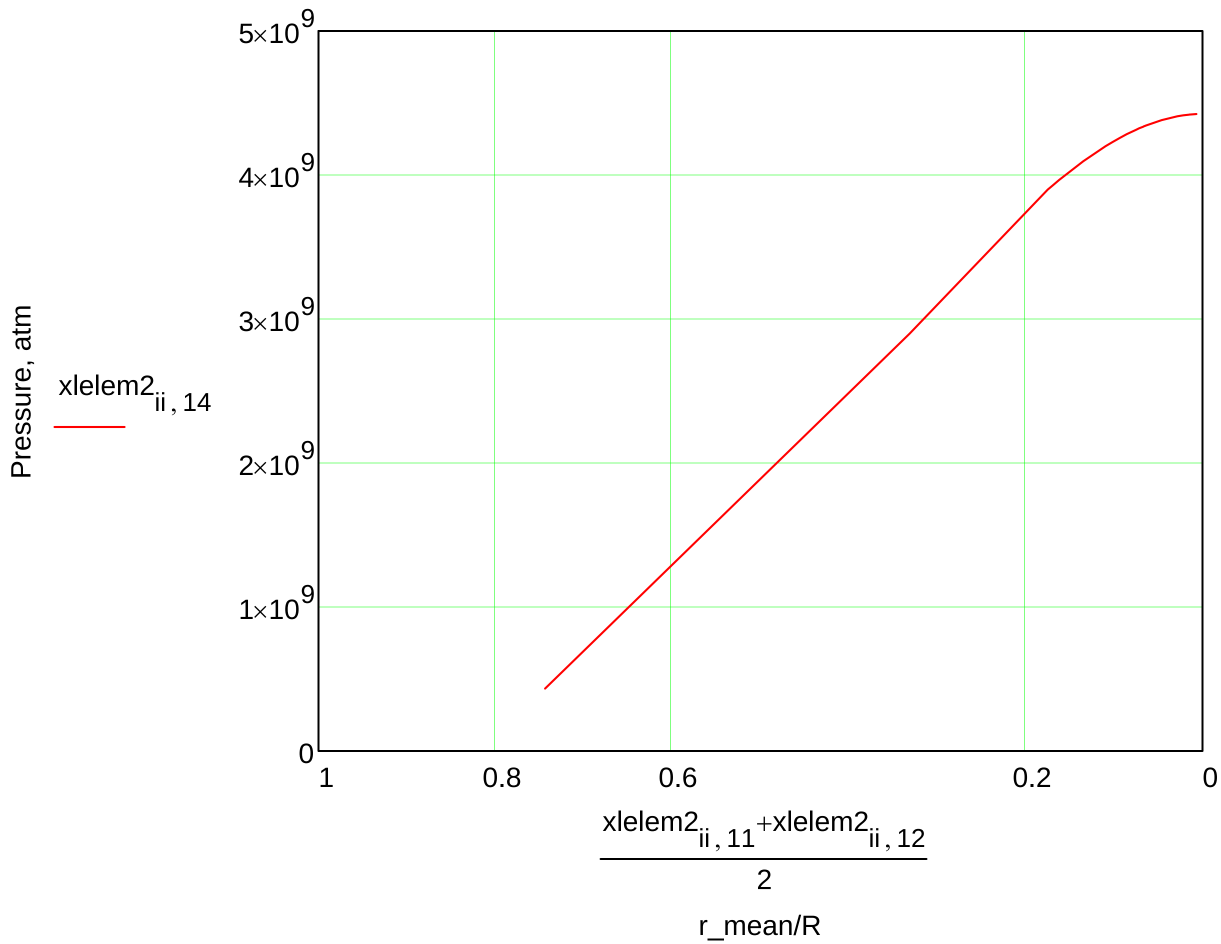


Figure 4. Pressure vs. r_{mean}/R for the Sun

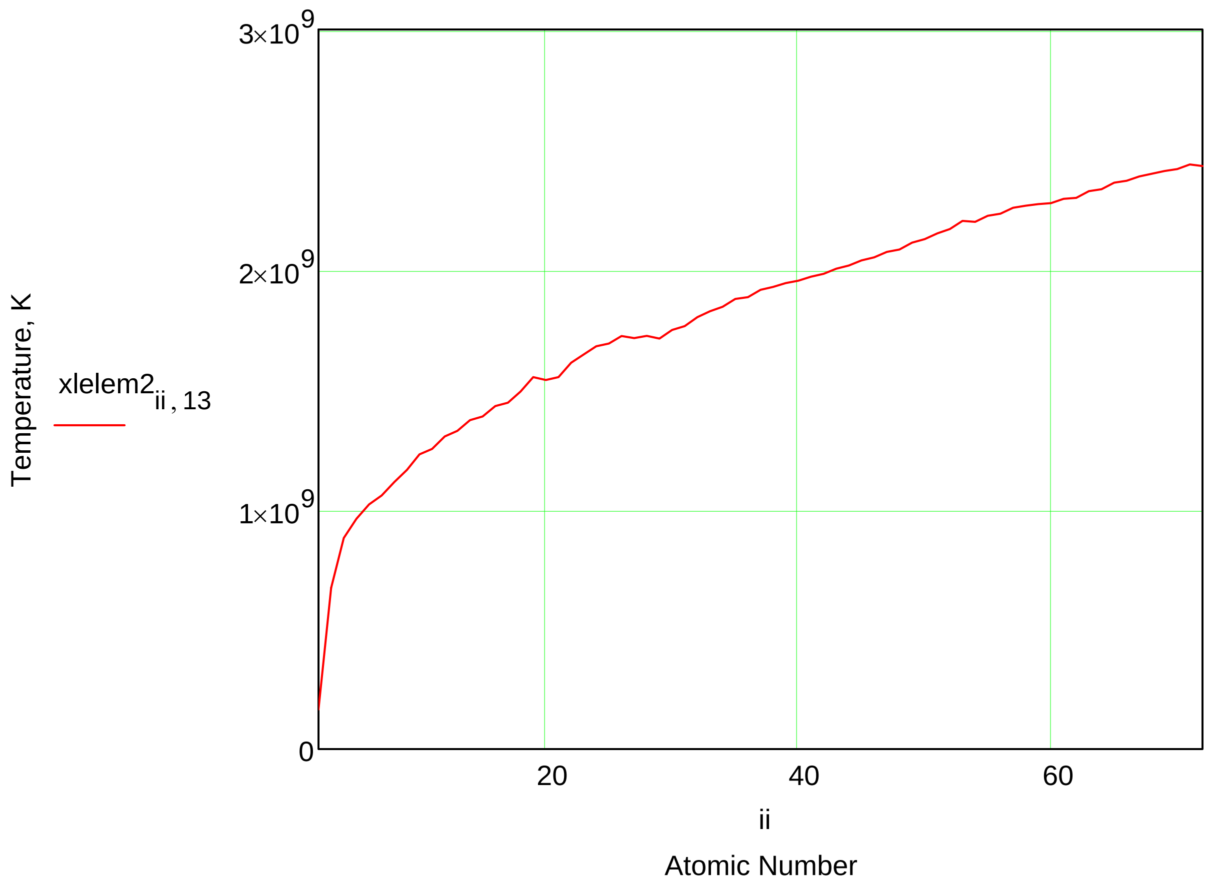


Figure 5. Temperature vs. Atomic Number for the Sun

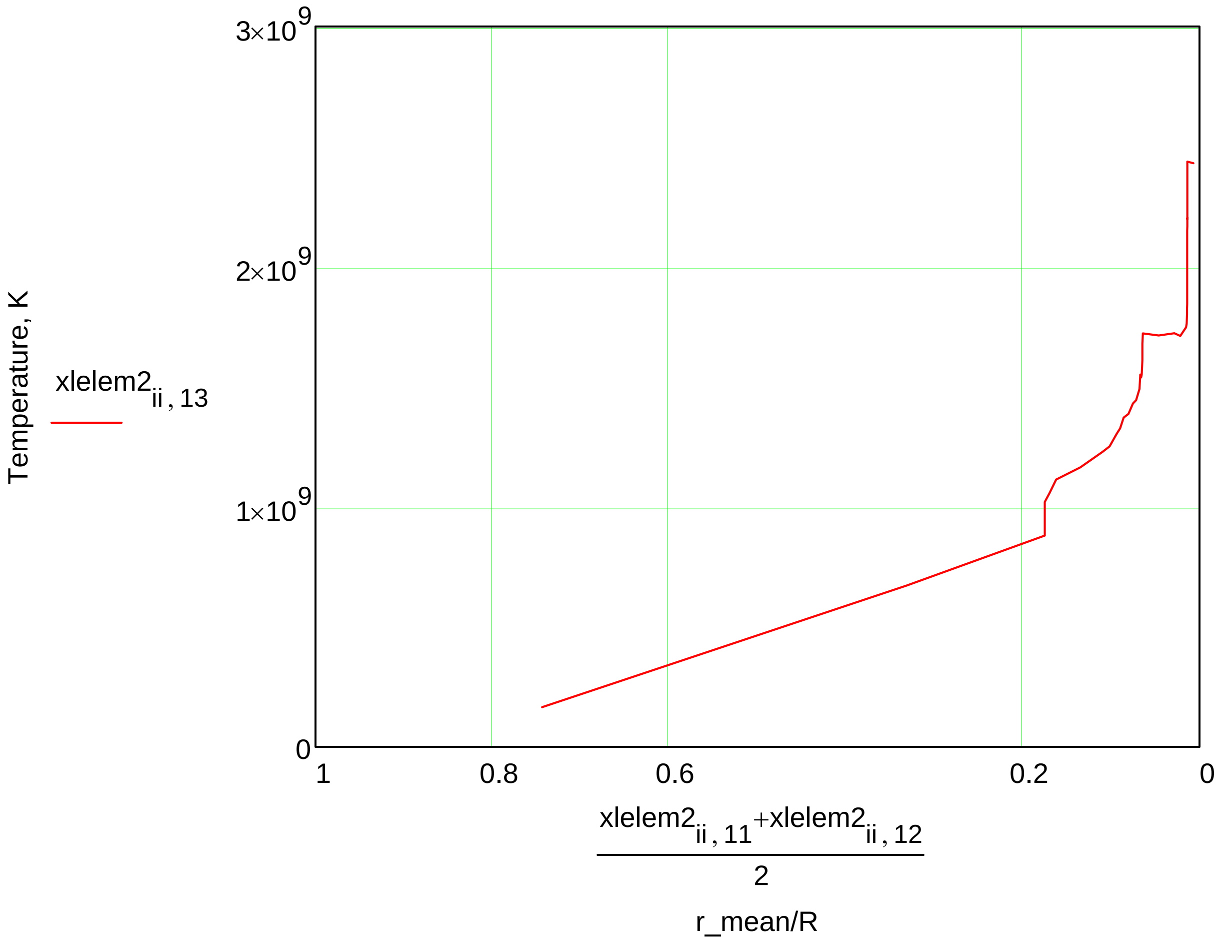


Figure 6. Temperature vs. r_{mean}/R for the Sun

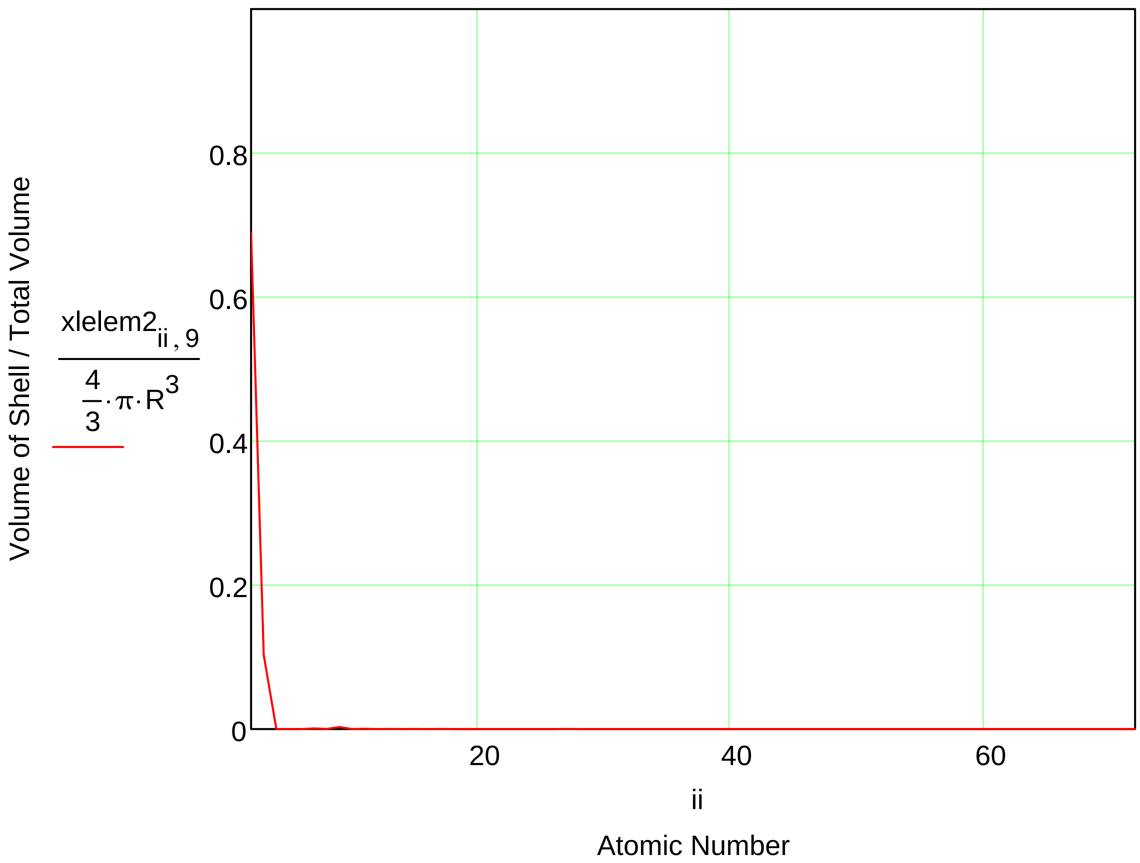


Figure 7. Volume of Shell / Total Volume vs. Atomic Number for the Sun

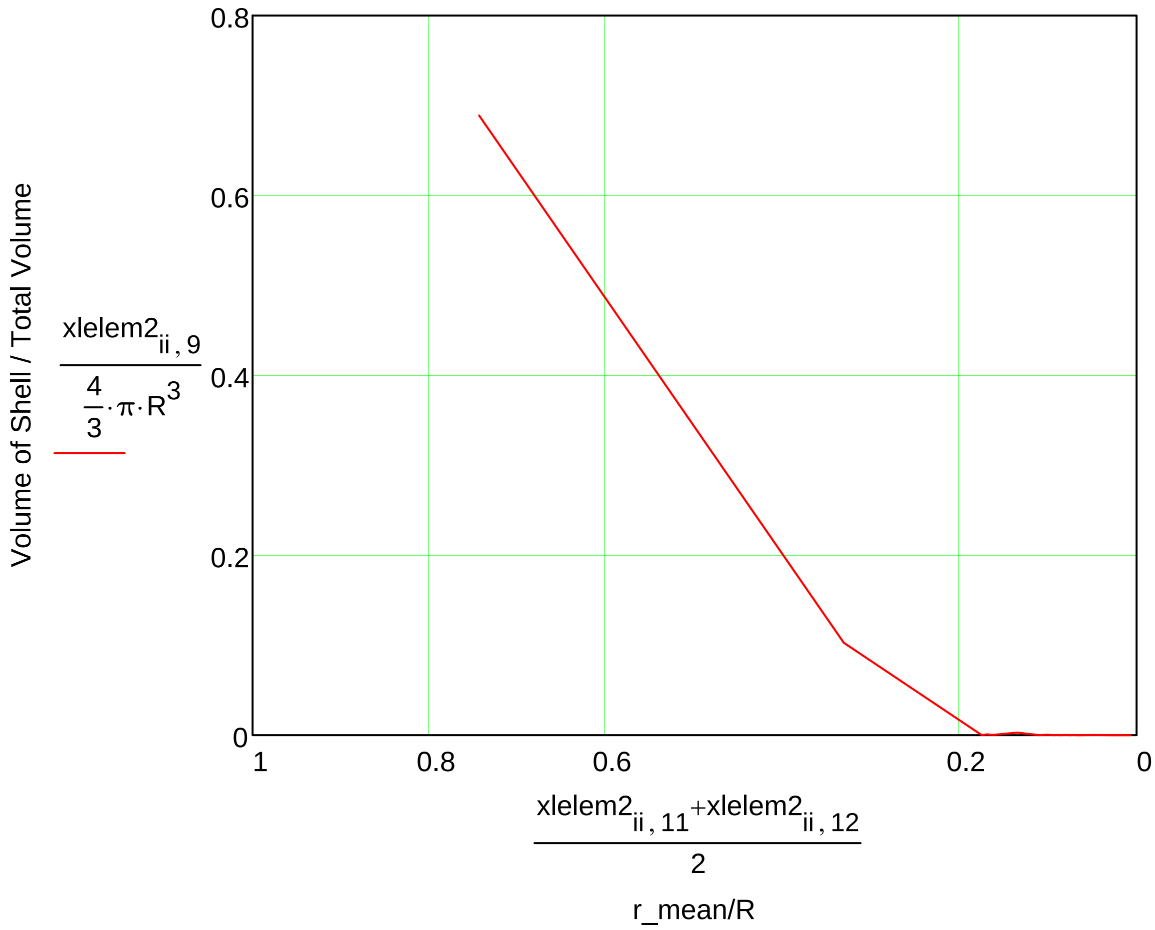


Figure 8. Volume of Shell / Total Volume vs. r_{mean}/R for the Sun

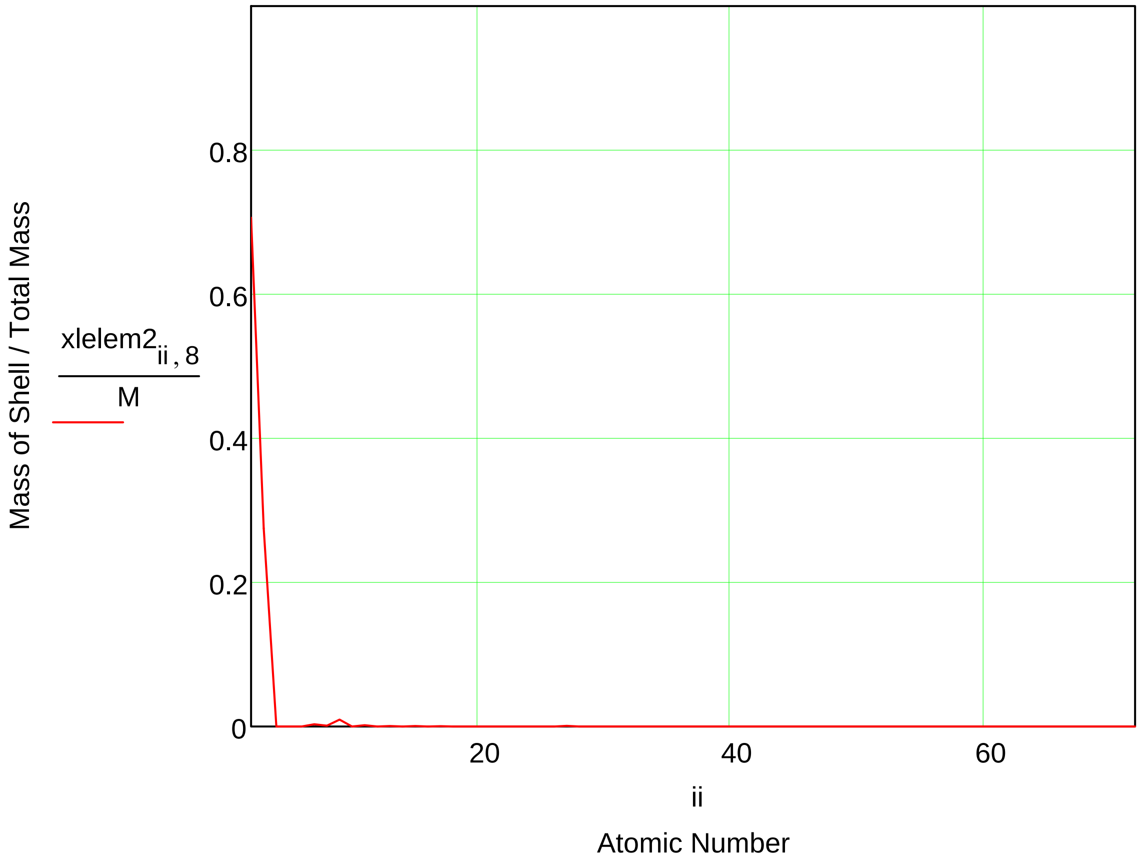


Figure 9. Mass of Shell / Total Mass vs. Atomic Number for the Sun

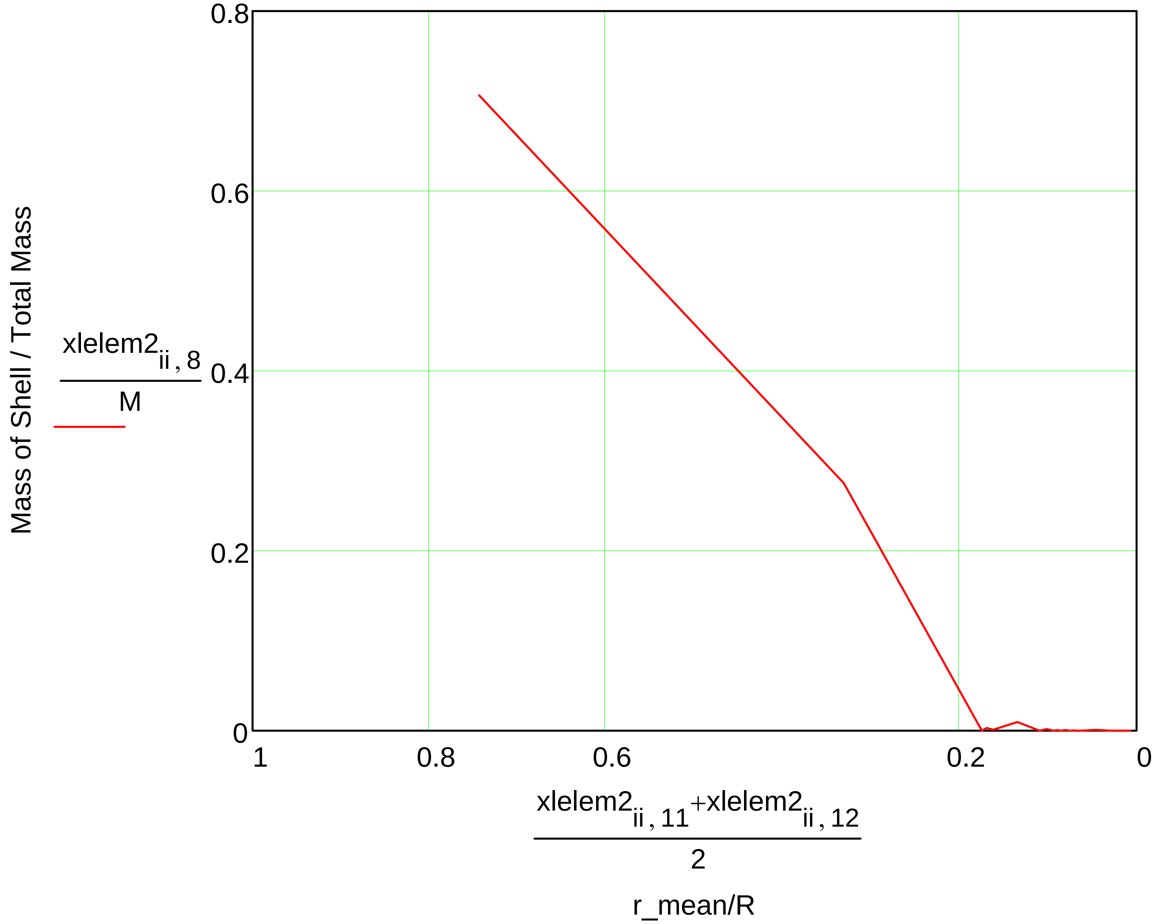


Figure 10. Mass of Shell / Total Mass vs. r_{mean}/R for the Sun

worked example: Proxima Centauri (spectral class M6, cycle 2B)

end_elem := 95 (Am, current fissioning element) metallicity := .21 I := 0 (very early Main Sequence)

$M := 0.123 \cdot M_{S\_cgs}$ $R := 0.141 \cdot R_{S\_cgs}$

xlelem2 := r2_calc_loop(R, M, end_elem, metallicity, adjFe, I)

The results are below.

xlelem2 =

	1	2
1	"Z"	"Element"
2	1	"H"
3	2	"He"
4	3	"Li"
5	4	"Be"
6	5	"B"
7	6	"C"
8	7	"N"
9	8	"O"
10	9	"F"
11	10	"Ne"
12	11	"Na"
13	12	"Mg"
14	13	"Al"
15	14	"Si"
16	15	...

$ii := 2 .. end\_elem + 1$ index for plots

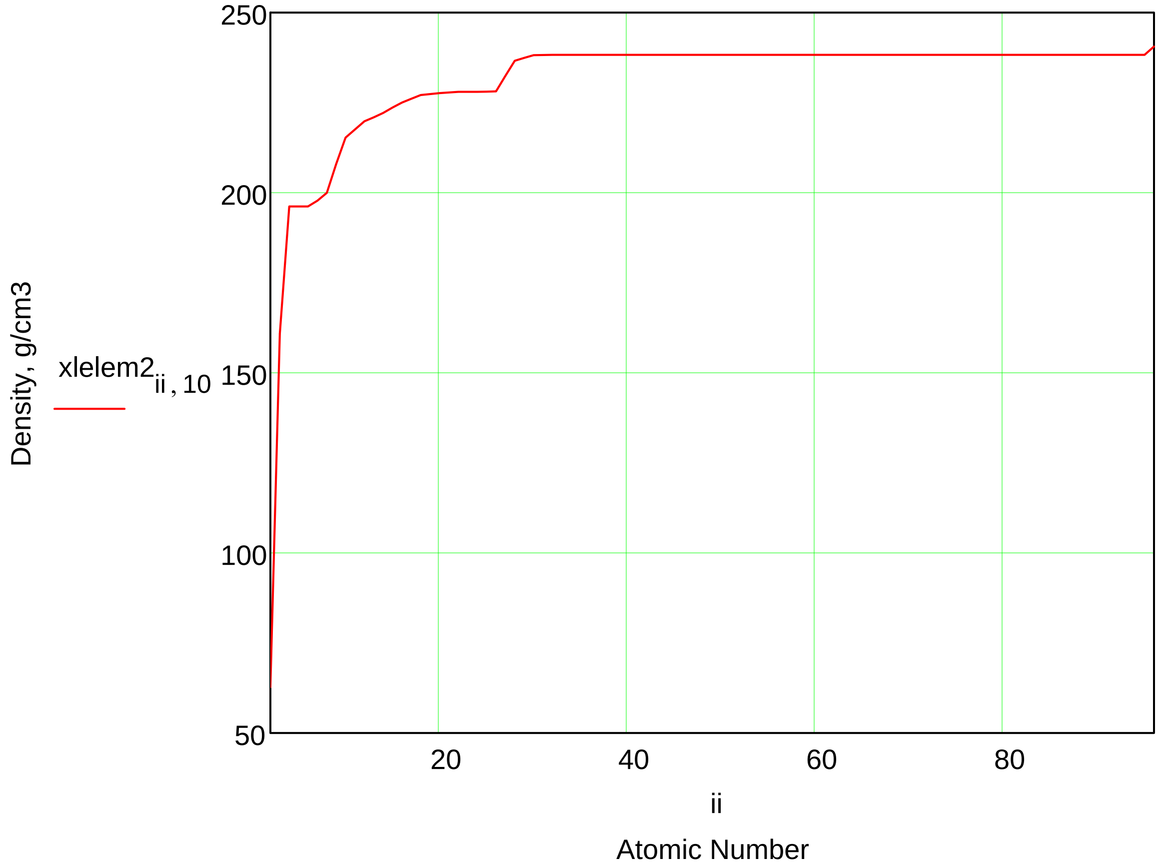


Figure 11. Density vs. Atomic Number for Proxima Centauri

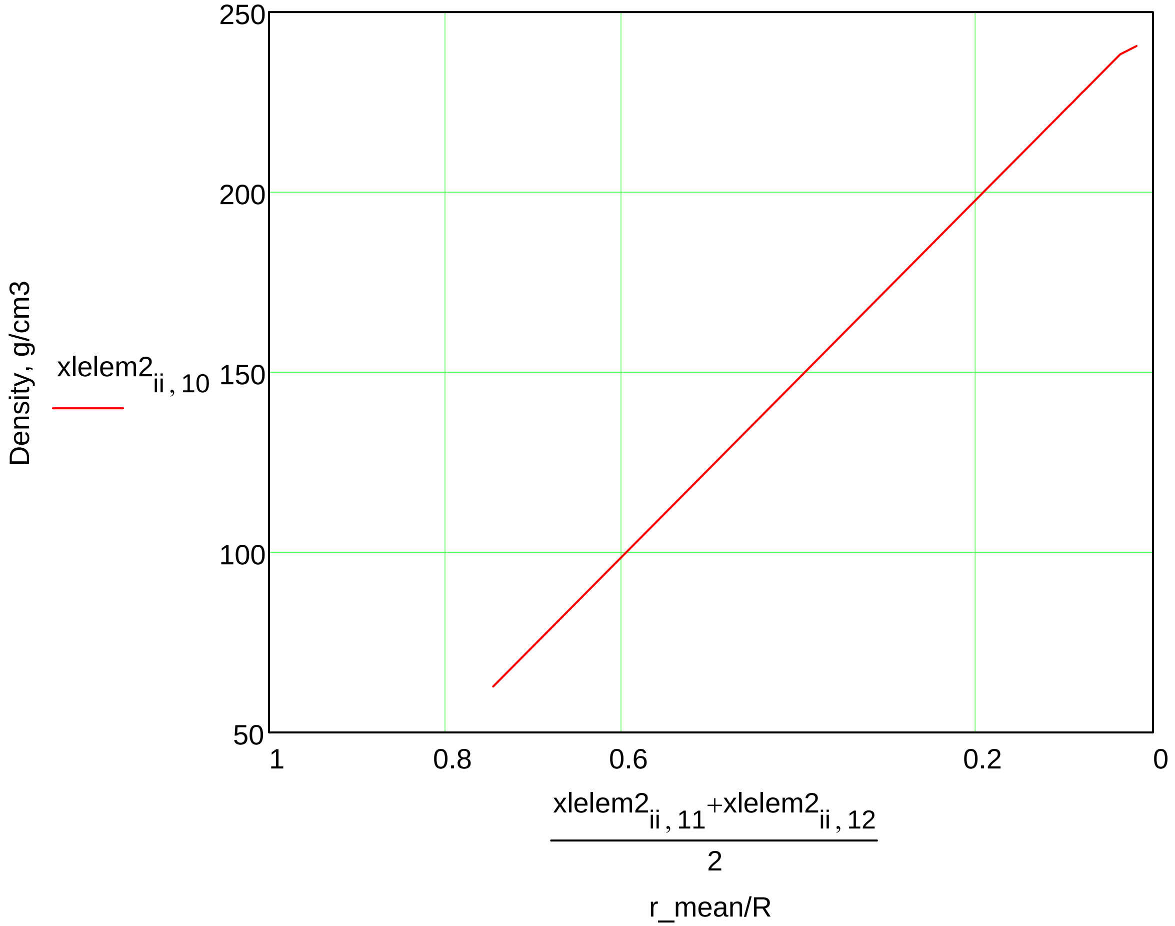


Figure 12. Density vs. r_{mean}/R for Proxima Centauri

Proxima Centauri must be a previous White Dwarf!

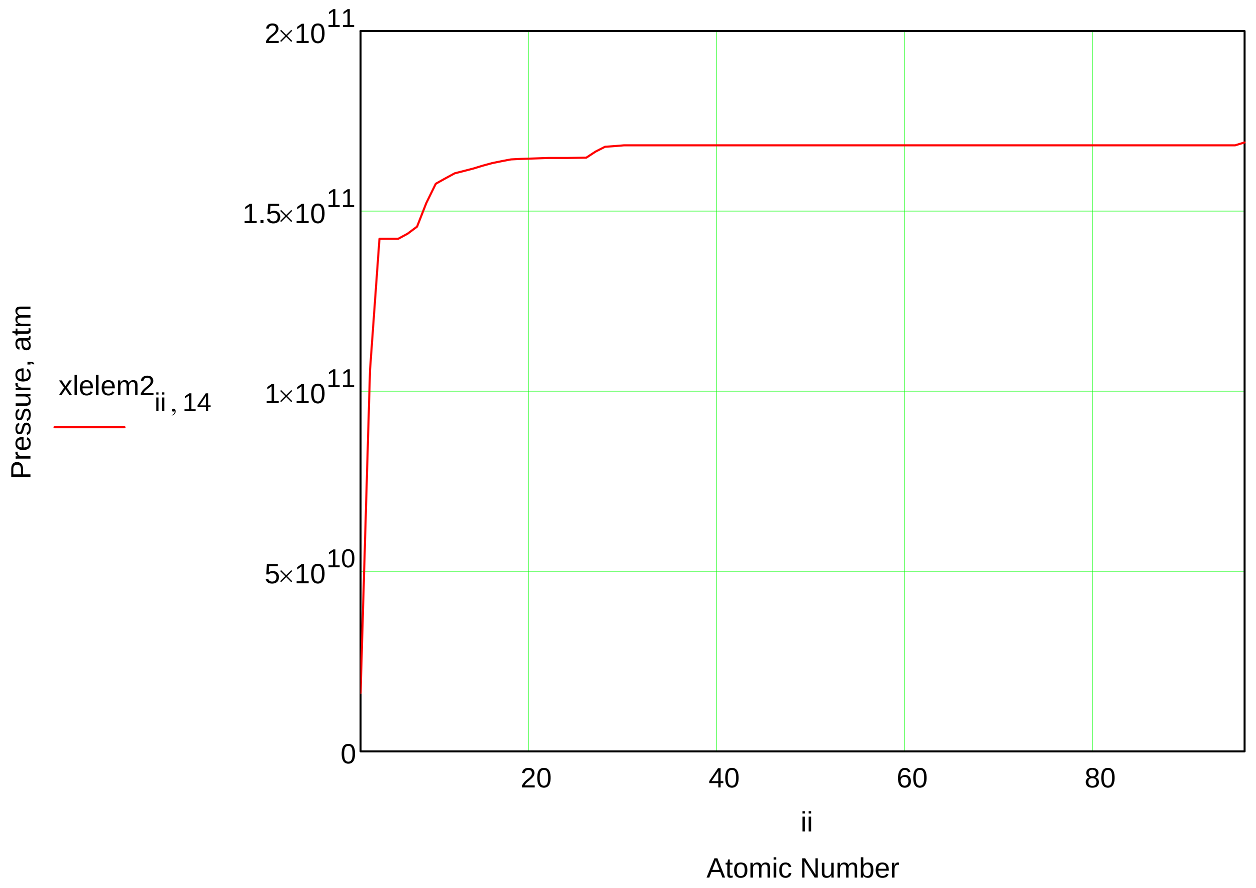


Figure 13. Pressure vs. Atomic Number for Proxima Centauri

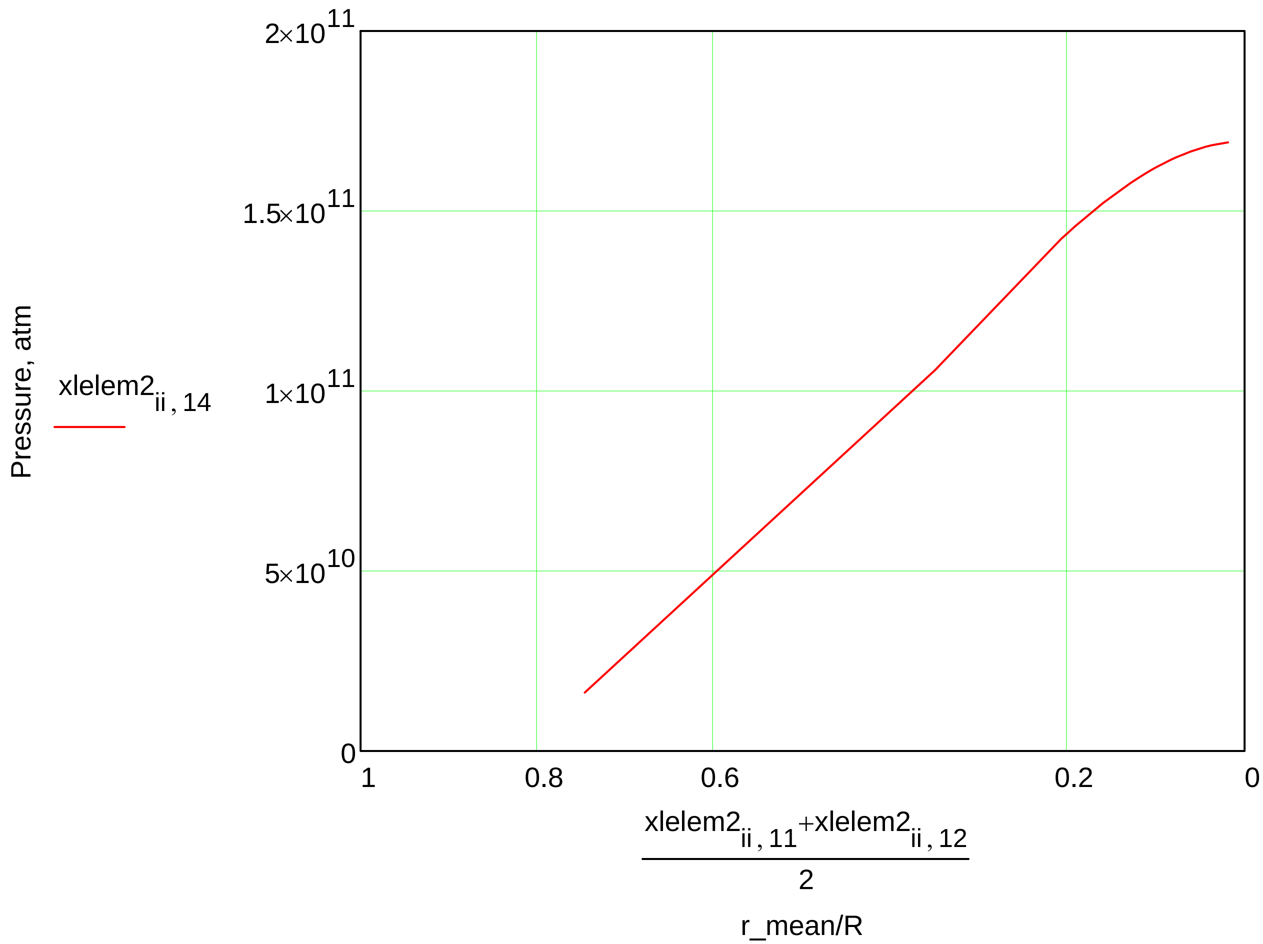


Figure 14. Pressure vs. r_{mean}/R for Proxima Centauri

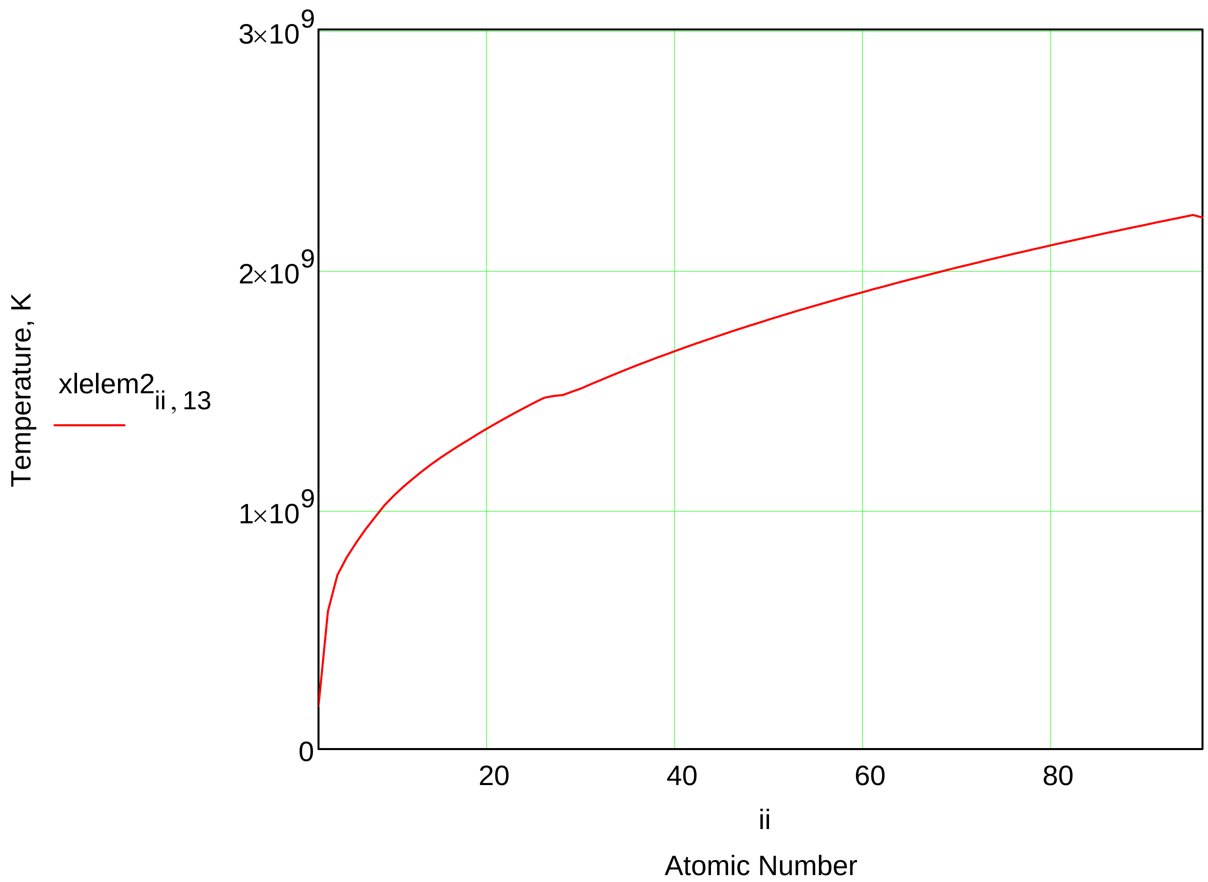


Figure 15. Temperature vs. Atomic Number for Proxima Centauri

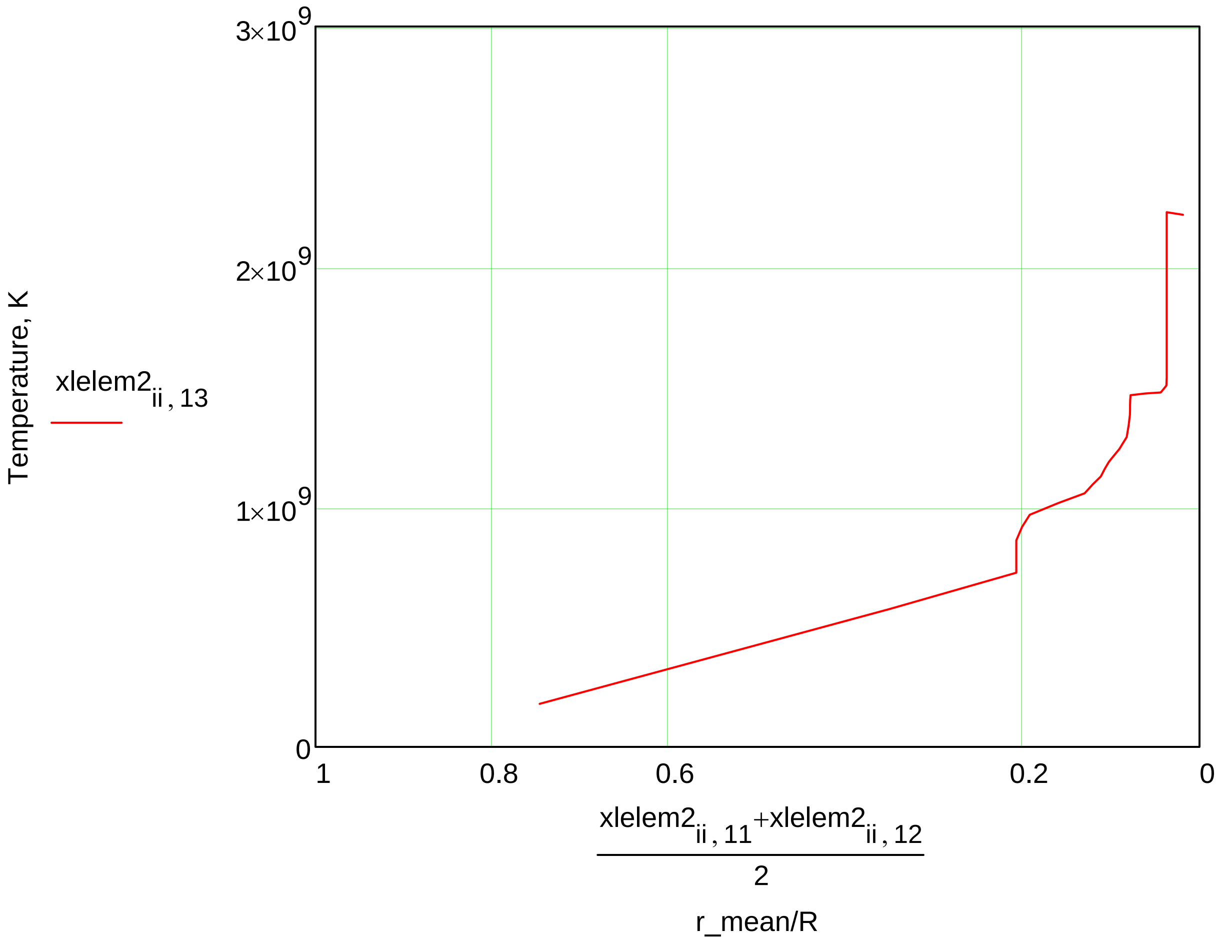


Figure 16. Temperature vs. r_{mean}/R for Proxima Centauri

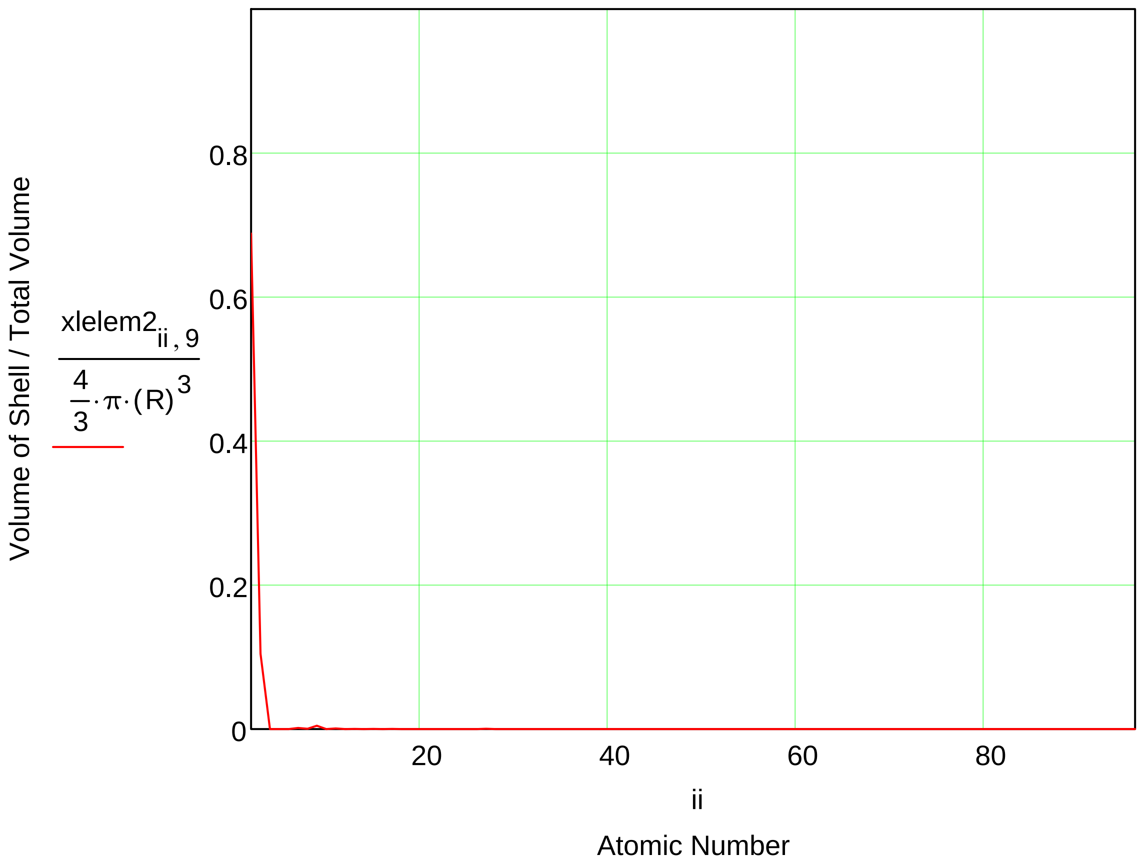


Figure 17. Volume of Shell / Total Volume vs. Atomic Number for Proxima Centauri

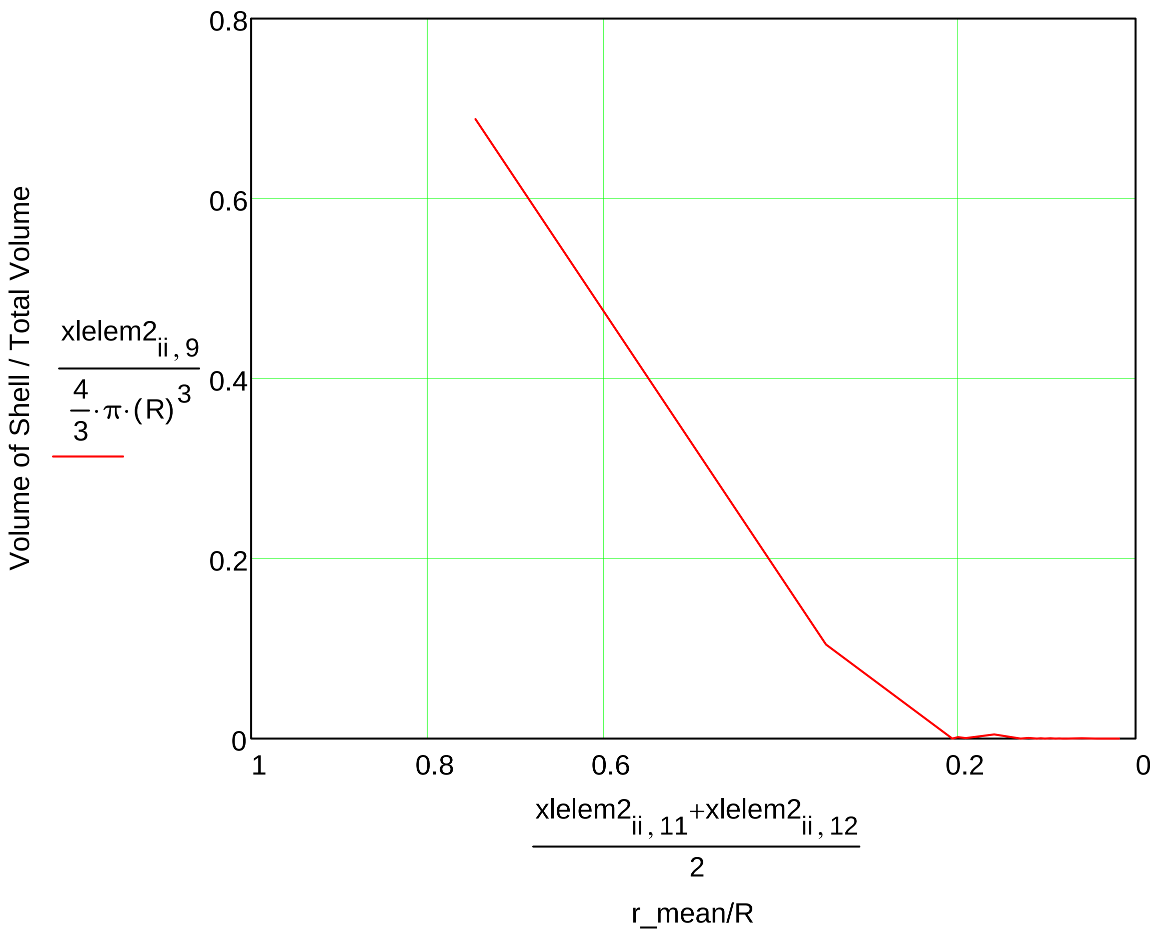


Figure 18. Volume of Shell / Total Volume vs. r_{mean}/R for Proxima Centauri

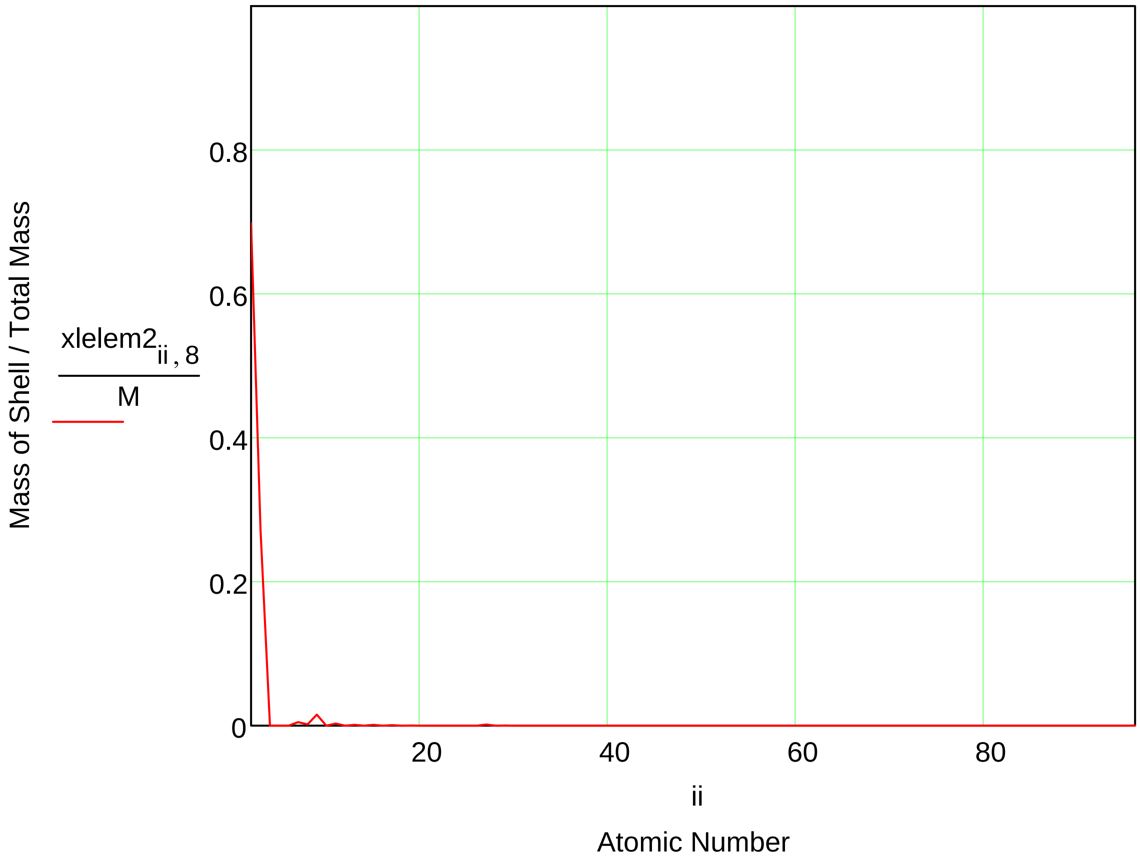


Figure 19. Mass of Shell / Total Mass vs. Atomic Number for Proxima Centauri

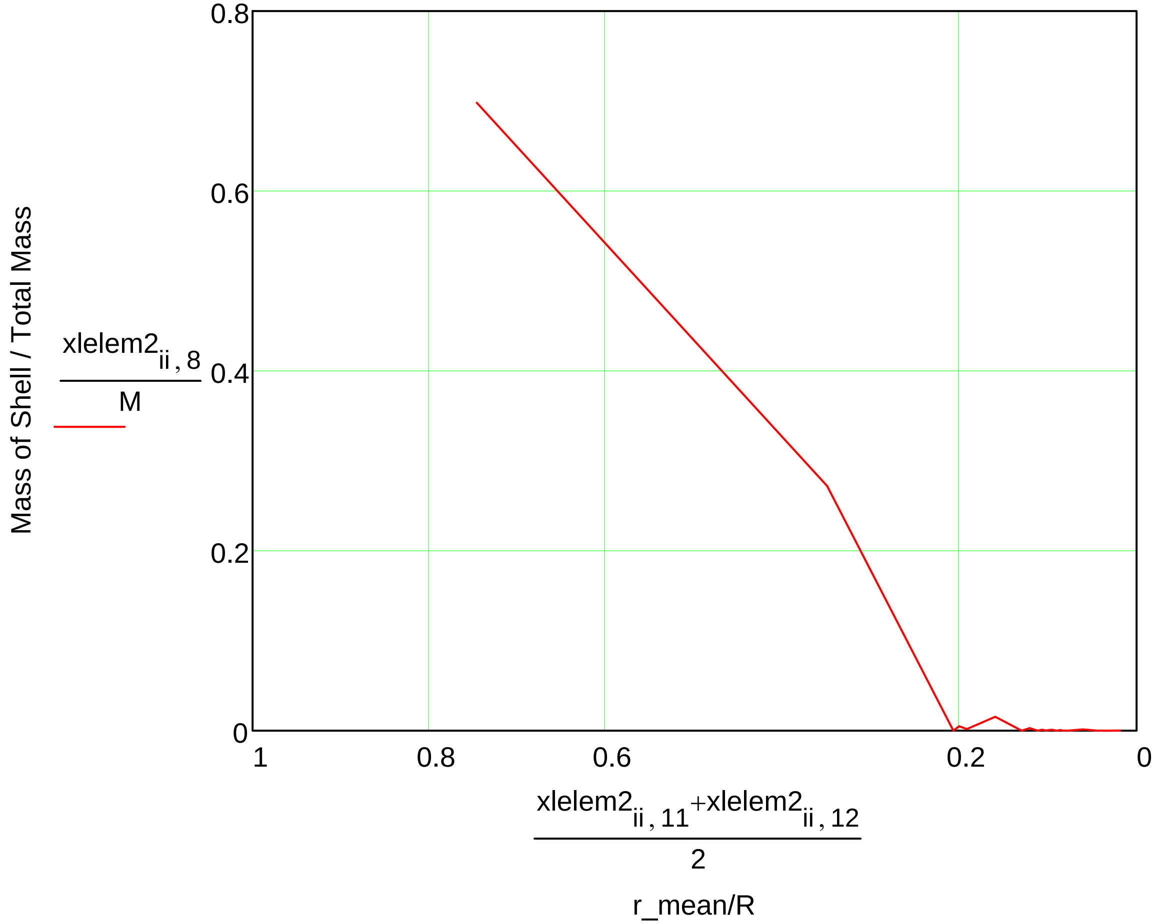


Figure 20. Mass of Shell / Total Mass vs. r_{mean}/R for Proxima Centauri

B. Red Giants

The ideal gas or perfect gas law works for Red Giants (and Orange Giants, as well), because their density is so *low*. Thus, a different equation for temperature than that for Main Sequence stars is used:

$$R_{gas} := 82.057 \quad \text{atm cm}^3 \text{ / mol K} \qquad \text{(gas constant)} \qquad (83)$$

$$T_{mean\_el} := \frac{P_{mean\_el} \cdot w_{el}}{R_{gas} \cdot \rho_{el}} \qquad \text{K} \qquad (84)$$

where the constant, 82.057, is the gas constant in appropriate units. With this one change we now have the internal *Mathcad* program for Red Giants:

$$\text{r2\_calc\_loop\_RG}(R, M, \text{end\_elem}, \text{metallicity}, \text{adjFe}, l) :=$$

$$\text{xlelem2} \leftarrow \text{xlelem}$$

$$\text{mass\_rel\_to\_Sun} \leftarrow \frac{M}{M_{S\_cgs}}$$

$$\text{mH} \leftarrow \left(\text{xlelem2}_{2,4}\right)\cdot \text{mass\_rel\_to\_Sun}$$

$$\text{mHe} \leftarrow \left(\text{xlelem2}_{3,4}\right)\cdot \text{mass\_rel\_to\_Sun}$$

$$\text{factormet} \leftarrow \frac{\left(1 - \frac{1}{10^{\text{metallicity}}\cdot .019288 + 1}\right)}{.018923}$$

$$\text{factorHplusHe} \leftarrow \frac{\frac{1}{10^{\text{metallicity}}\cdot .019288+1}}{.981077}$$

$$\rho_c \leftarrow \frac{3\cdot M}{\pi\cdot R^3}$$

$$G \leftarrow 6.67259\cdot 10^{-8}$$

$$\text{conv}_{\text{dynescm2toatm}} \leftarrow 9.8716683\cdot 10^{-7}$$

$$\text{mass\_tot} \leftarrow 0$$

$$\text{for } i \in 2 .. \text{end\_elem}$$

$$r_1 \leftarrow R \quad \text{if } i = 2$$

$$\Delta\text{M\_actual} \leftarrow \text{xlelem2}_{i,4}$$

$$\Delta\text{M\_actual} \leftarrow \Delta\text{M\_actual}\cdot\text{mass\_rel\_to\_Sun}\cdot\text{factormet} \quad \text{if } i > 3$$

$$\Delta\text{M\_actual} \leftarrow \text{mH}\cdot\text{factorHplusHe} \quad \text{if } i = 2$$

$$\Delta\text{M\_actual} \leftarrow \text{mHe}\cdot\text{factorHplusHe} \quad \text{if } i = 3$$

$$\Delta\text{M\_actual} \leftarrow \text{adjFe}\cdot\Delta\text{M\_actual} \quad \text{if } i = 27$$

$$r_2 \leftarrow r_{2\_calc}\left(R, r_1, \Delta\text{M\_actual}, \rho_c\right)$$

$$r_2 \leftarrow \sqrt{{r_2}^2}$$

$$r_{mean} \leftarrow .5\cdot(r_1 + r_2)$$

$$\rho_{current} \leftarrow \rho_c\cdot\left(1 - \frac{r_{mean}}{R}\right)$$

$$P_{mean} \leftarrow \frac{\pi}{36}\cdot G\cdot\rho_c^{\,2}\cdot R^2\cdot\left(5 - 24\cdot\frac{r_{mean}^{\,2}}{R^2} + 28\cdot\frac{r_{mean}^{\,3}}{R^3} - 9\cdot\frac{r_{me}}{R}\right.$$

$$xlelem2_{i,5} \leftarrow \Delta M\_actual$$

$$xlelem2_{i,6} \leftarrow r_1$$

$$xlelem2_{i,7} \leftarrow r_2$$

$$xlelem2_{i,8} \leftarrow \Delta M\_actual$$

$$xlelem2_{i,9} \leftarrow \frac{\Delta M\_actual}{\rho_{current}}$$

$$xlelem2_{i,10} \leftarrow \rho_{current}$$

$$w \leftarrow 2\cdot(i-1)\cdot conv_{amu\_to\_u} \quad \text{if} \quad l = 0$$

$$w \leftarrow xlelem2_{i,3} \quad \text{if} \quad l = 1$$

$$w \leftarrow \left[2\cdot(i-1) + \frac{(i-1)^2}{I_R}\right]\cdot conv_{amu\_to\_u} \quad \text{if} \quad l > 1$$

$$T_{mean} \leftarrow \frac{P_{mean}\cdot w}{82.057\cdot xlelem2_{i,10}}$$

$$xlelem2_{i,11} \leftarrow \frac{r_1}{R}$$

$$xlelem2_{i,12} \leftarrow \frac{r_2}{R}$$

$$xlelem2_{i,13} \leftarrow T_{mean}$$

$xlelem2_{i,14} \leftarrow P_{mean}$

$r_1 \leftarrow r_2$

$\rho_H \leftarrow xlelem2_{i,10} \quad \text{if} \quad i = 2$

$\rho_{prev} \leftarrow xlelem2_{i,10}$

$mass\_tot \leftarrow mass\_tot + \Delta M\_actual$

$mass\_remaining \leftarrow M - mass\_tot$

$j \leftarrow end\_elem + 1$

$r_2 \leftarrow 0$

$xlelem2_{j,5} \leftarrow mass\_remaining$

$$xlelem2_{j,6} \leftarrow r_1$$

$$xlelem2_{j,7} \leftarrow r_2$$

$$r_{mean} \leftarrow .5\cdot(r_1)$$

$$P_{mean} \leftarrow \frac{\pi}{36}\cdot G\cdot \rho_c{}^2\cdot R^2\cdot\left(5 - 24\cdot\frac{r_{mean}{}^2}{R^2} + 28\cdot\frac{r_{mean}{}^3}{R^3} - 9\cdot\frac{r_{mean}{}^4}{R^4}\right.$$

$$xlelem2_{j,8} \leftarrow xlelem2_{j,5}$$

$$xlelem2_{j,9} \leftarrow \frac{4}{3}\cdot\pi\cdot r_1{}^3$$

$$xlelem2_{j,10} \leftarrow \frac{mass\_remaining}{xlelem2_{j,9}}$$

$$w \leftarrow 2\cdot(j-1)\cdot conv_{amu\_to\_u} \quad \text{if} \quad I = 0$$

$$w \leftarrow xlelem2_{j,3} \quad \text{if} \quad I = 1$$

$$w \leftarrow \left[2\cdot(j-1) + \frac{(j-1)^2}{I_R}\right]\cdot conv_{amu\_to\_u} \quad \text{if} \quad I > 1$$

$$T_{mean} \leftarrow \frac{P_{mean}\cdot w}{82.057\cdot xlelem2_{j,10}}$$

$$xlelem2_{j,11} \leftarrow \frac{xlelem2_{j,6}}{R}$$

$$xlelem2_{j,12} \leftarrow 0$$

$$xlelem2_{j,13} \leftarrow T_{mean}$$

$$xlelem2_{j,14} \leftarrow P_{mean}$$

$$xlelem2$$

worked example: Epsilon Lep (spectral class K4-K3, cycle 2C)

Most Red and Orange Giants (like Epsilon Lep) are *variable* stars; they *oscillate between two different fissioning elements*. For a K4-K3 star, like Epsilon Lep, the fissioning elements are Bi (Z = 83) and Pb (Z = 82). We'll do Bi first.

$end\_elem := 83$ (Bi, current fissioning element) $metallicity := -0.02$ $I := 0$ (assumed) $adjFe := .6$

$M := 1.7 \cdot M_{S\_cgs}$ $R := 40.1 \cdot R_{S\_cgs}$

(set so that r_1 and r_2 are real, not imaginary)

$xlelem2 := r2\_calc\_loop\_RG(R, M, end\_elem, metallicity, adjFe, I)$

The results are below.

xlelem2 =

	1	2
1	"Z"	"Element"
2	1	"H"
3	2	"He"
4	3	"Li"
5	4	"Be"
6	5	"B"
7	6	"C"
8	7	"N"
9	8	"O"
10	9	"F"
11	10	"Ne"
12	11	"Na"
13	12	"Mg"
14	13	"Al"
15	14	"Si"
16	15	...

$ii := 2 .. end\_elem + 1$ index for plots

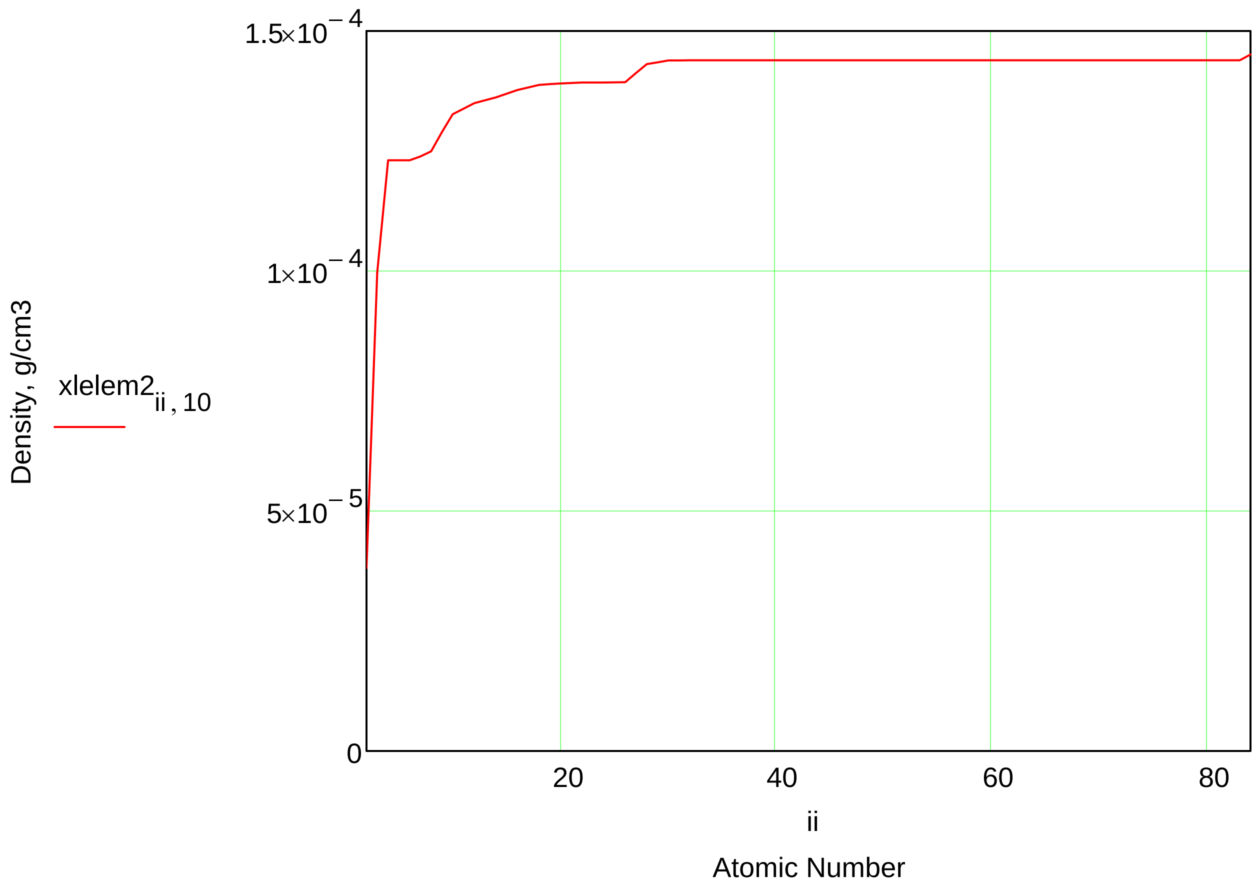


Figure 21. Density vs. Atomic Number for Epsilon Lep (Bi)

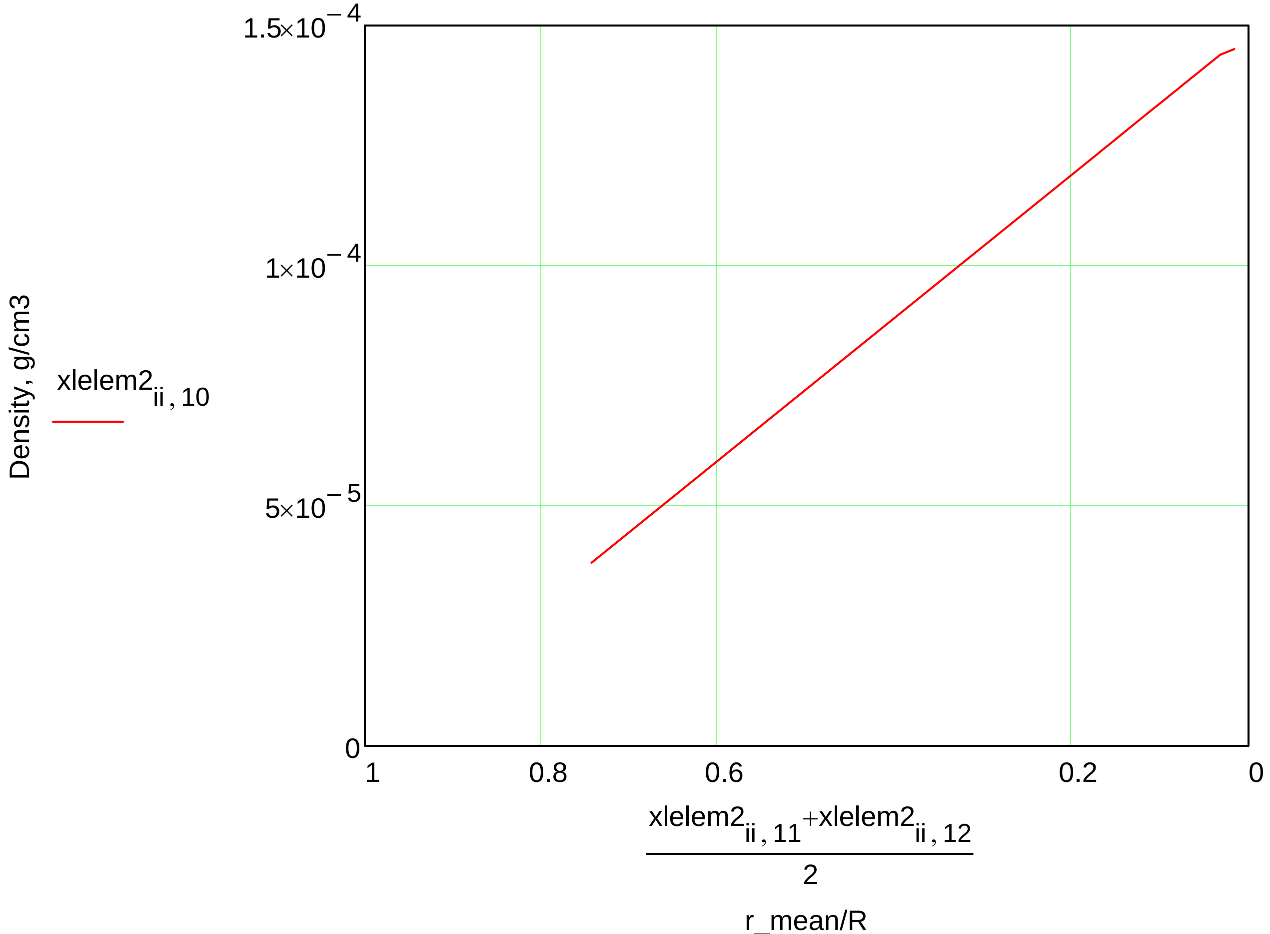


Figure 22. Density vs. r_{mean}/R for Epsilon Lep (Bi)

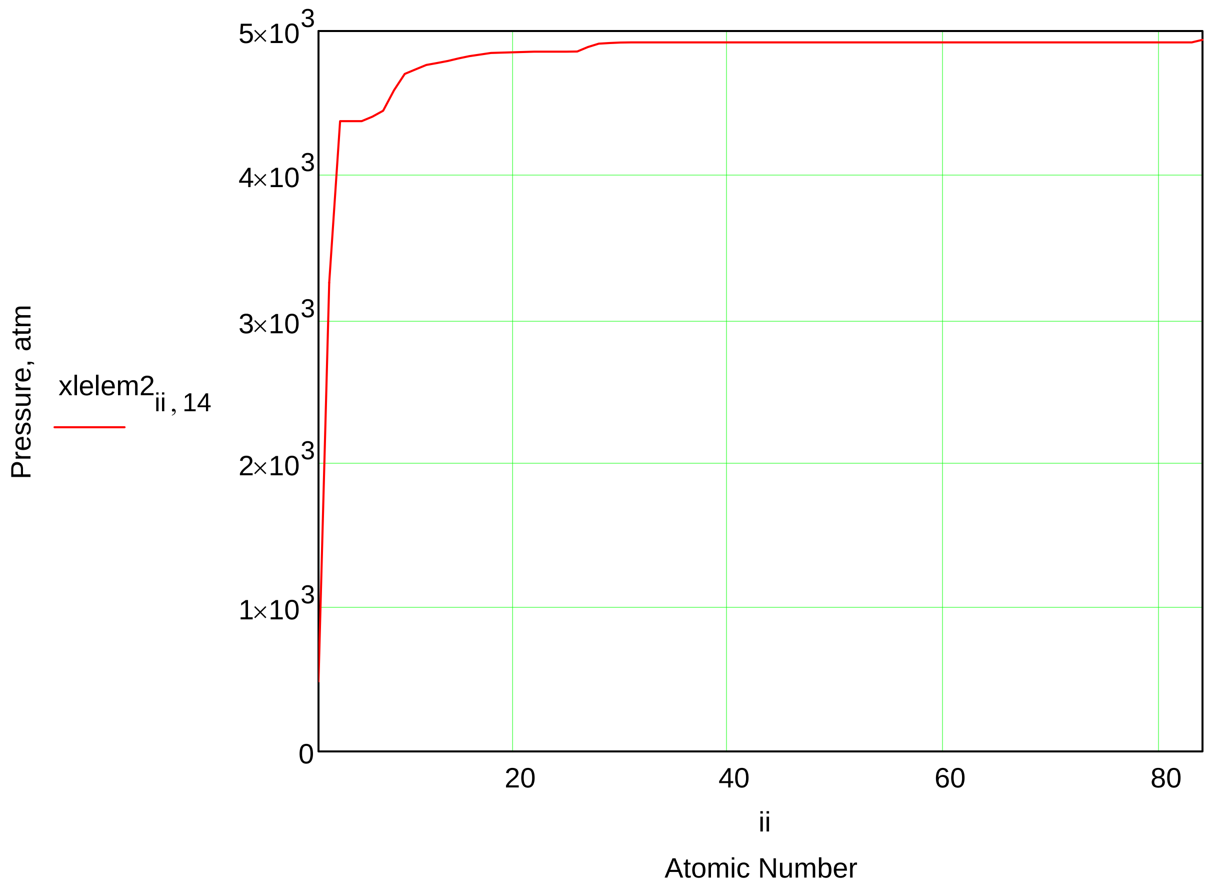


Figure 23. Pressure vs. Atomic Number for Epsilon Lep (Bi)

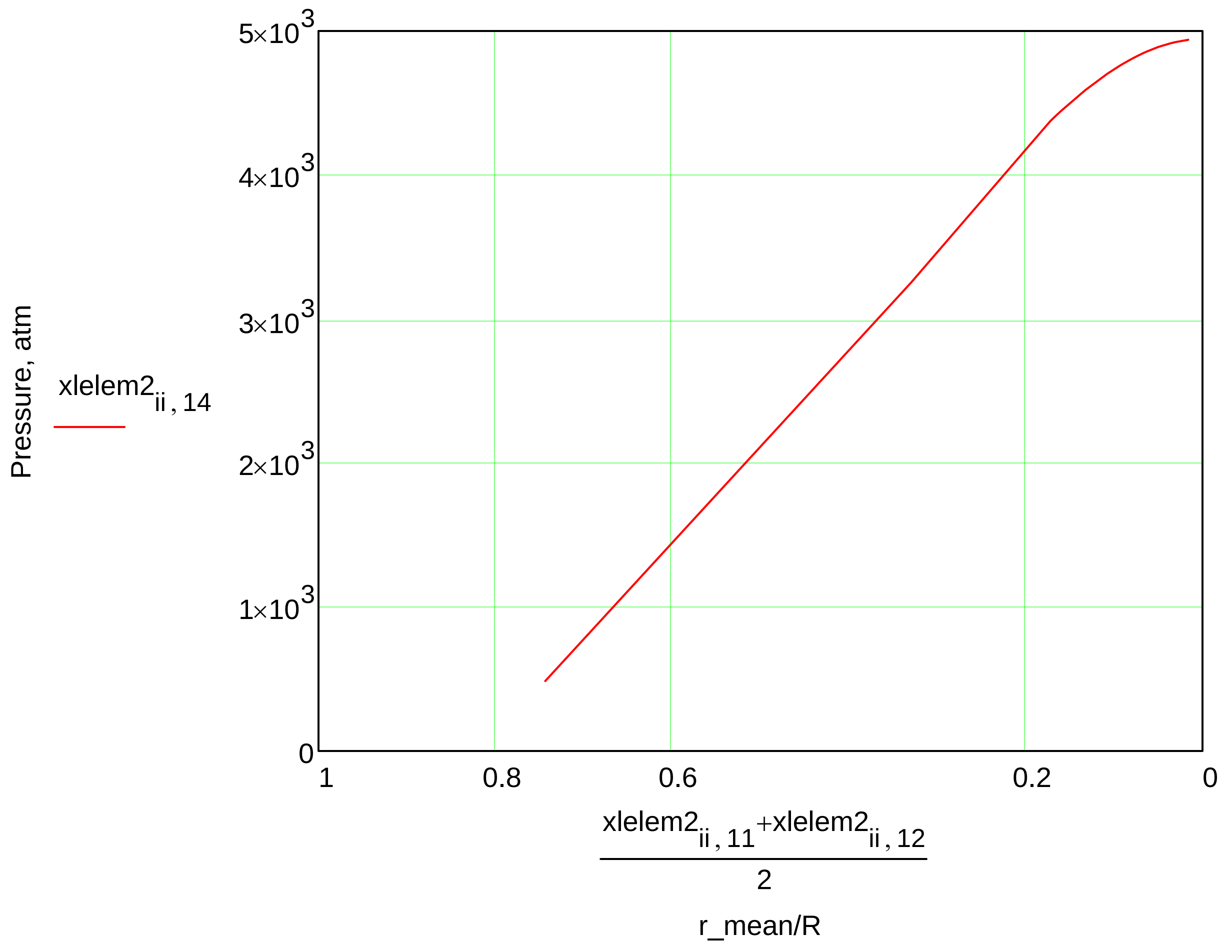


Figure 24. Pressure vs. r_{mean}/R for Epsilon Lep (Bi)

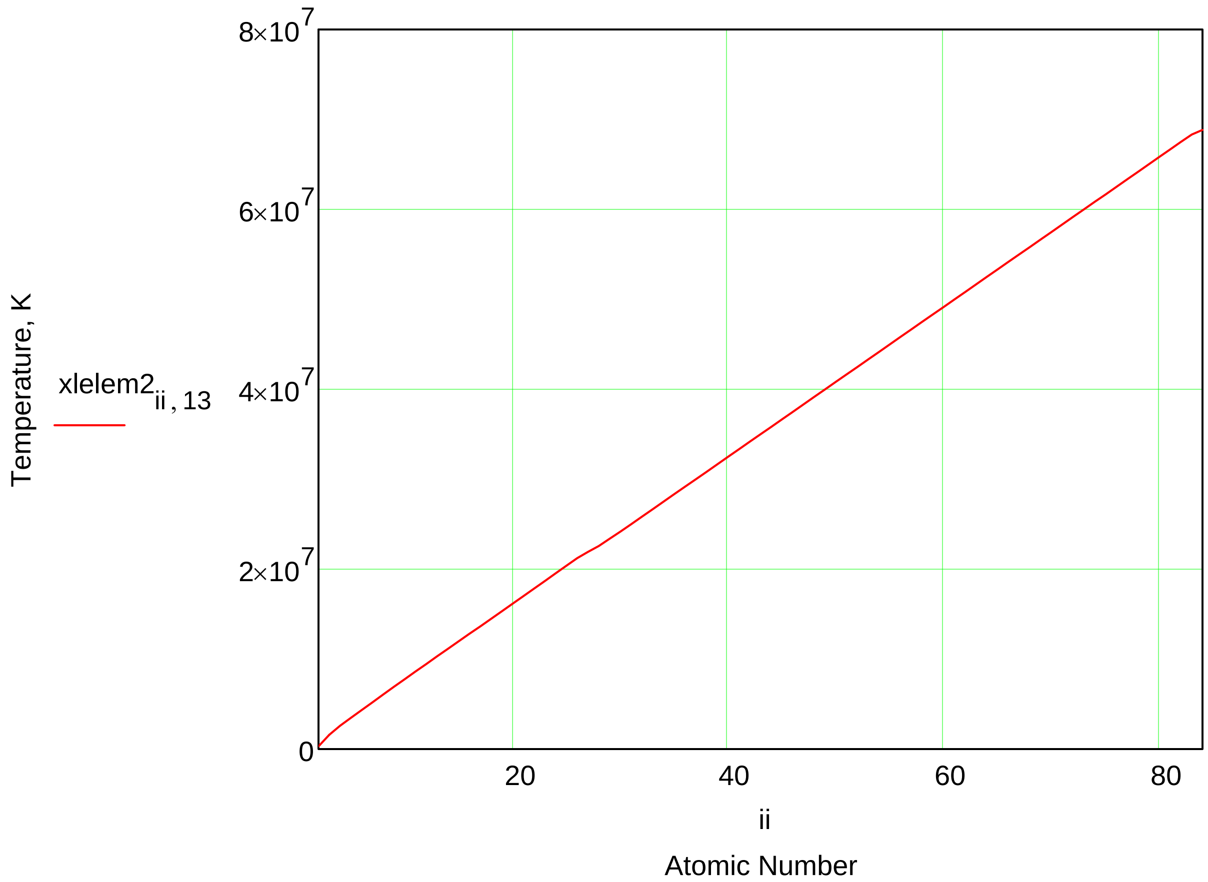


Figure 25. Temperature vs. Atomic Number for Epsilon Lep (Bi)

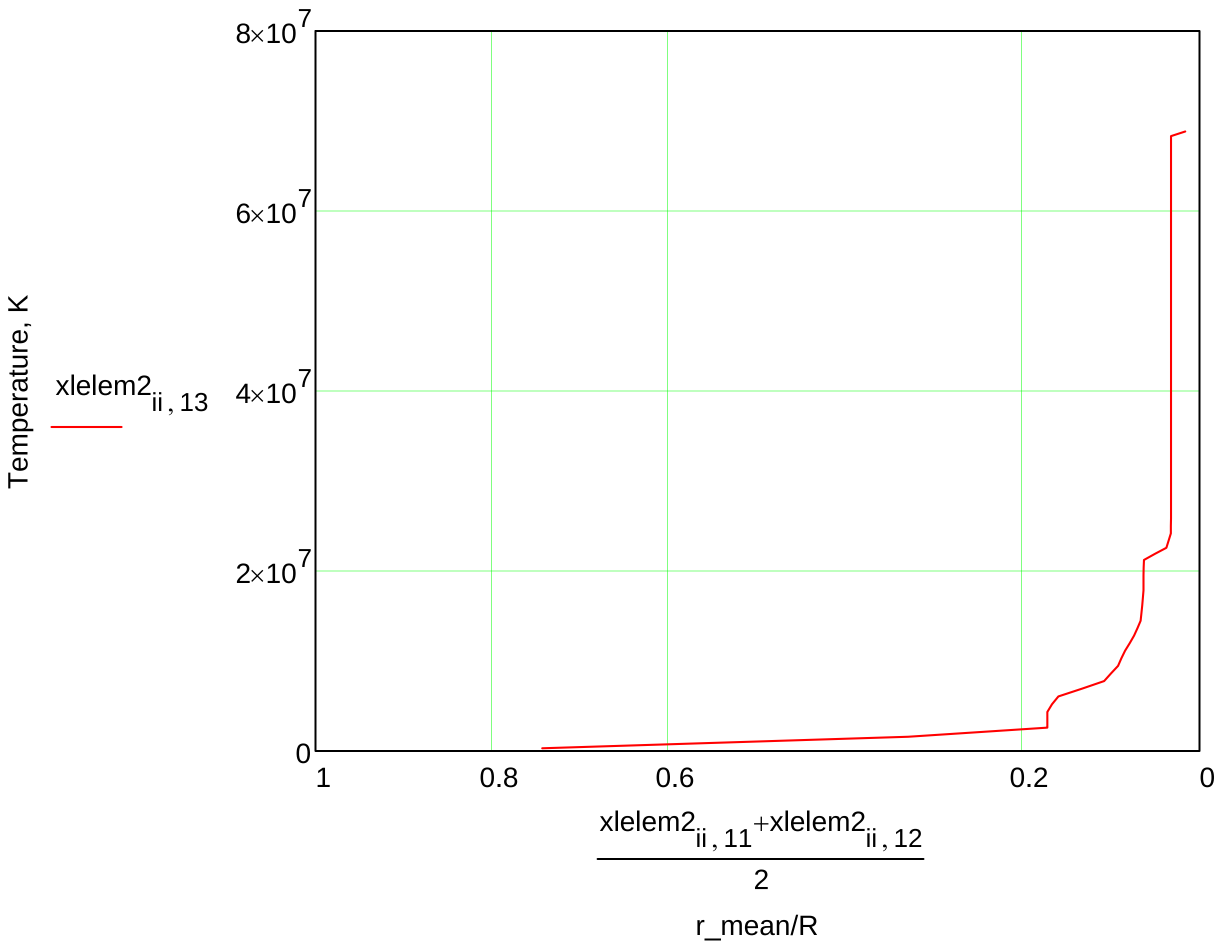


Figure 26. Temperature vs. r_{mean}/R for Epsilon Lep (Bi)

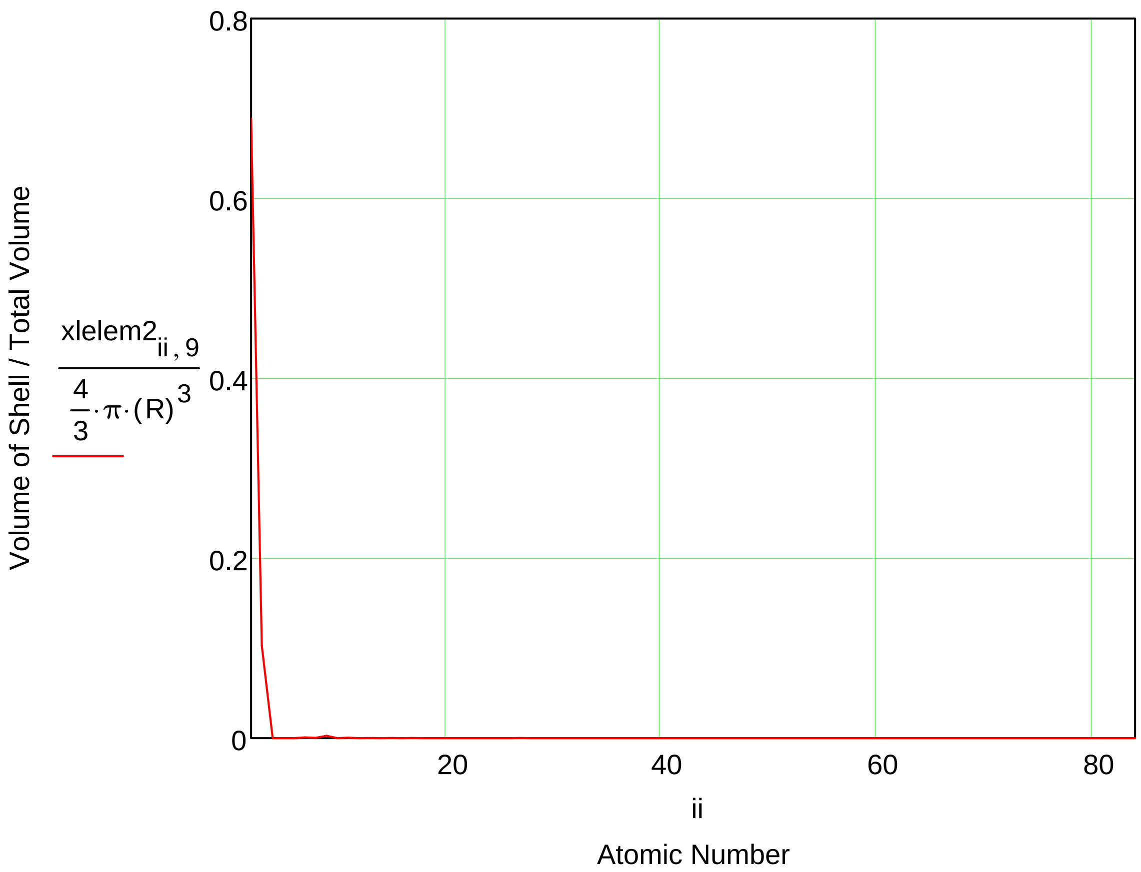


Figure 27. Volume of Shell / Total Volume vs. Atomic Number for Epsilon Lep (Bi)

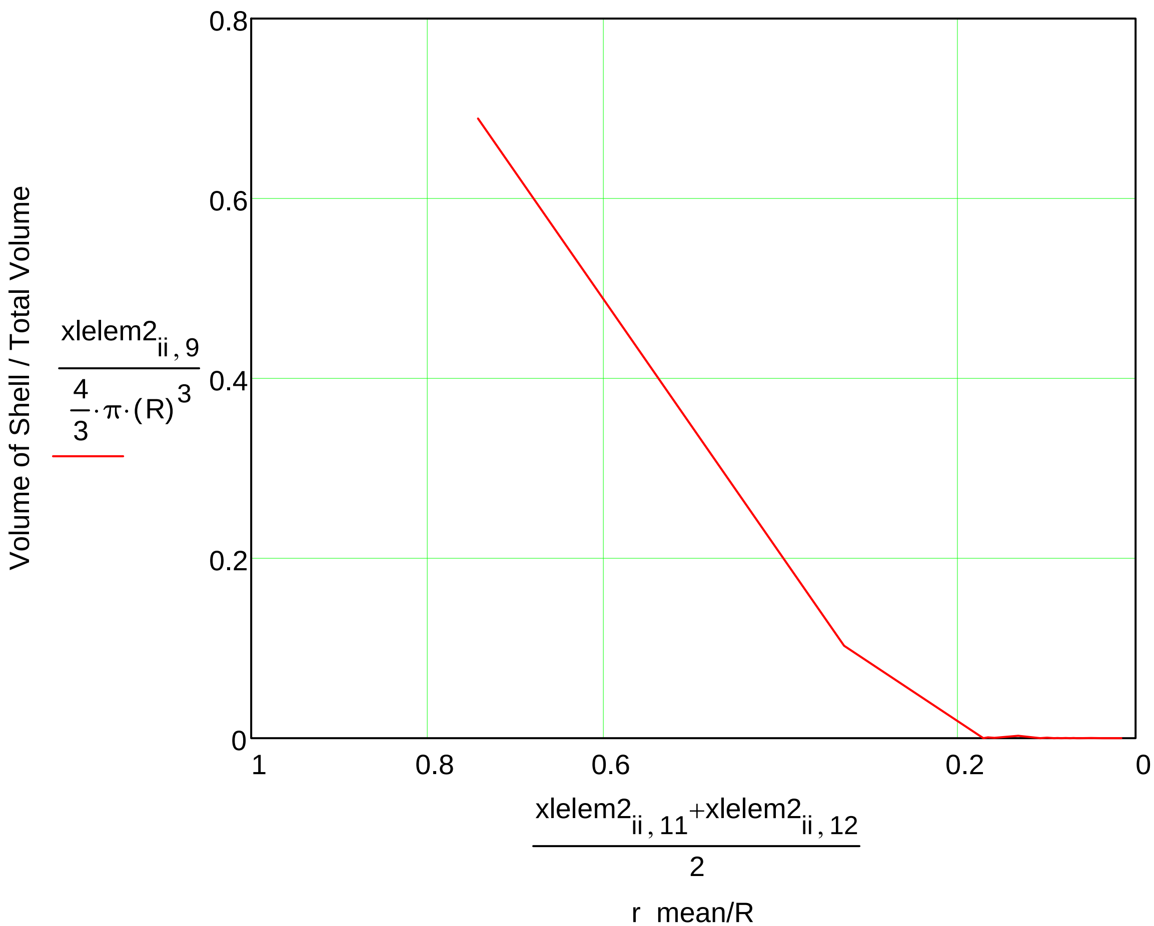


Figure 28. Volume of Shell / Total Volume vs. r_{mean}/R for Epsilon Lep (Bi)

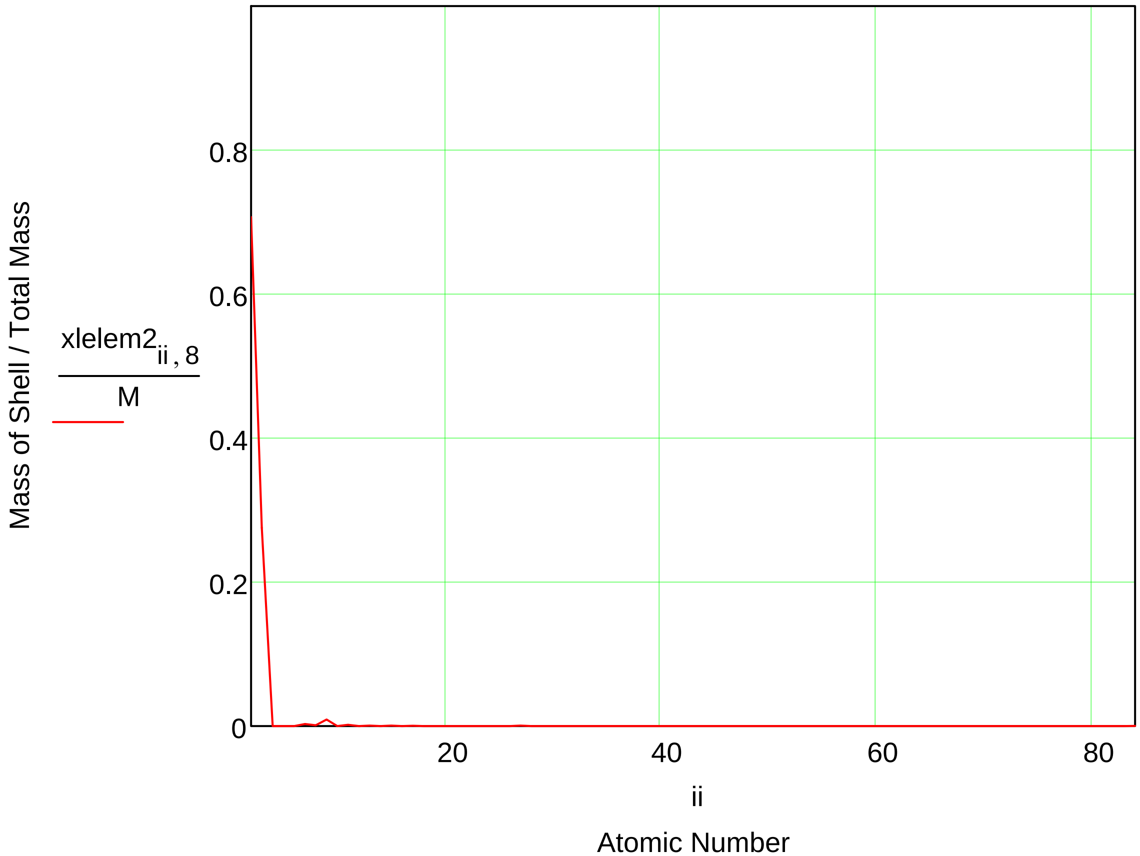


Figure 29. Mass of Shell / Total Mass vs. Atomic Number for Epsilon Lep (Bi)

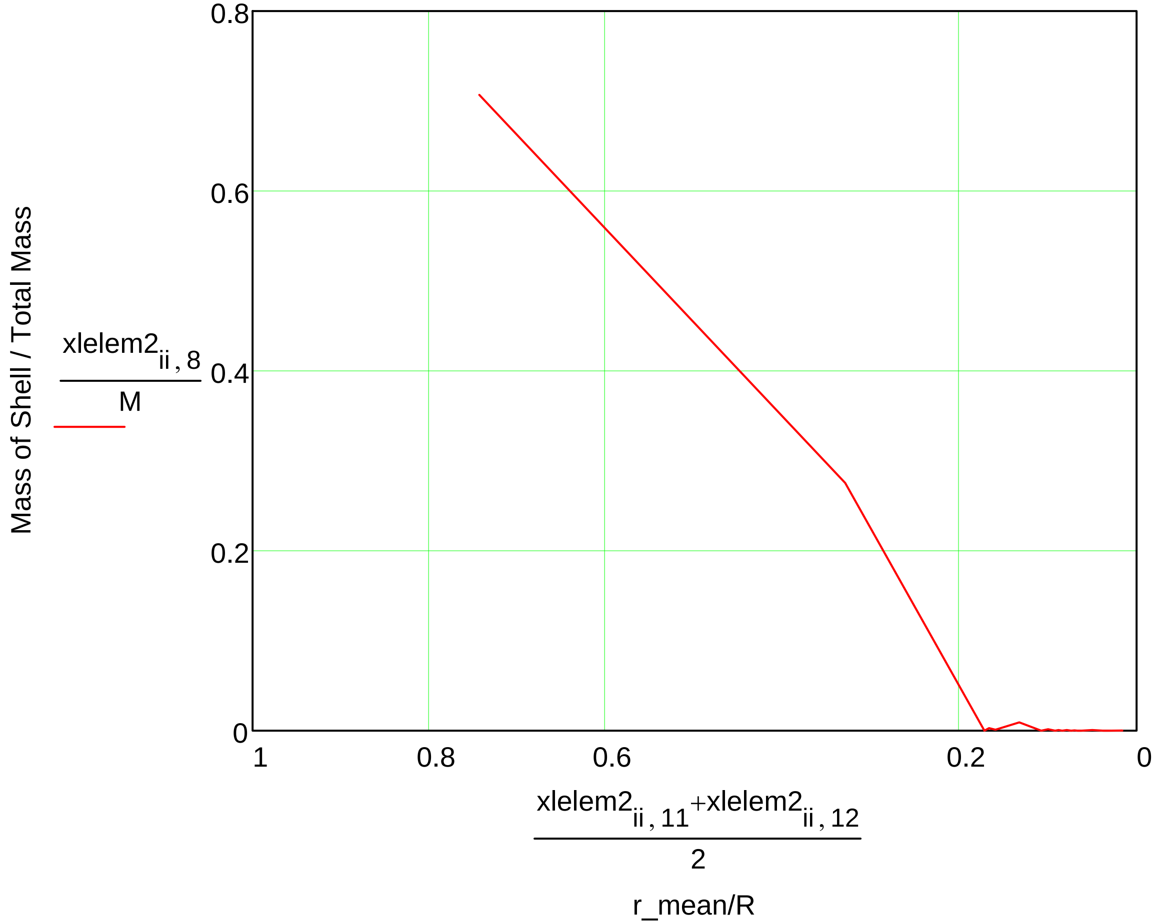


Figure 30. Mass of Shell / Total Mass vs. r_{mean}/R for Epsilon Lep (Bi)

end_elem := 82 (Pb, current fissioning element) metallicity := −0.02 I := 0 (assumed) adjFe := .6 (as before)

$M := 1.7 \cdot M_{S\_cgs}$ $R := 40.1 \cdot R_{S\_cgs}$

xlelem2 := r2_calc_loop_RG(R, M, end_elem, metallicity, adjFe, I)

The results are below.

xlelem2 =

	1	2
1	"Z"	"Element"
2	1	"H"
3	2	"He"
4	3	"Li"
5	4	"Be"
6	5	"B"
7	6	"C"
8	7	"N"
9	8	"O"
10	9	"F"
11	10	"Ne"
12	11	"Na"
13	12	"Mg"
14	13	"Al"
15	14	"Si"
16	15	...

$ii := 2 .. end\_elem + 1$ index for plots

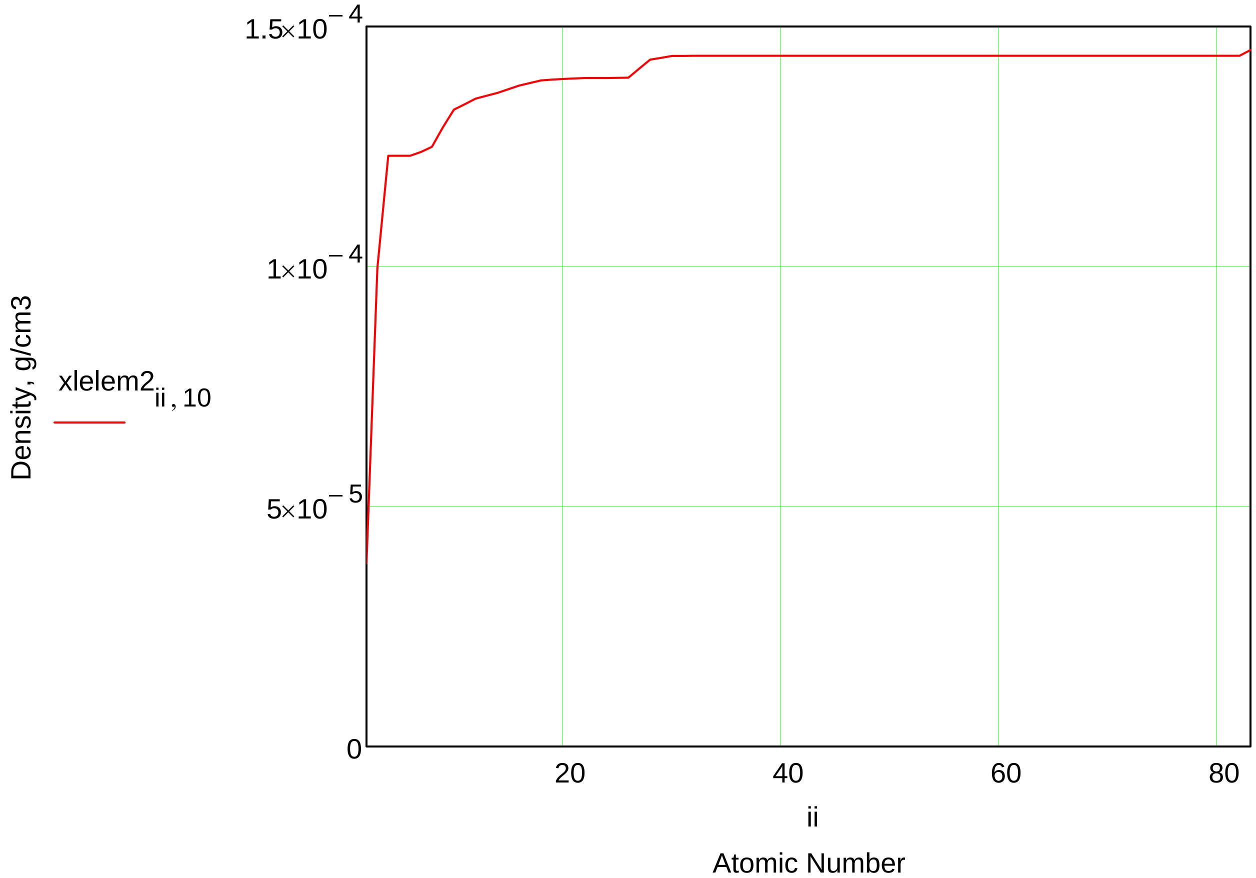


Figure 31. Density vs. Atomic Number for Epsilon Lep (Pb)

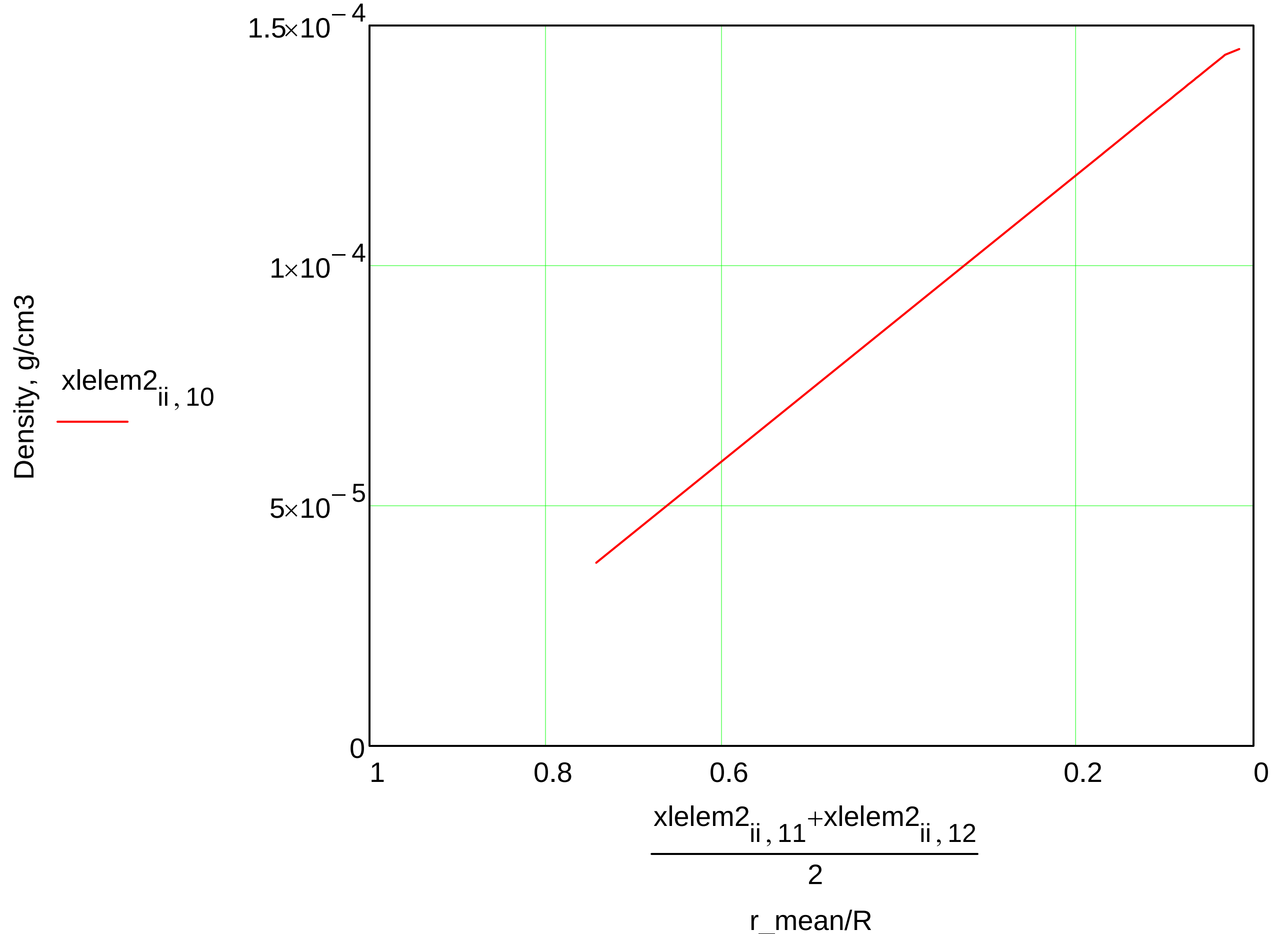


Figure 32. Density vs. r_{mean}/R for Epsilon Lep (Pb)

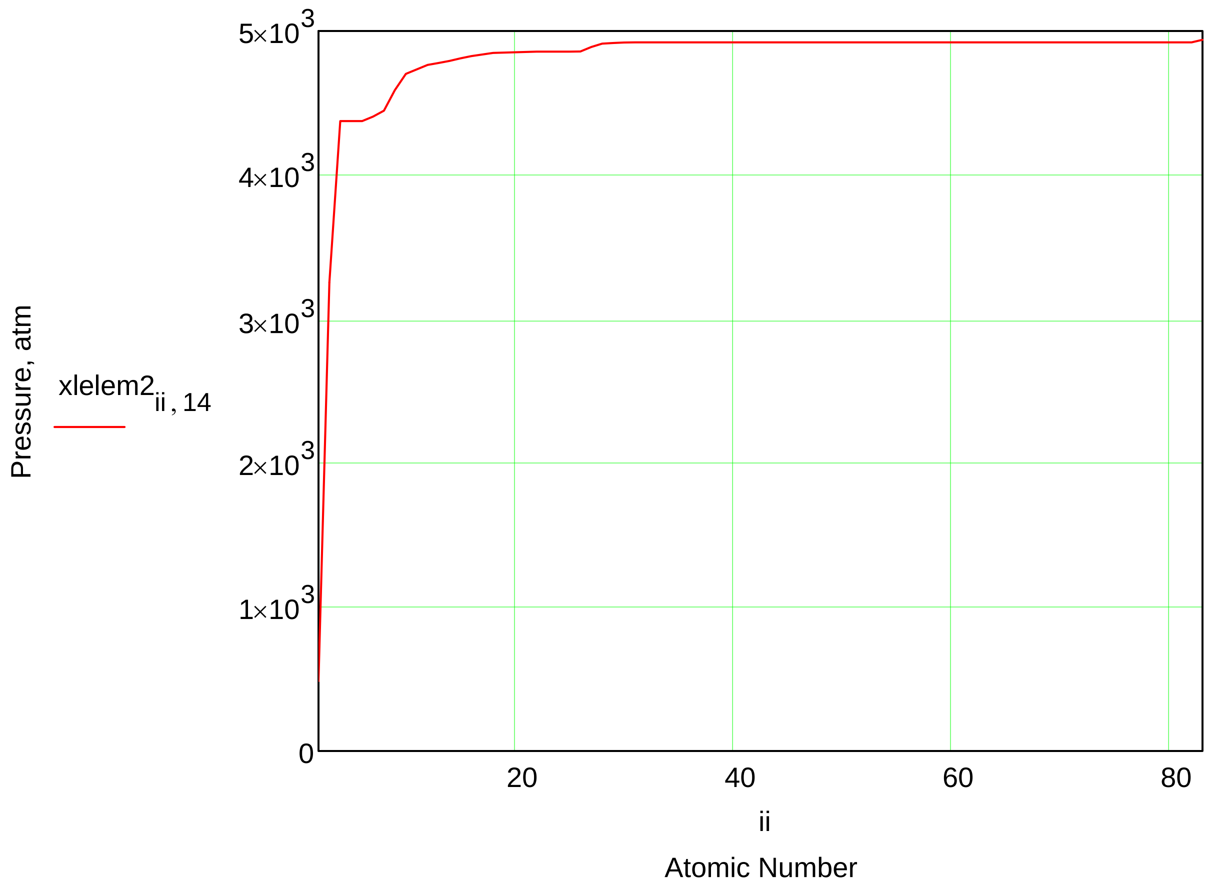


Figure 33. Pressure vs. Atomic Number for Epsilon Lep (Pb)

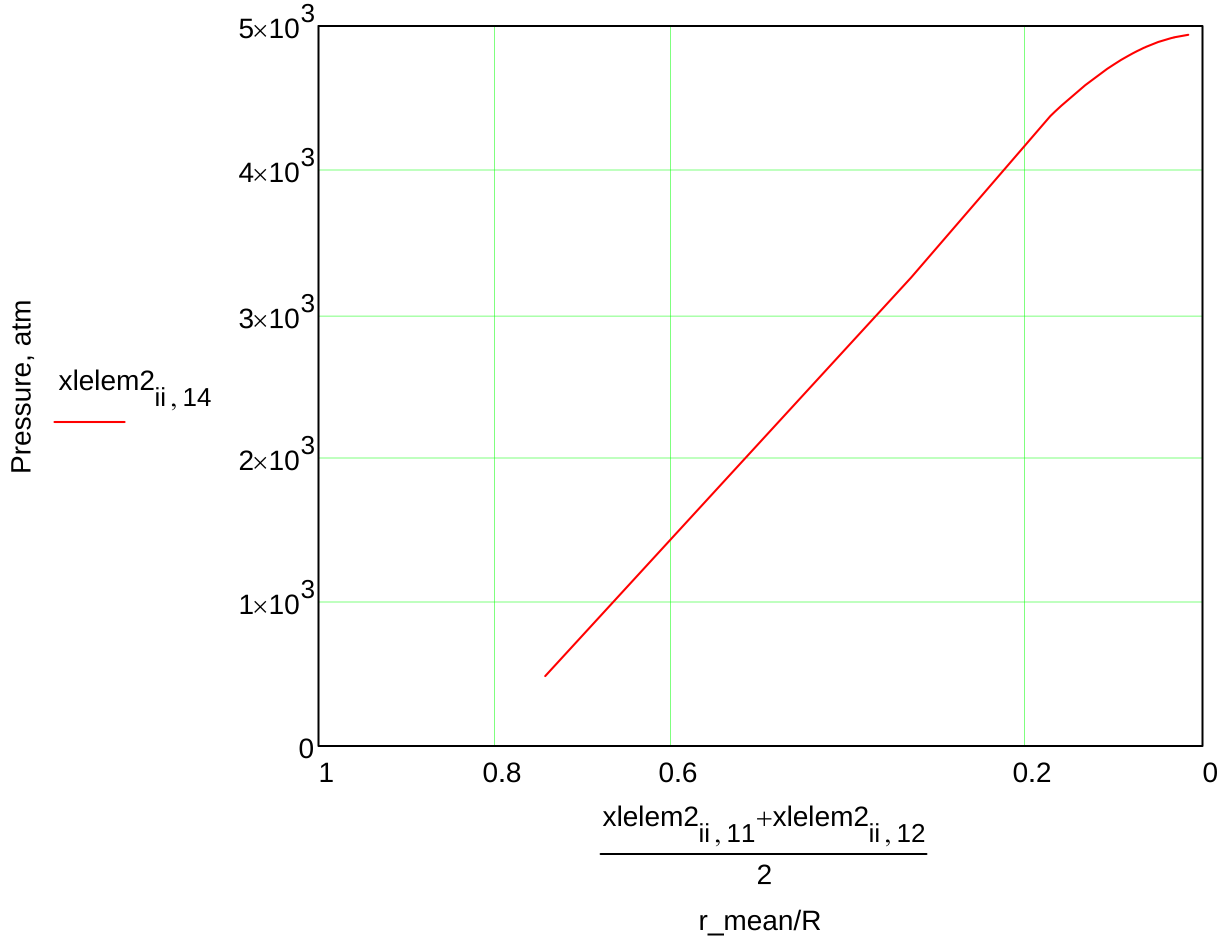


Figure 34. Pressure vs. r_{mean}/R for Epsilon Lep (Pb)

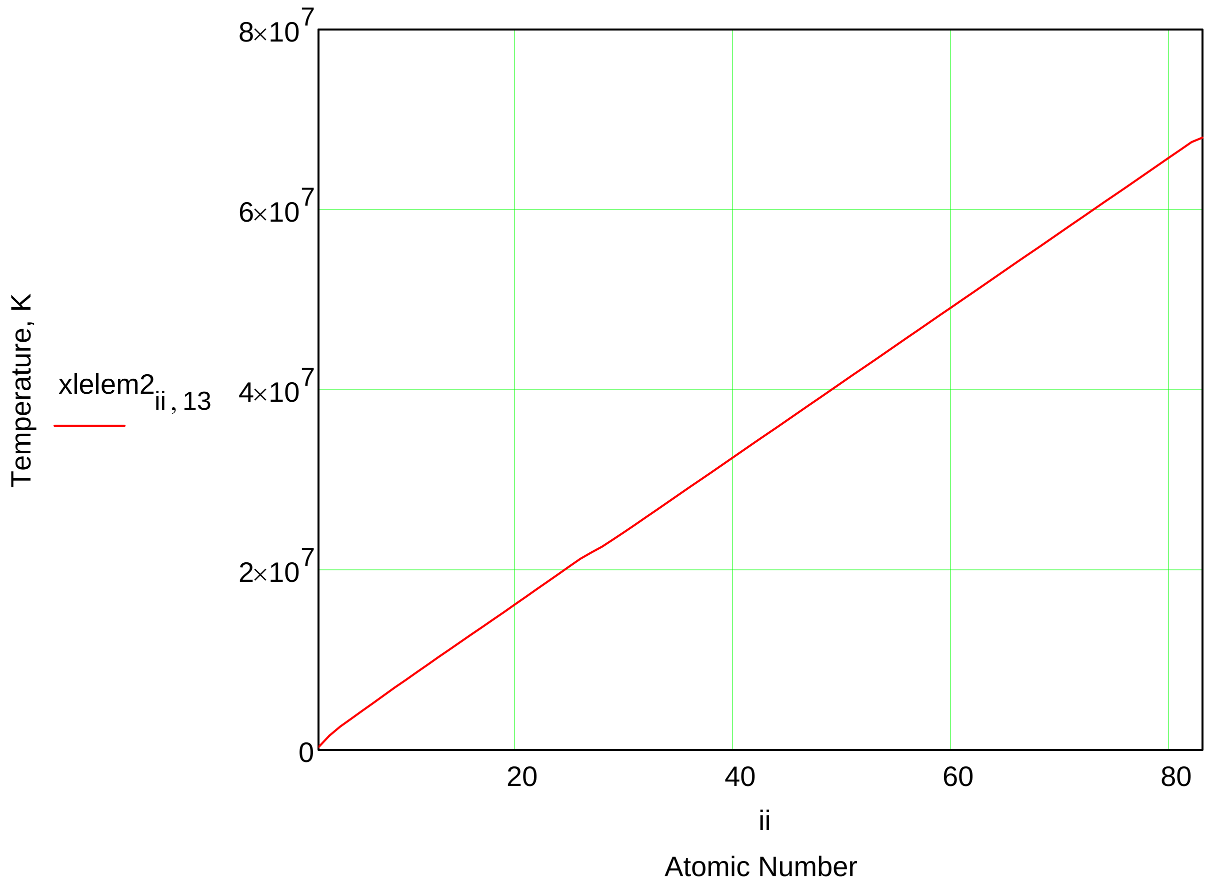


Figure 35. Temperature vs. Atomic Number for Epsilon Lep (Pb)

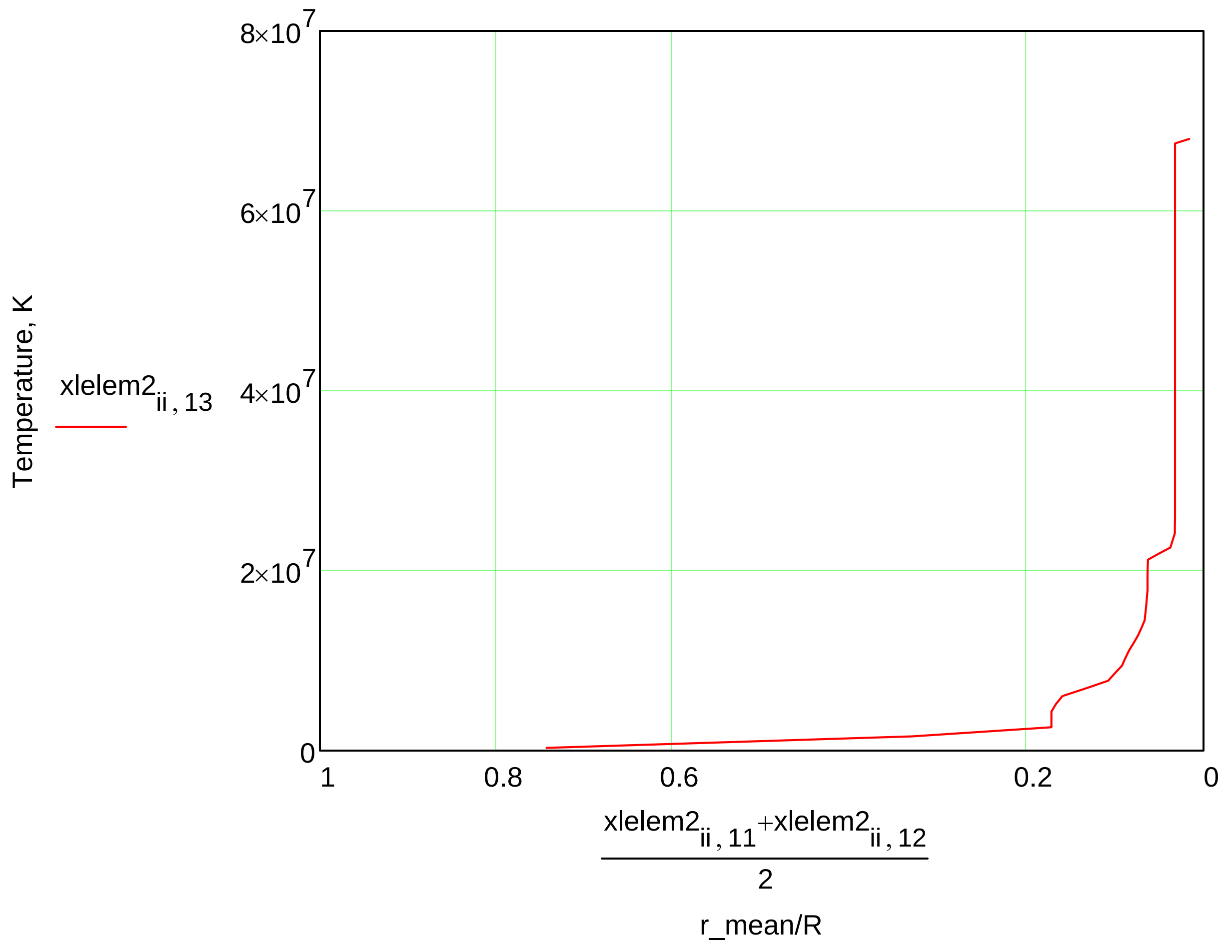


Figure 36. Temperature vs. r_{mean}/R for Epsilon Lep (Pb)

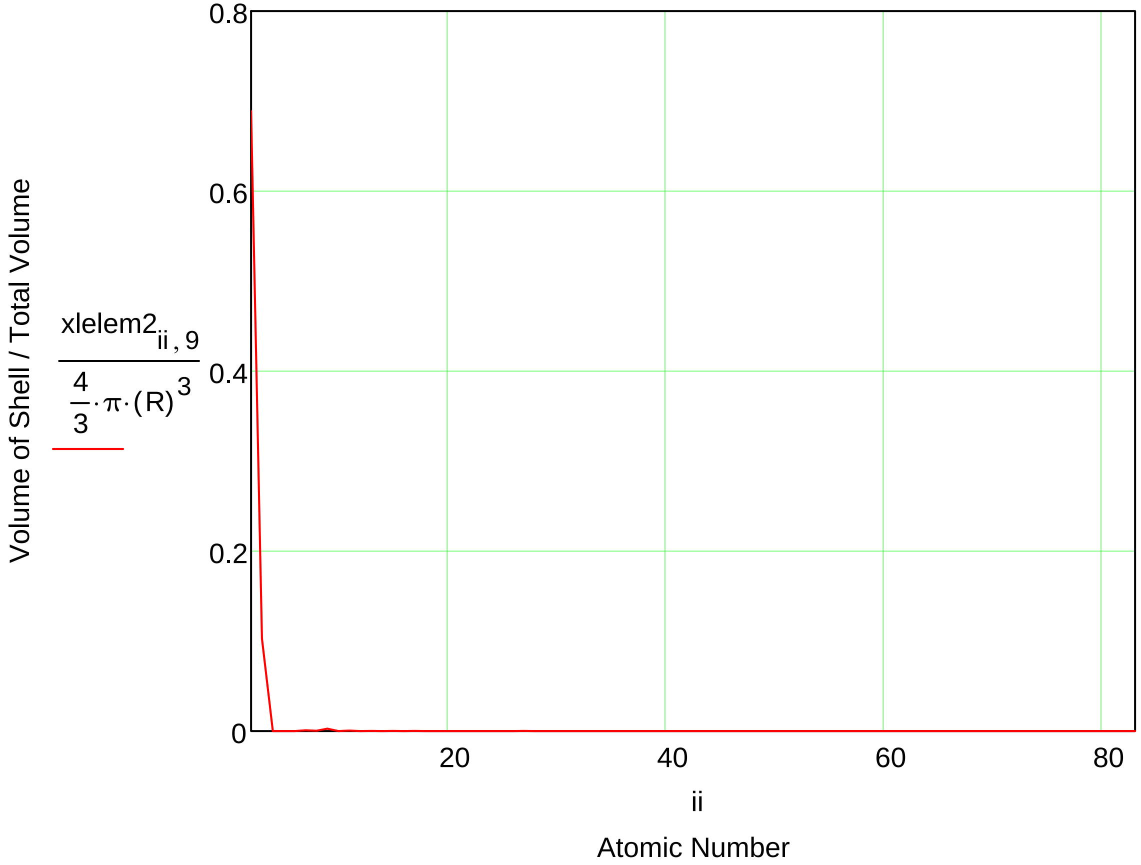


Figure 37. Volume of Shell / Total Volume vs. Atomic Number for Epsilon Lep (Pb)

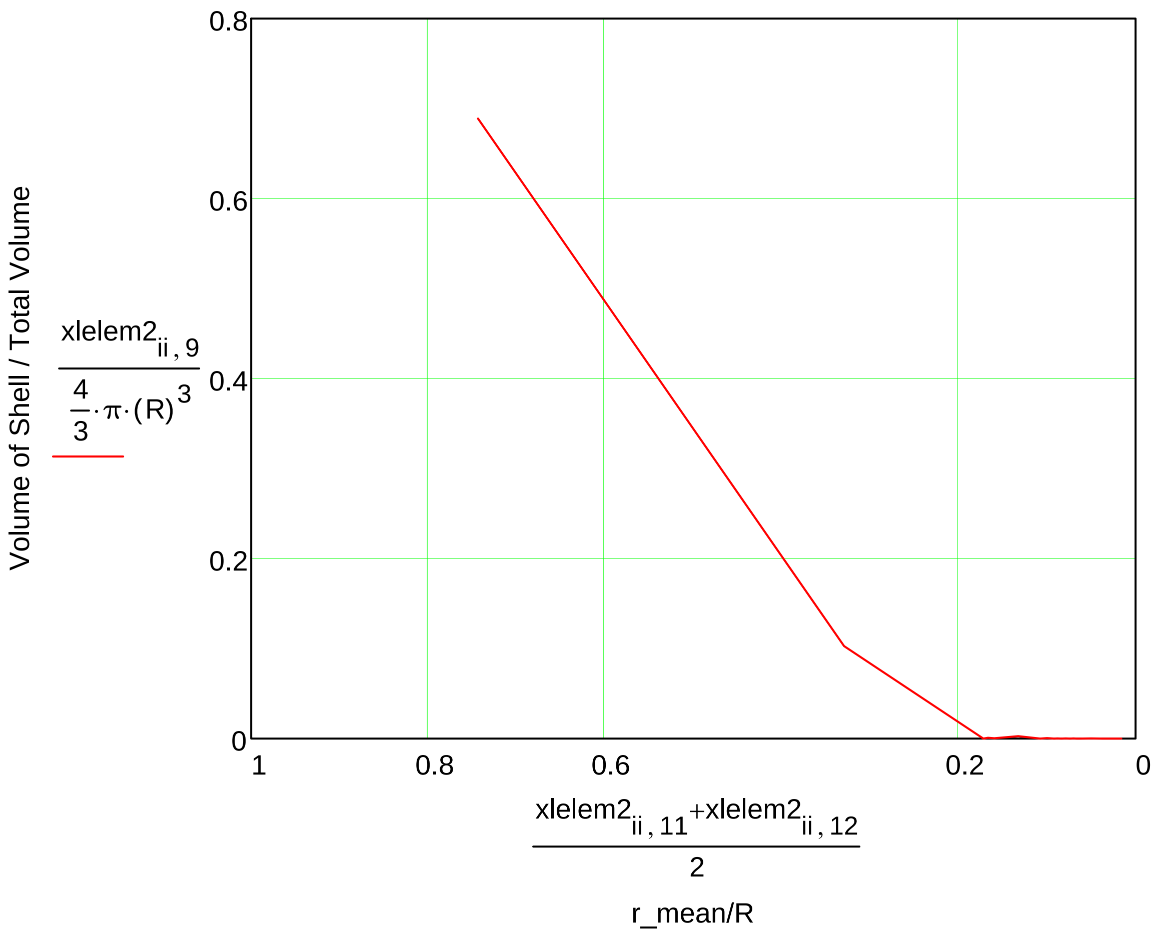


Figure 38. Volume of Shell / Total Volume vs. r_{mean}/R for Epsilon Lep (Pb)

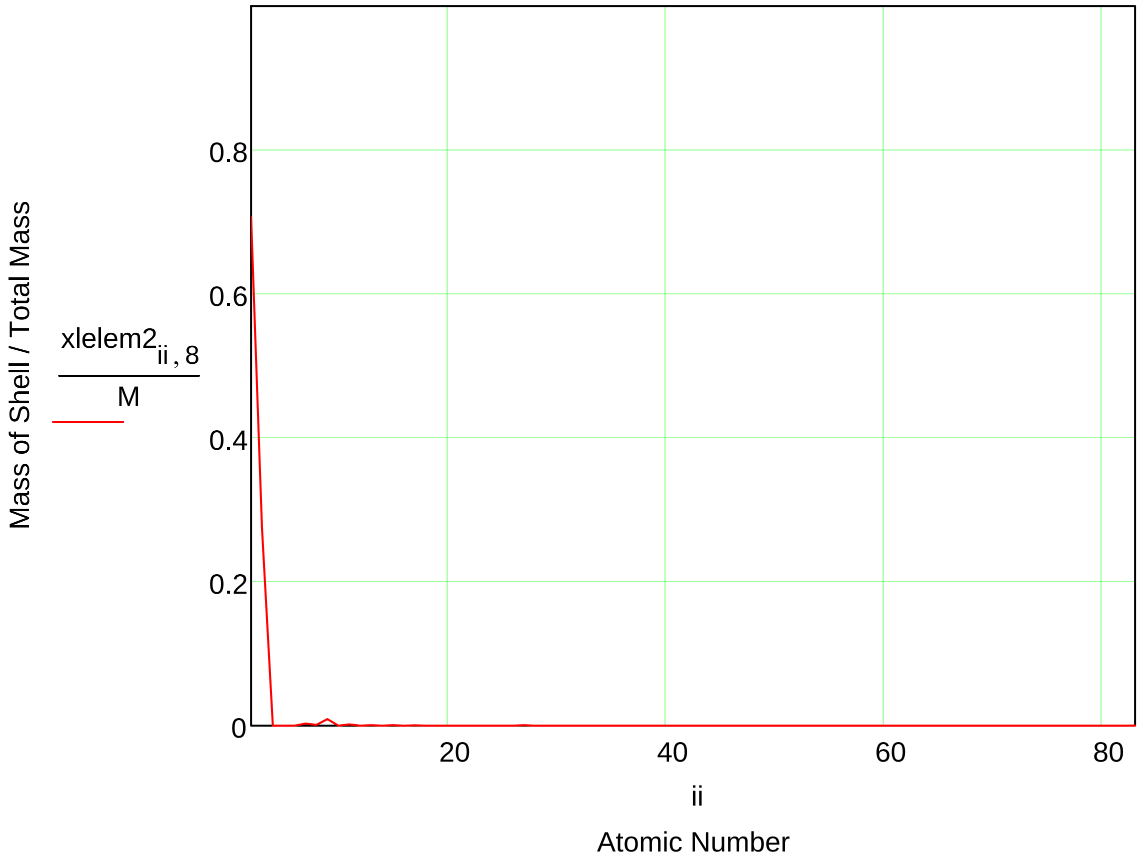


Figure 39. Mass of Shell / Total Mass vs. Atomic Number for Epsilon Lep (Pb)

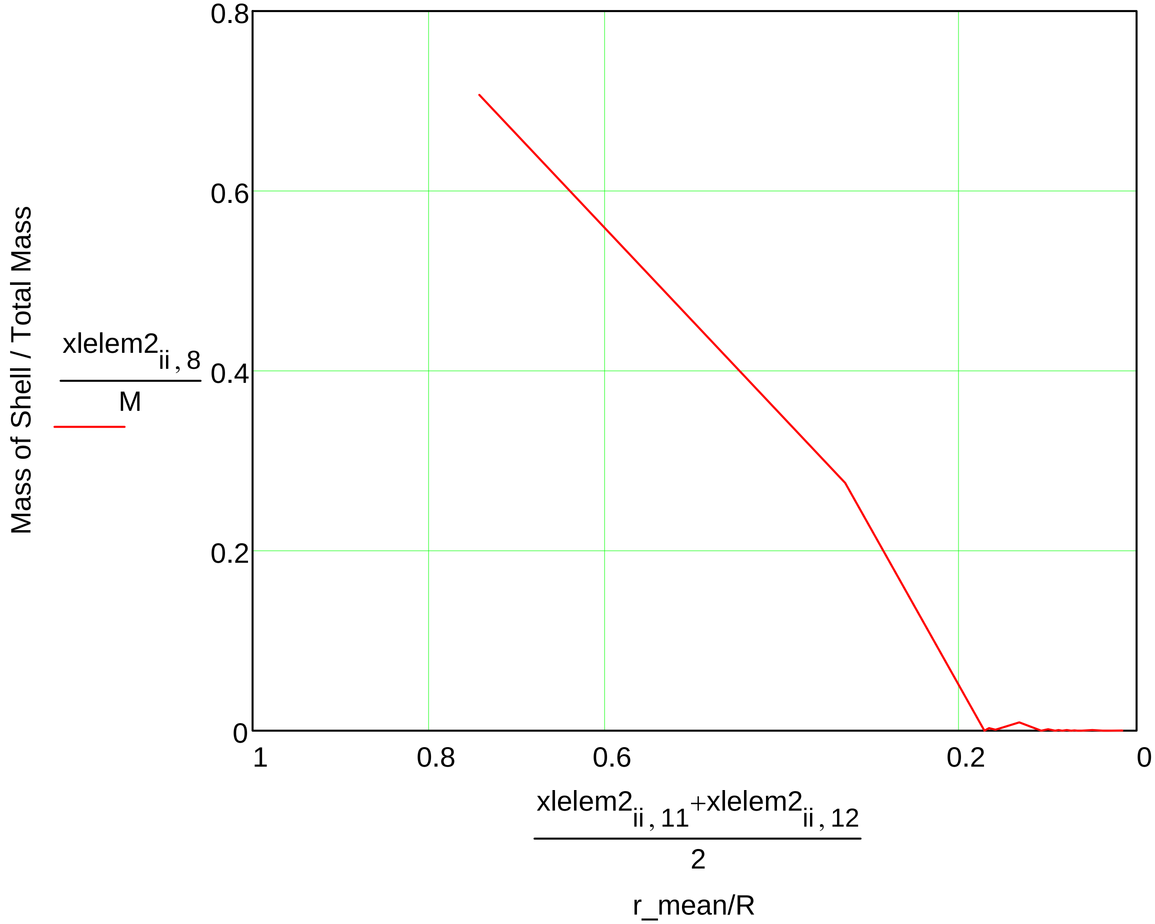


Figure 40. Mass of Shell / Total Mass vs. r_{mean}/R for Epsilon Lep (Pb)

From Ref. [1], p. 63: "The particular element that is involved cannot be positively identified without further investigation, but since lead is not only the first moderately abundant element in the descending order of atomic mass, but also the only such element in the upper portion of the atomic series, we may, at least tentatively, correlate the destructive thermal limit of this element number 82 with the central temperature corresponding to the mass range of the Cepheid zone." So many Cepheids will oscillate between element Bi and Pb as the fissioning element. However, there are many Cephids with a spectral class between F and K, so there are also oscillations in stars whose fissioning elements are lower than Pb.

Note that the return of White Dwarfs to the Main Sequence takes longer than Red Giants; their spectral class may thus only be in the M range when they have arrived at the Main Sequence (as in Proxima Centauri); Red Giants or Orange Giants may have a higher spectral class (such as K or G or F) before they reach the Main Sequence.

Epsilon Lep should theoretically have a partner star, a White Dwarf, or a retinue of planets--but there is no observation of this yet--so this should be considered to be a prediction.

The energy generation and luminosity equations for Red Giants are the same as those for Main Sequence stars and so need not be repeated here.

C. White Dwarfs / Pulsars

White Dwarfs result from Type I Supernovae, and Pulsars result from Type II Supernovae. By the Reciprocal Postulate, they are *inverse* to the nebulae expanding outward in space; instead they are *expanding outward in coordinate time*, which means *contracting inward in space* (until the maximum surface density is reached). White Dwarfs are *not* comprised of "degenerate matter" nor are Pulsars comprised of "neutrons." They are both comprised of regular atoms! The Reciprocal System atom is approximately the *same size* as the "nucleus" of the conventional atom.

Tentative Theorem 6: The density of White Dwarfs and Pulsars is *inverse* with distance from the *surface*.

The equation for density is then:

$$\rho(r) := \rho_{surf} \cdot \left(1 - \frac{r}{R}\right) \quad g/cm^3 \qquad (85)$$

(*here* r *starts at the surface, not at the center!*)

The mass of the star as a function of radius *r* is

$$M(r) := \int_0^r 4 \cdot \pi \cdot r^2 \cdot \rho(r)\, dr \qquad (86a)$$

Carrying out the integration:

$$M(r) := \int_0^r 4 \cdot \pi \cdot r^2 \cdot \left[\rho_{surf} \cdot \left(1 - \frac{r}{R}\right)\right] dr \qquad (86b)$$

$$M(r) := \frac{\pi \cdot r^3 \cdot \rho_{surf} \cdot (4 \cdot R - 3 \cdot r)}{3 \cdot R} \qquad (86c)$$

When r = R and M = total mass,

$$M(R) := \frac{\pi \cdot R^3 \cdot \rho_{surf} \cdot (R)}{3 \cdot R} \qquad (87a)$$

or

$$M := \frac{\pi \cdot R^3 \cdot \rho_{surf}}{3} \qquad (87b)$$

Solving for ρ_{surf}:

$$\rho_{surf} := \frac{3 \cdot M}{\pi \cdot R^3} \qquad (88)$$

Now, what about temperature? White Dwarfs and Pulsars have *no internal source* of energy generation! They are *cooling* only! Also, they are so compact, that we can assume that the temperature is approximately *constant* throughout the star; of course, there must be a small temperature gradient, because the stars are radiating and thus losing energy at the surface, which is made up by thermal motion coming from below--but we will neglect this.

Tentative Theorem 7: The temperature of White Dwarfs and Pulsars is very roughly constant from surface to center at any point in time. The thermal current is carried by massless, chargeless electrons which are able to go easily from atom to atom.

(Of course, White Dwarfs gradually cool and eventually reach the Main Sequence, when their density relation *flips* to that of Main Sequence stars.)

We can make use of the same temperature relation we used for Main Sequence stars.

$$T := \left(48.45665552 \cdot \frac{1}{\rho} \cdot w + 0.55041157 \cdot P \cdot \frac{1}{\rho} \cdot w^{\frac{1}{3}}\right)$$

Solving for P and putting into functional form:

$$P(w, T, \rho) := -\frac{1.6 \cdot 10^{-27} \cdot \left(5.5023207 \cdot 10^{28} \cdot w - 1.1355139 \cdot 10^{27} \cdot T \cdot \rho\right)}{w^{\frac{1}{3}}} \qquad (89)$$

Eq. (75) has to be reformulated for White Dwarfs and Pulsars.

$$M(r) := \frac{4}{3}\cdot\pi\cdot\rho_{surf}\cdot\left(R - r_1\right)^3\cdot\left[1 - \frac{3}{4}\cdot\frac{(R - r)}{R}\right] \quad (90a)$$

$$M\left(r_1\right) := \frac{4}{3}\cdot\pi\cdot\rho_{surf}\cdot\left(R - r_1\right)^3\cdot\left[1 - \frac{3}{4}\cdot\frac{\left(R - r_1\right)}{R}\right] \quad (90b)$$

$$M\left(r_2\right) := \frac{4}{3}\cdot\pi\cdot\rho_{surf}\cdot\left(R - r_2\right)^3\cdot\left[1 - \frac{3}{4}\cdot\frac{\left(R - r_2\right)}{R}\right] \quad (90c)$$

Then, for each shell:

$$\Delta M\_el := \frac{4}{3}\cdot\left(\pi\cdot\rho_{surf}\right)\cdot\left[\left(R - r_2\right)^3\cdot\left[1 - \frac{3}{4}\cdot\frac{\left(R - r_2\right)}{R}\right] - \left(R - r_1\right)^3\cdot\left[1 - \frac{3}{4}\cdot\frac{\left(R - r_1\right)}{R}\right]\right] \quad (91)$$

$$\Delta M\_el = \frac{4}{3}\cdot\left(\pi\cdot\rho_{surf}\right)\cdot\left[\left(R - r_1\right)^3\cdot\left[1 - \frac{3}{4}\cdot\frac{\left(R - r_1\right)}{R}\right] - \left(R - r_2\right)^3\cdot\left[1 - \frac{3}{4}\cdot\frac{\left(R - r_2\right)}{R}\right]\right]$$

To use use the r2_calc equation, we must call it with $(R - r_1)$ instead of R. The result then must be subtracted from R.

$$r2\_calc\_loop\_WD(R, M, end\_elem, metallicity, adjFe, I, T) :=$$

$$xlelem2 \leftarrow xlelem$$

$$mass\_rel\_to\_Sun \leftarrow \frac{M}{M_{S\_cgs}}$$

$$mH \leftarrow (xlelem2_{2,4}) \cdot mass\_rel\_to\_Sun$$

$$mHe \leftarrow (xlelem2_{3,4}) \cdot mass\_rel\_to\_Sun$$

$$factormet \leftarrow \frac{\left(1 - \frac{1}{10^{metallicity} \cdot .019288 + 1}\right)}{.018923}$$

$$factorHplusHe \leftarrow \frac{\frac{1}{10^{metallicity} \cdot .019288 + 1}}{.981077}$$

$$\rho_{surf} \leftarrow \frac{3 \cdot M}{\pi \cdot R^3}$$

$$G \leftarrow 6.67259 \cdot 10^{-8}$$

$$conv_{dynescm2toatm} \leftarrow 9.8716683 \cdot 10^{-7}$$

$$mass\_tot \leftarrow 0$$

for $i \in end\_elem + 1 .. 3$

$$r_1 \leftarrow 0 \quad \text{if} \quad i = end\_elem + 1$$

$$\Delta M\_actual \leftarrow xlelem2_{i,4}$$

$$\Delta M\_actual \leftarrow \Delta M\_actual \cdot mass\_rel\_to\_Sun \cdot factormet \quad i$$

$$\Delta M\_actual \leftarrow mHe \cdot factorHplusHe \quad \text{if} \quad i = 3$$

$$\Delta M\_actual \leftarrow adjFe \cdot \Delta M\_actual \quad \text{if} \quad i = 27$$

[illegible]

$r_2 \leftarrow r_{2\_calc}(R, R - r_1, \Delta M\_actual, \rho_{surf})$

$r_2 \leftarrow \sqrt{r_2^{\,2}}$

$r_2 \leftarrow R - r_2$

$r_{mean} \leftarrow .5 \cdot (r_1 + r_2)$

$\rho_{current} \leftarrow \rho_{surf} \cdot \left(1 - \frac{r_{mean}}{R}\right)$

$xlelem2_{i,5} \leftarrow \Delta M\_actual$

$xlelem2_{i,6} \leftarrow r_1$

$xlelem2_{i,7} \leftarrow r_2$

$xlelem2_{i,8} \leftarrow \Delta M\_actual$

$xlelem2_{i,9} \leftarrow \frac{\Delta M\_actual}{\rho_{current}}$

$xlelem2_{i,10} \leftarrow \rho_{current}$

$w \leftarrow 2 \cdot (i - 1) \cdot conv_{amu\_to\_u} \quad \text{if} \quad l = 0$

$w \leftarrow xlelem2_{i,3} \quad \text{if} \quad l = 1$

$w \leftarrow \left[2 \cdot (i - 1) + \frac{(i - 1)^2}{l_R}\right] \cdot conv_{amu\_to\_u} \quad \text{if} \quad l > 1$

$P_{mean} \leftarrow -\frac{1.6 \cdot 10^{-27} \cdot \left(5.5023207 \cdot 10^{28} \cdot w - 1.1355139\right.}{w^{\frac{1}{3}}}$

$T_{mean} \leftarrow T$

$xlelem2_{i,11} \leftarrow \frac{r_1}{R}$

r_2

$$xlelem2_{i,12} \leftarrow \frac{r_2}{R}$$

$$xlelem2_{i,13} \leftarrow T_{mean}$$

$$xlelem2_{i,14} \leftarrow P_{mean}$$

$$r_1 \leftarrow r_2$$

$$\rho_H \leftarrow xlelem2_{i,10} \quad \text{if} \quad i = 2$$

$$\rho_{prev} \leftarrow xlelem2_{i,10}$$

$$mass\_tot \leftarrow mass\_tot + \Delta M\_actual$$

$$mass\_remaining \leftarrow M - mass\_tot$$

$$j \leftarrow 2$$

$$r_2 \leftarrow R$$

$$xlelem2_{j,5} \leftarrow mass\_remaining$$

$$xlelem2_{j,6} \leftarrow r_1$$

$$xlelem2_{j,7} \leftarrow r_2$$

$$r_{mean} \leftarrow .5 \cdot (r_1 + r_2)$$

$$\rho_{current} \leftarrow \rho_{surf} \cdot \left(1 - \frac{r_{mean}}{R}\right)$$

$$xlelem2_{j,8} \leftarrow xlelem2_{j,5}$$

$$xlelem2_{j,9} \leftarrow \frac{xlelem2_{j,5}}{\rho_{current}}$$

$$xlelem2_{j,10} \leftarrow \frac{mass\_remaining}{xlelem2_{j,9}}$$

$$w \leftarrow 2 \cdot (j - 1) \cdot conv_{amu\_to\_u} \quad \text{if} \quad I = 0$$

$$w \leftarrow xlelem2_{j,3} \quad \text{if} \quad I = 1$$

$$w \leftarrow \left\lfloor 2\cdot(j-1) + \frac{(j-1)^2}{I_R} \right\rfloor \cdot conv_{amu\_to\_u} \quad \text{if} \quad I > 1$$

$$P_{mean} \leftarrow -\frac{1.6\cdot 10^{-27}\cdot\left(5.5023207\cdot 10^{28}\cdot w - 1.1355139\cdot 10^{2}\right.}{w^{\frac{1}{3}}}$$

$$T_{mean} \leftarrow T$$

$$xlelem2_{j,11} \leftarrow \frac{xlelem2_{j,6}}{R}$$

$$xlelem2_{j,12} \leftarrow 0$$

$$xlelem2_{j,13} \leftarrow T$$

$$xlelem2_{j,14} \leftarrow P_{mean}$$

$$xlelem2$$

worked example: Sirius B (spectral class DA2, cycle 2D)

$T := 30000$ K $end\_elem := 27$ (Co) $adjFe := .71$ $metallicity := .5$ $I := 0$

$M := .978 \cdot M_{S\_cgs}$ $R := .0084 \cdot R_{S\_cgs}$

xlelem2 := r2_calc_loop_WD(R, M, end_elem, metallicity, adjFe, I, T)

xlelem2 =

	1	2
1	"Z"	"Element"
2	1	"H"
3	2	"He"
4	3	"Li"
5	4	"Be"
6	5	"B"
7	6	"C"
8	7	"N"
9	8	"O"
10	9	"F"
11	10	"Ne"
12	11	"Na"
13	12	"Mg"
14	13	"Al"
15	14	"Si"
16	15	...

$ii := end\_elem + 1 .. 2$ index for plots

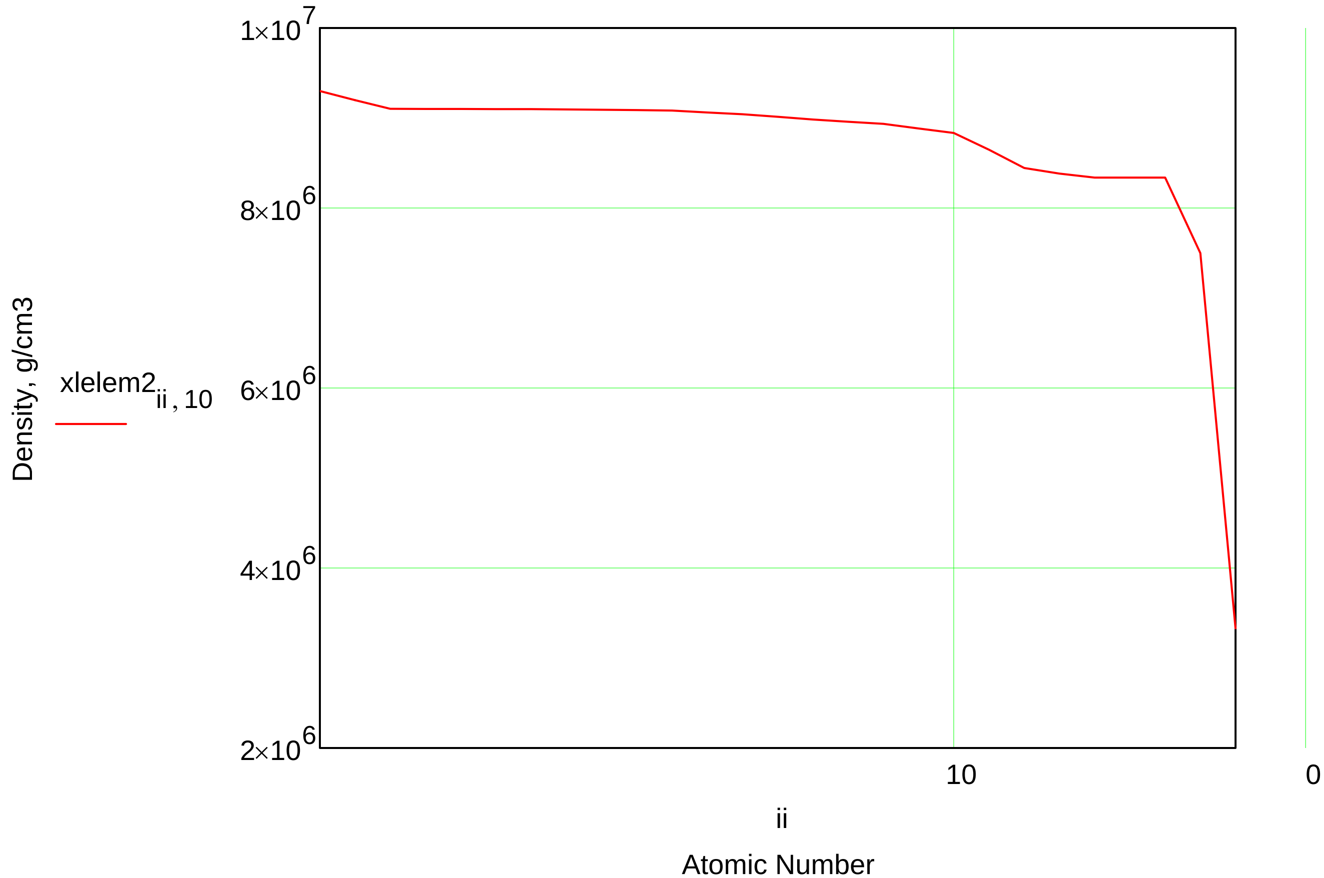


Figure 41. Density vs. Atomic Number for Sirius B

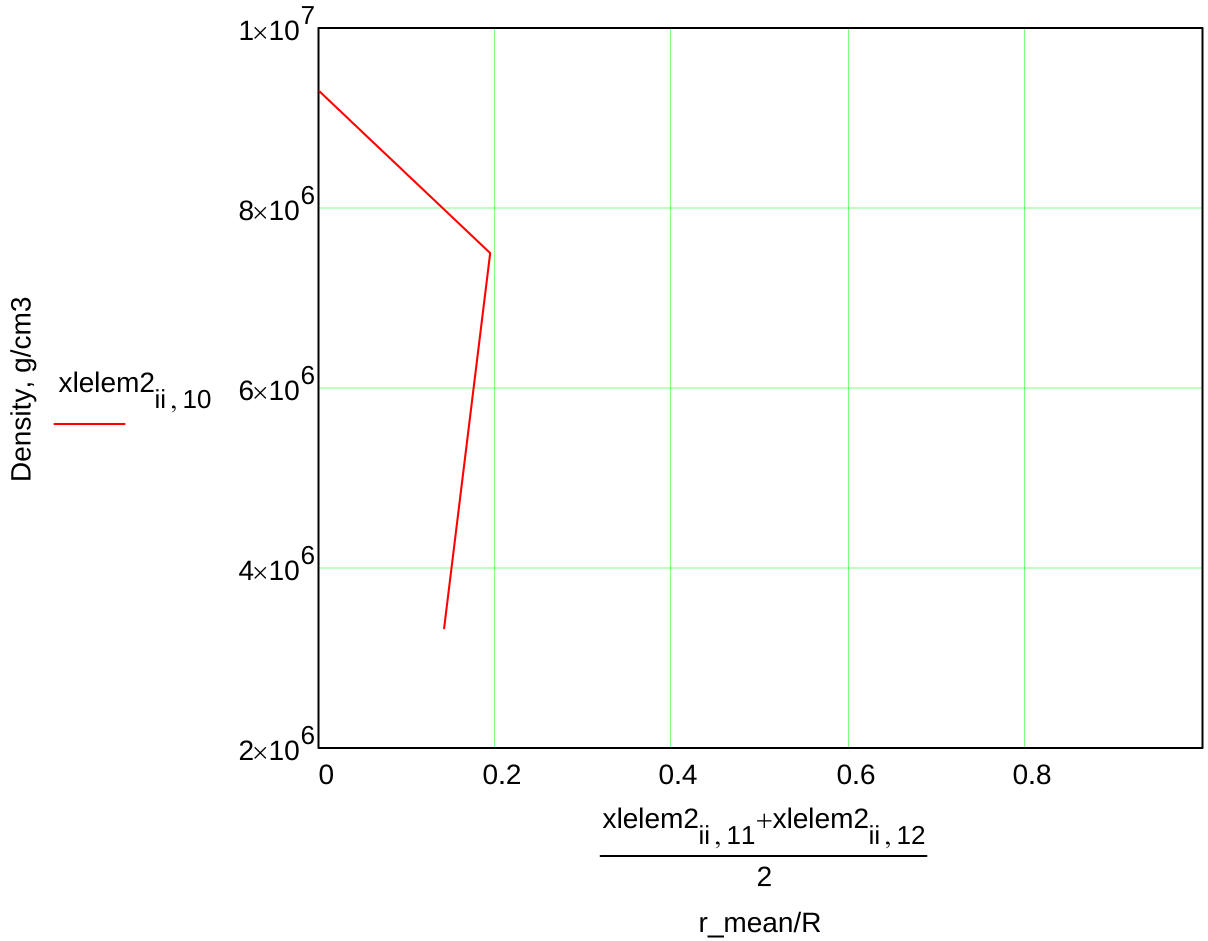


Figure 42. Density vs. r_{mean}/R for Sirius B

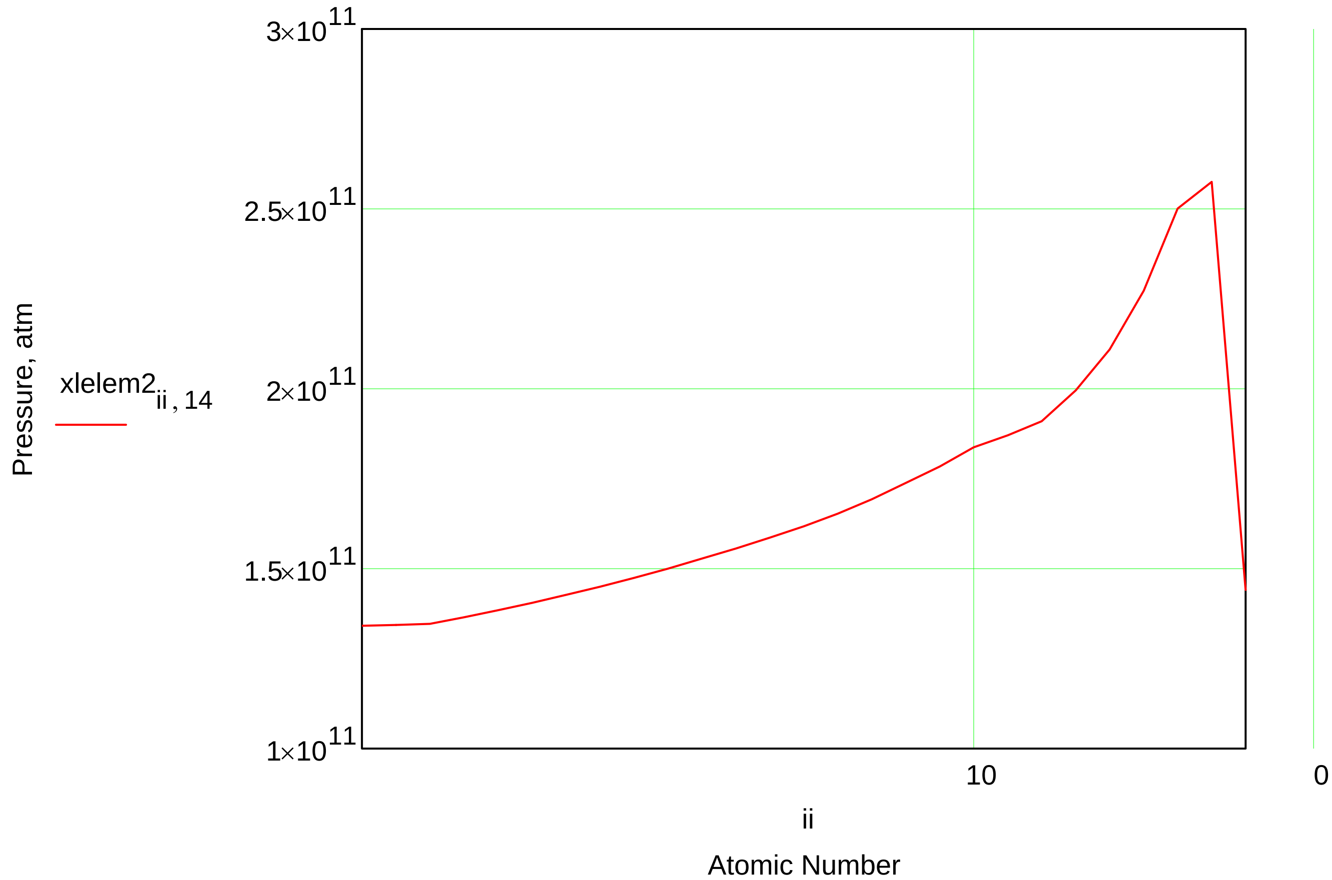


Figure 43. Pressure vs. Atomic Number for Sirius B

The internal pressure near the core eventually causes a Nova! Hot material from the inside breaks through the "crust"--causing an eruption.

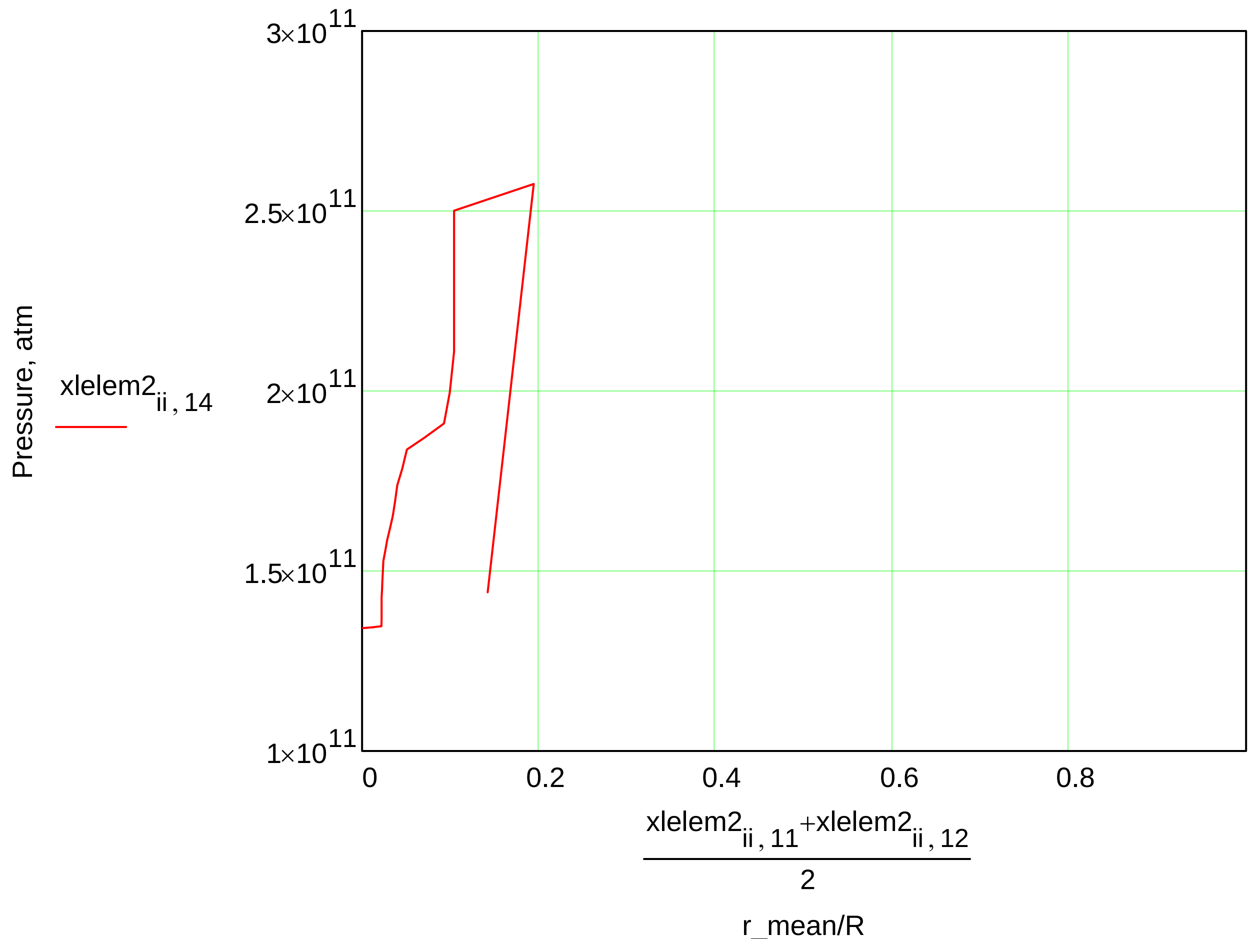


Figure 44. Pressure vs. r_{mean}/R for Sirius B

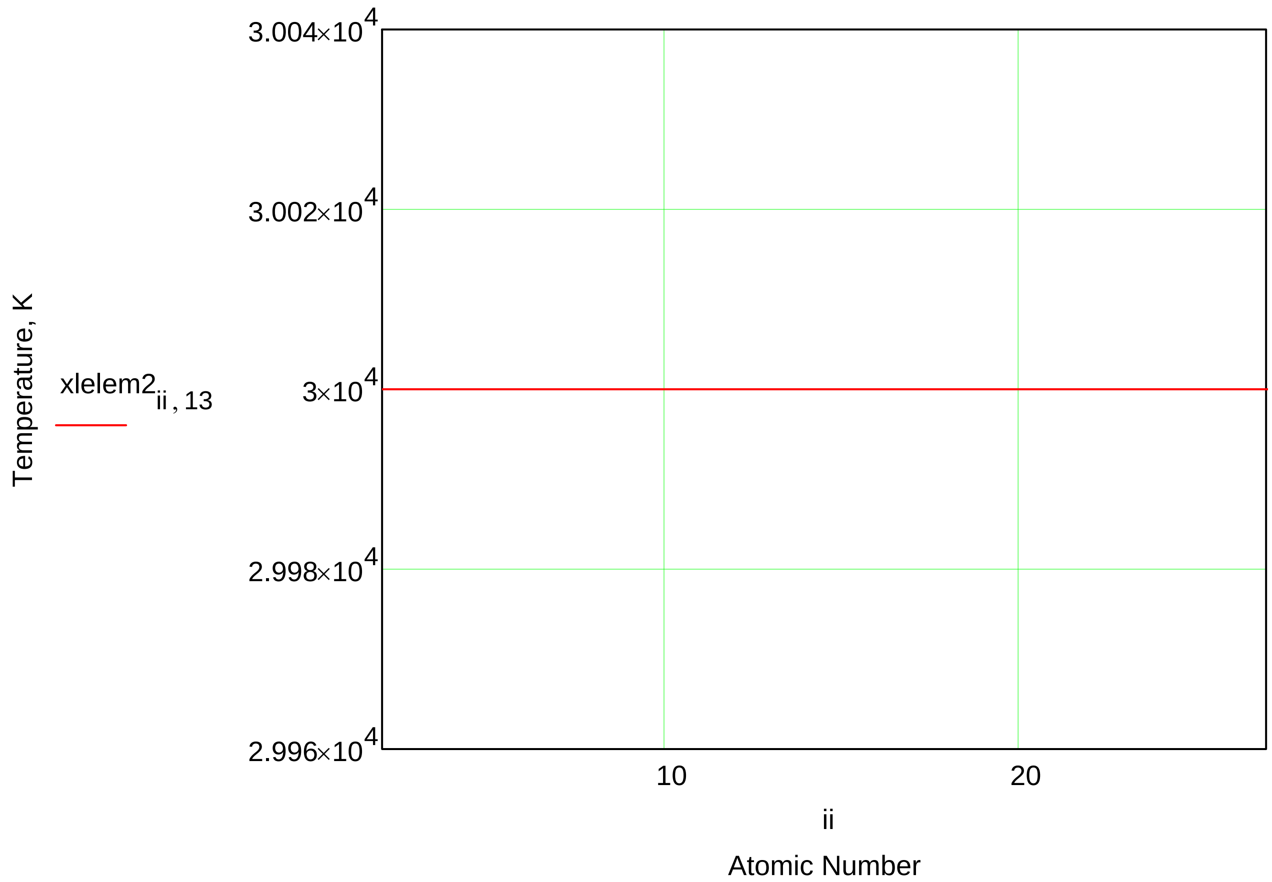


Figure 45. Temperature vs. Atomic Number for Sirius B

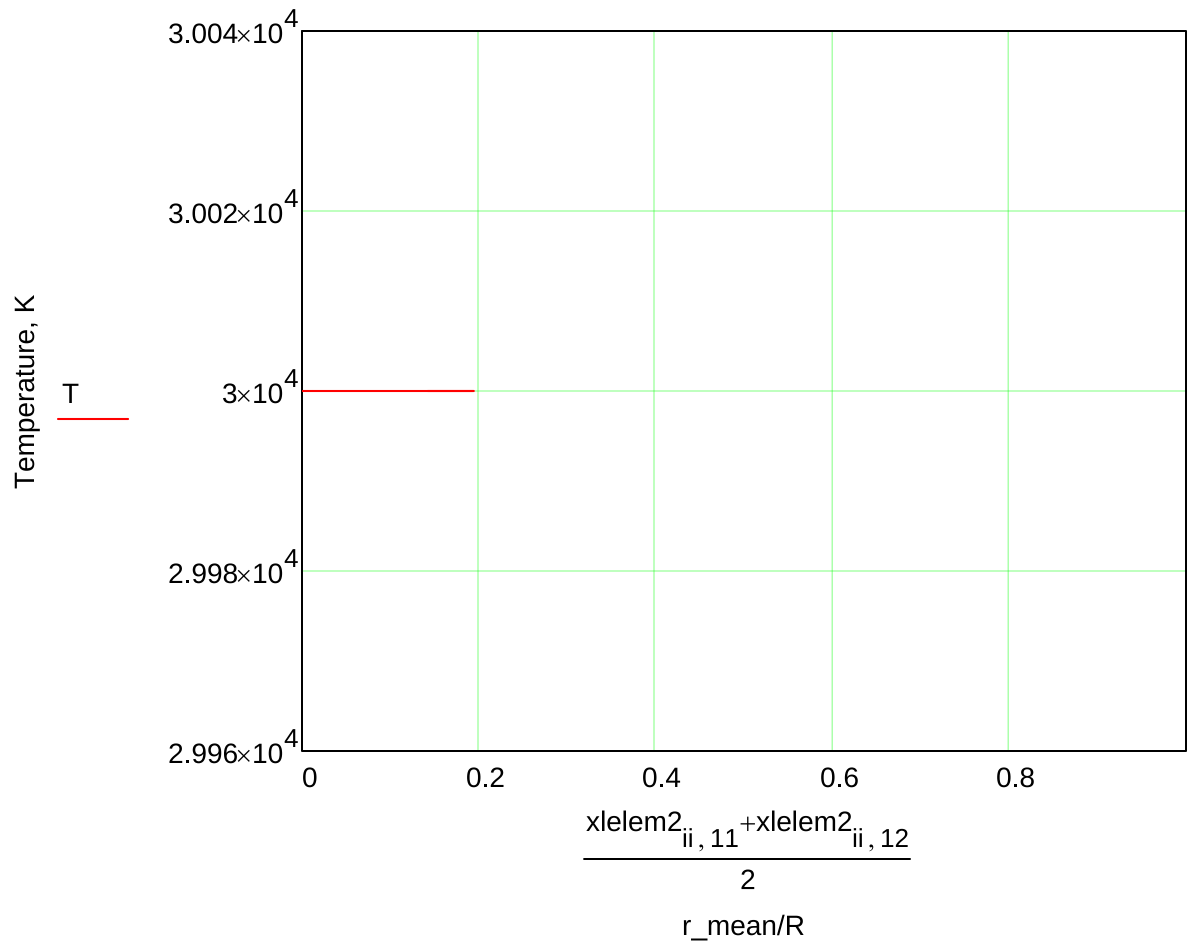


Figure 46. Temperature vs. r_{mean}/R for Sirius B

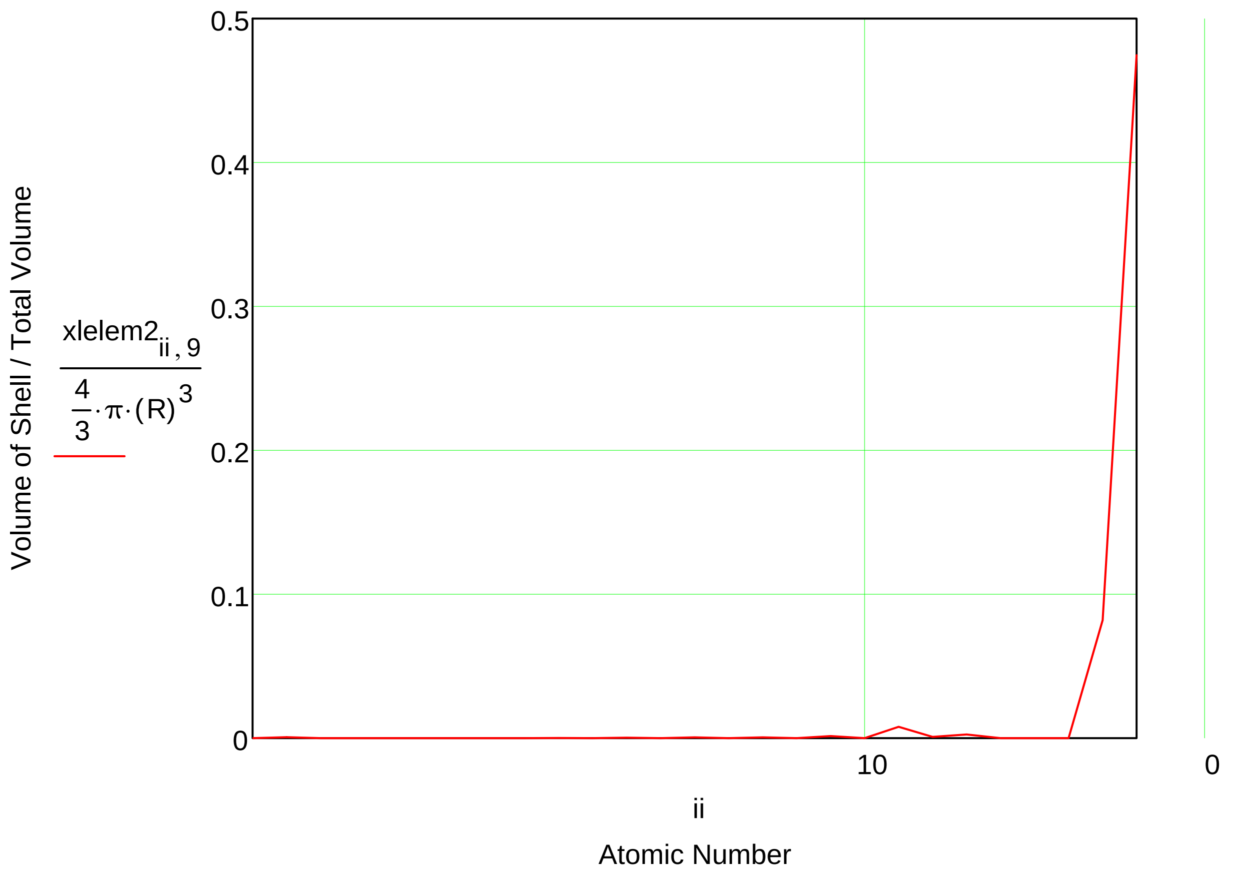


Figure 47. Volume of Shell / Total Volume vs. Atomic Number for Sirius B

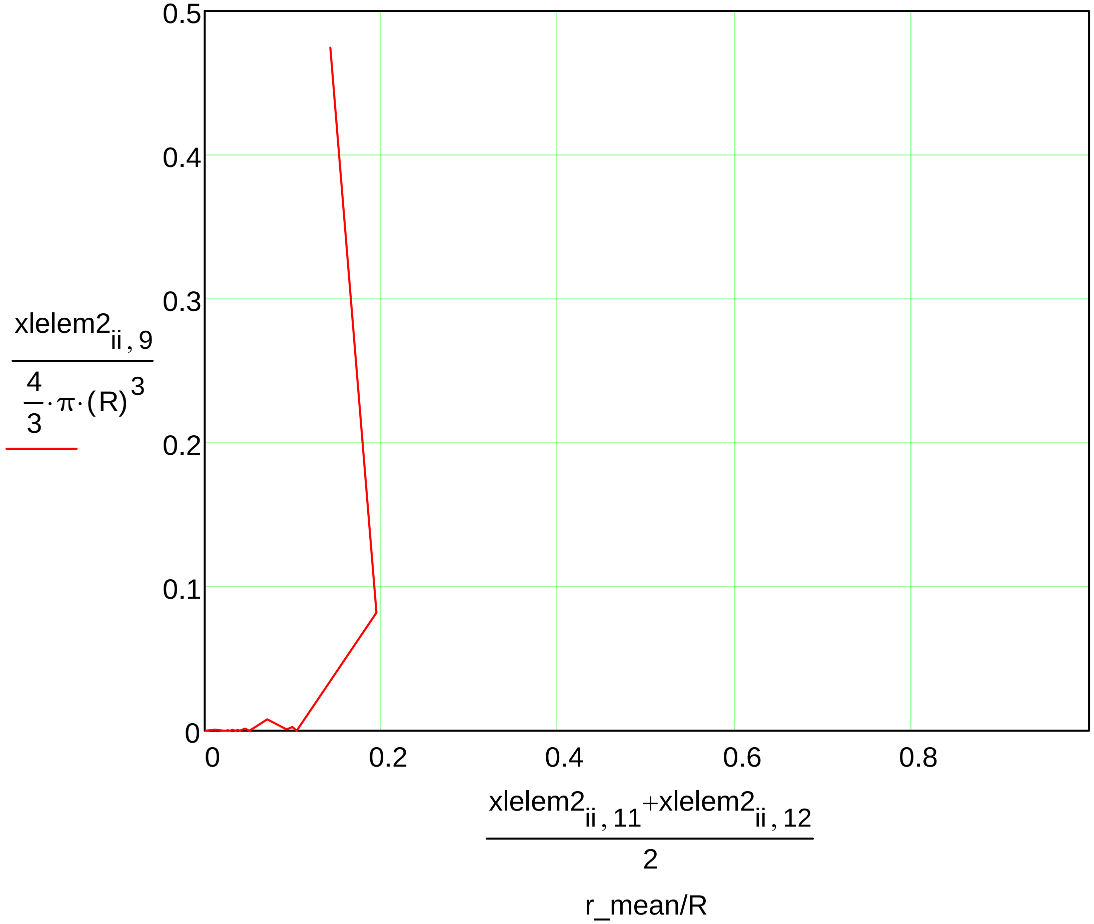


Figure 48. Volume of Shell / Total Volume vs. r_{mean}/R for Sirius B

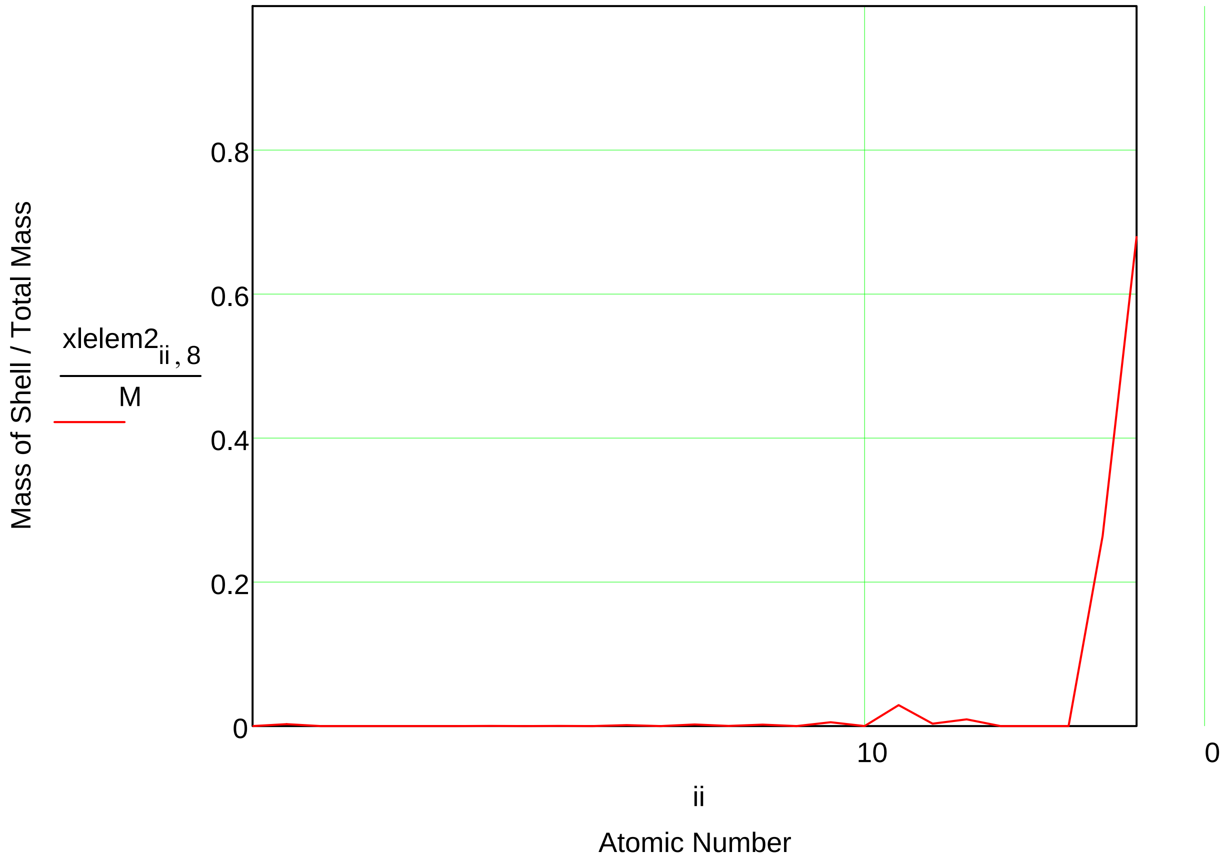


Figure 49. Mass of Shell / Total Mass vs. Atomic Number for Sirius B

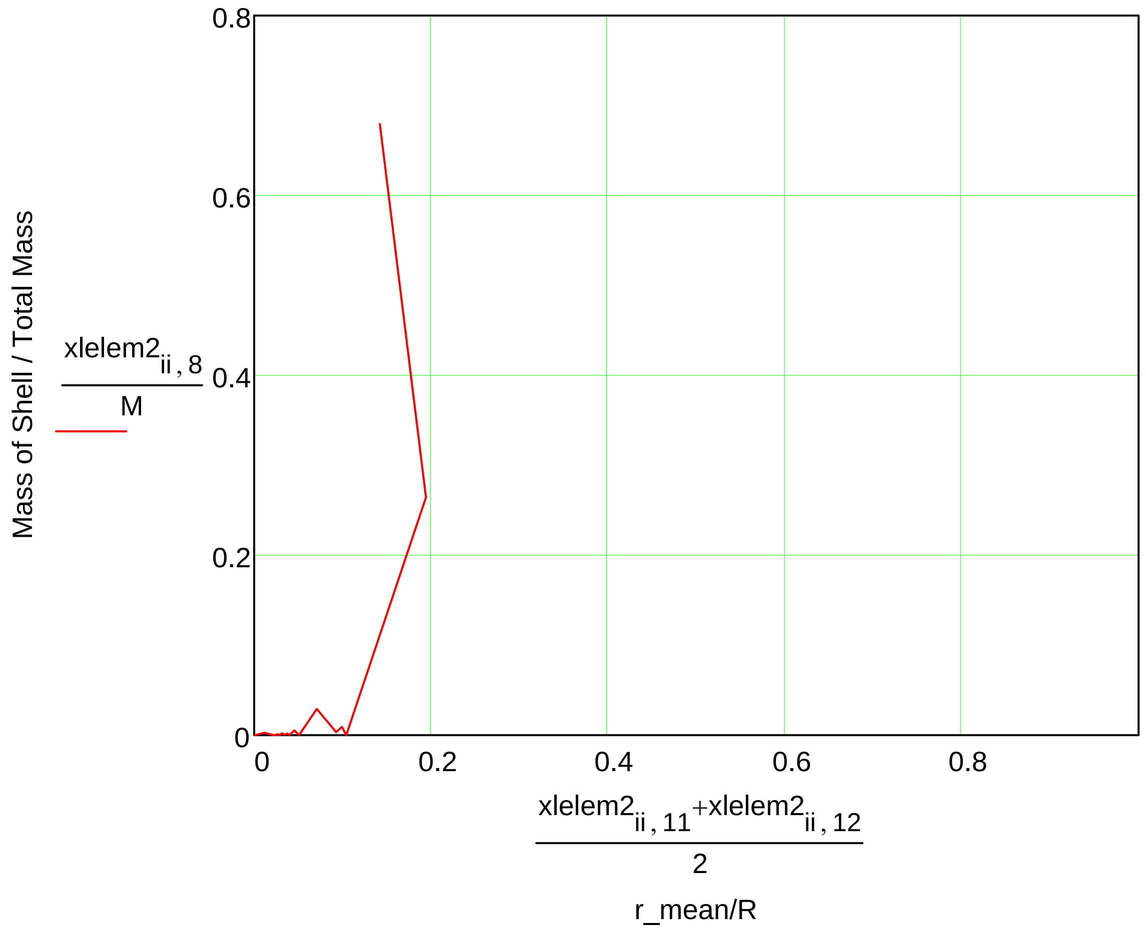
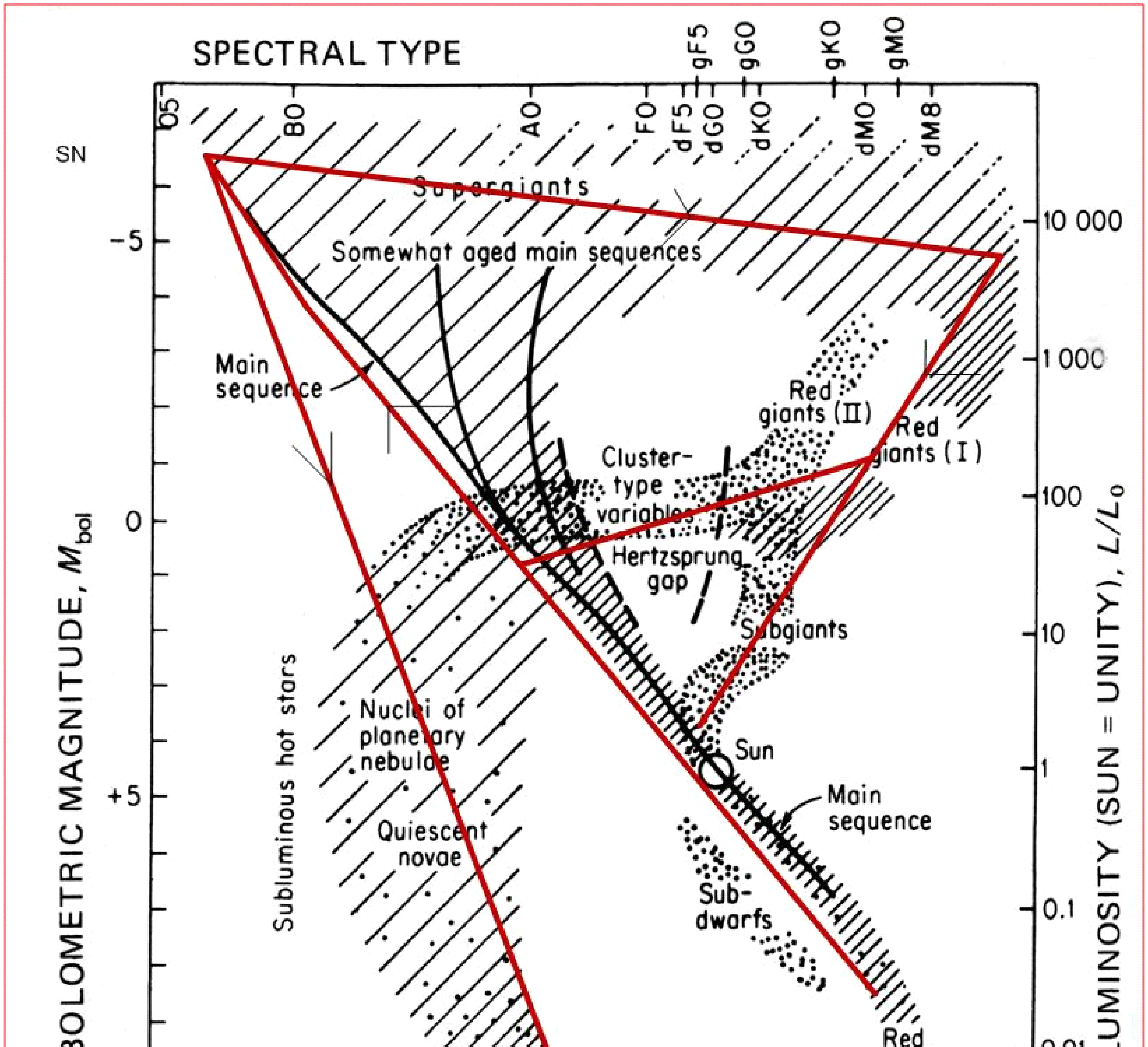


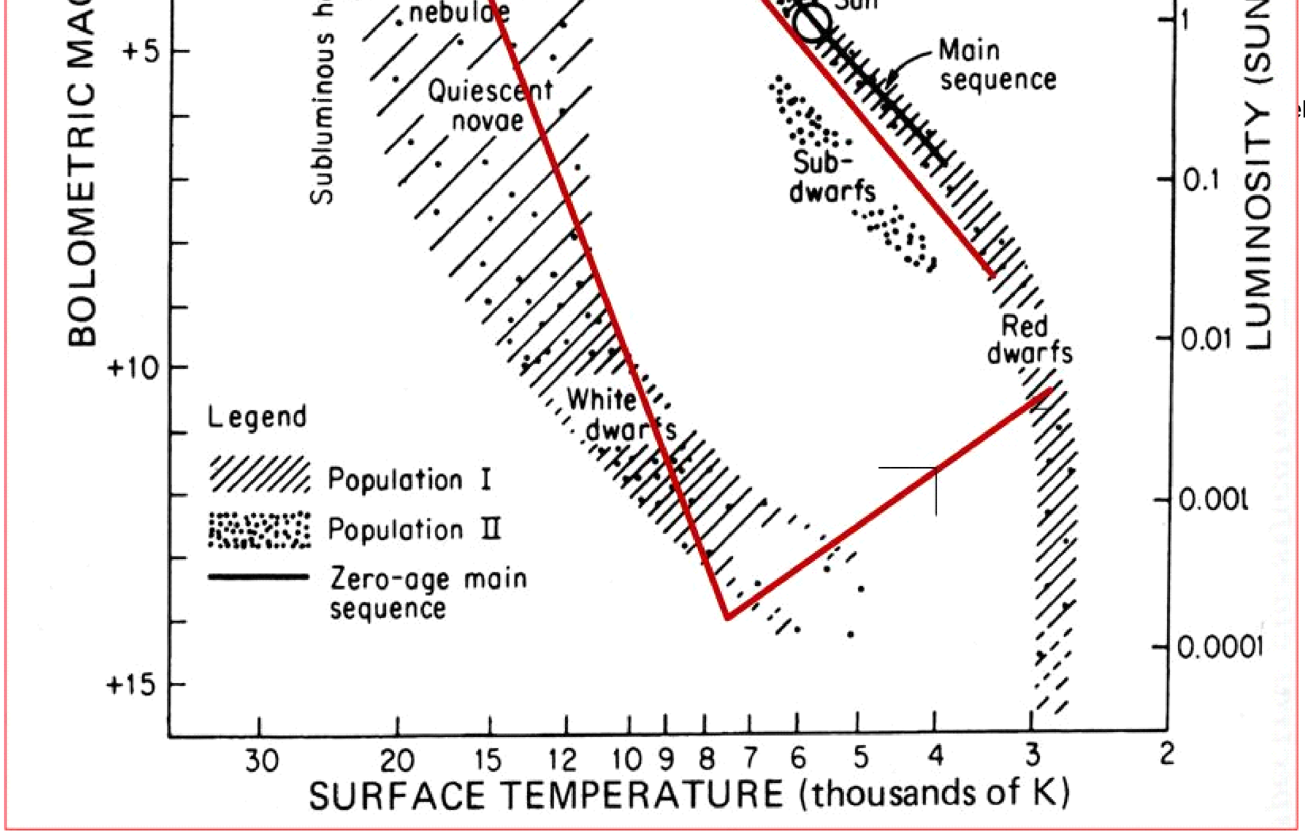
Figure 50. Mass of Shell / Total Mass vs. r_{mean}/R for Sirius B (Pb)

White Dwarfs and Pulsars do not have *internal energy generation*, so the equations for such do not apply. The luminosity equations can be used and would be based on the effective temperature T or T_{eff}.

II. Stellar Evolution

To follow the step-by-step evolutionary paths of the various classes of stars given in this part of the paper (based on pages 29 to 267 of Ref. [1]), carefully study the following two color-magnitude diagrams.





Adapted from M. Zombeck*, Handbook of Space Astronomy and Astrophysics, 3rd ed*. (Cambridge, UK: Cambridge University Press, 2007), p. 102. The red lines have been *added* to show the star paths according to the Reciprocal System.. Here, on the Main Sequences, stars *accrete more mass than they lose*, and so they move UP the Main Sequence. When they reach the O5 or higher spectral class, they explode as a Supernova Type I. This results in a White Dwarf-Red Giant *pair* or a set of planets and a Red Giant (if the explosion wasn't strong enough to create a White Dwarf). Eventually, the White Dwarf *expands* back to the Main Sequence, and the Red Giant *contracts* back to the Main Sequence, either on the *constant growth line* (higher) or the *constant mass line* (lower). (If planets were created, they expand to obtain gravitational equilibrium.) The sequence of the number of stars, at each cycle, if planets are not created, is 1, 2, 4, 8. It's probable that after the fourth cycle, the isotopic mass limit is reached, and the stars of a multiple star system explode as Supernovae Type II, creating nebulae (whose matter is eventually absorbed into other stars) and Pulsars (which usually leave the site of the explosion). The predecessor star to the Sun exploded 4.6 billion years ago, as a Supernova Type I, after being on the Main Sequence for about 10 billion years. Prior to that it was a smallish dust cloud and then a smallish Red Giant, then a Red Dwarf, and then began to slowly move up the Main Sequence. Our Sun will explode in about 5.4 billion years from now. Originally the Sun was simply a nebula, then a Red Giant, then it contracted back to the Main Sequence, but *lower* than where it is now; this explains the so-called "Faint-Young-Sun Paradox." Note that stars on color-magnitude diagrams given in textbooks may actually be in *different* cycles. Also, it's not too difficult to distinguish between Red Dwarfs which were previously White Dwarfs, and Red Dwarfs which were originally Red Giants; the former have much *higher density*, like Proxima Centauri, which has a density of 62 g/cm^3. Most of the stars in our local region are in the second cycle, and so many have planets (if they're not binary). Most of the stars in globular clusters are in the first cycle, and thus *young*. Novae result from eruptions of White Dwarfs as their centers have higher pressure; they have an *inverse density gradient*, but by the time they reach the Main Sequence, this switches back to the normal density gradient. A planetary nebula is a White Dwarf with ejected incoming material (which will eventually form the Red Giant). There are no Black Dwarfs in the Reciprocal System, and no "heat death" for the universe. Pulsars leaving the Galactic plane emit radio waves; those that do not disappear into the cosmic sector come back down toward the Galactic plane, emitting x-rays.

Figure 51. Color-Magnitude Diagram 1

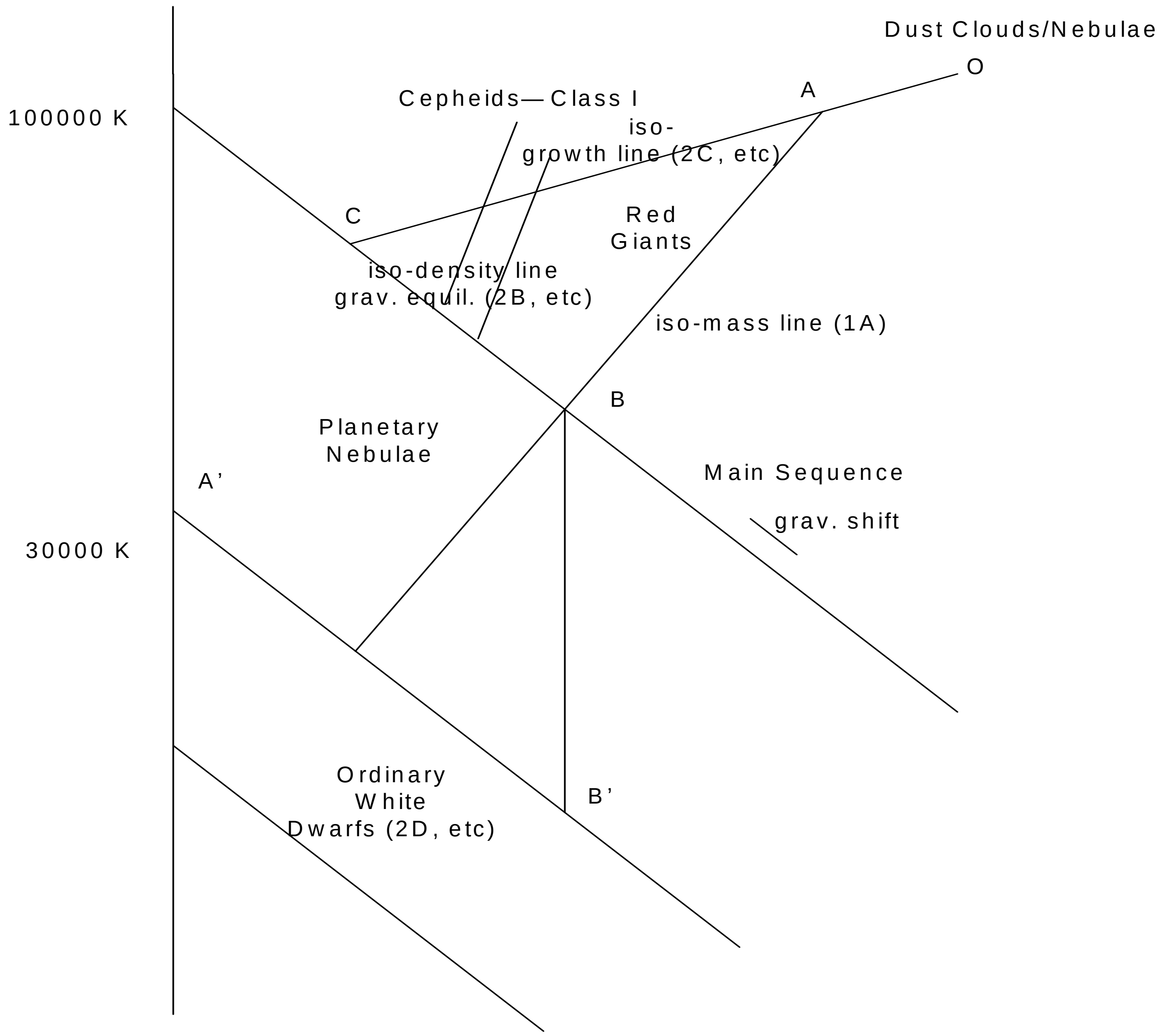


Figure 52. Color-Magnitude Diagram 2 (adapted from Ref. [1], p. 174)

A. Gas and Dust Clouds--Proto-Globular Star Clusters

1. Primitive dispersed matter (either that initially formed or coming from the cosmic sector) eventually condenses into *globular clusters*; however, galaxies may capture *immature* globular clusters, which then become the so-called "star-forming regions" of the galaxy--these are *pre-stellar* aggregates; they probably have less fragmentary old material to work with (e.g., stray older stars). Immature globular clusters which are not captured evolve into *mature* globular clusters in intergalactic space.

2. Rather than entering our Galaxy as a set of stars, immature globular clusters come in as *clouds of particles*; these particles are able to penetrate the *interstellar space of the disk* (unlike the stars of a *mature* globular cluster); these particles eventually *envelope* individual stars; many of the identified *nebulae* or "star forming" regions are portions of much *larger aggregates*--the captured immature globular clusters; the density of these regions is 100x that of the interstellar medium; individual *clumps* of the proto-cluster are *outside the gravitational limit* of other clumps; the proto-clusters are originally *spherical* but become *irregular* due to the *tidal* gravitational attraction of the stars in the Galaxy's disk.

3. The so-called "new stars" of the clouds are *not* being formed out of the clouds; rather the clouds are accreting to *pre-existing stars* in the Galaxy disk; O and T association stars are *pre-existing*, not formed from the clouds; the IRAS has shown that *many stars are surrounded by gas and dust*--which proves that *accretion* is a very important process of the universe--in fact, the accretion rate is *greater* than the mass loss rate ("stellar wind").

4. The presence of Red Giants in our neighbor, the LMC (Large Megallanic Cloud), indicates that it is an aggregate in which the *most advanced stars* have become 2C (second cycle, Red Giants); there are also quite a few 1B (first cycle, Main Sequence) stars in the LMC (hot blue stars); however, in another neighbor, the SMC (Small Megallanic Cloud), the stars are still in the first cycle--it has more Cepheid variables than the LMC; therefore, the SMC is comprised mostly of 1A (first cycle, Red Giants) and 1B stars, and is thus *younger* than the LMC; M67 is a *hybrid* cluster--somewhere between a globular cluster and a *galactic cluster* (which is a globular cluster in the process of disintegration due to the gravitational tidal forces of the Galaxy).

5. Open (galactic) clusters disintegrate *quite rapidly*--and this means that their *supply must be replenished*--this comes from the captured globular clusters, which are initially formed in intergalactic space.

6. Globular clusters are *not* the oldest objects in the universe; they are the *youngest*.

B. Globular Clusters vs. Galactic or Open Clusters; Capture by the Galaxy; Gravitational Shift; Galactic Color Change

1. *Mature* globular clusters are on their way to *capture* by the Galaxy (or other galaxies) (they do not "orbit" the Galaxy or other galaxies); tidal forces disrupt them, and so stars are *stripped off*; by the time the cluster reaches the *disk*, the stars are numbered in the *tens of thousands*, rather than *hundreds of thousands*; eventually the globular cluster breaks into a number of *open clusters* (from a dozen stars to a thousand) by the time it reaches the rotational disk of the Galaxy; open clusters are *expanding* at measurable rates, due to the space-time progression overpowering gravitation and the rotation of the Galaxy; examples: Pleiades and Hyades are, indeed disintegrating. Note: aggregates of stars have the characteristic of a *viscous liquid*--thus the entry of the cluster into the Galaxy is physically similar to the *impact* of one fluid aggregate on another.

2. There is thus an *age differential* between globular clusters and galactic clusters: those clusters *higher* above the Galactic plane are the *younger*; M67 is about 440 pc above the plane, whereas the Perseus cluster is in the *vicinity* of the plane--hence the stars of M67 are *younger* than those of Perseus; star density *decreases with age*; distant clusters contain only Class 1 stars--Infrared Stars, Red Giants, sub-giants, long-period Class 1A variables, and variables of the RR Lyrae and associated types; clusters much *closer* to the Galaxy may contain some Class 1B stars of the lower Main Sequence

3. Main Sequence stars are *not* in the gaseous state--they are in the *liquid* state or in the "condensed gas state" (as shown in Part I); the atoms are separated by less than a unit of space; the condensed gas volume can be computed by means of the relations that apply to the liquid state! Stars in a condensed gas or liquid state have a *surface*--like the Sun; as the star condenses from gas and dust, it eventually reaches a *critical density* at which point it becomes *self-gravitating*; Ref. [1], p. 111, shows the A-C or A-B paths of six globular clusters: M79, M3, M5, M22, M13, and Omega Centauri; p. 112 shows the AC, AB, and BC lines for M5 and M13 globular clusters; the difference in luminosity between point A and point B is 5.6 magnitudes, whereas the difference between point B and point C is 2.8 magnitudes, twice as much--this is a fixed relationship! This thus gives a new distance scale--see Ref. [1], p. 116 for *calculated* distances to a large set of globular clusters.

4. Globular clusters *most distant* from the Galactic center are the *most metal-poor*--so, again, they must be the *youngest,* not the "oldest" objects in the universe, as claimed by conventional theory. Globular clusters are the *fundamental building blocks* (i.e., aggregates of stars) of the macrocosmos.

5. M 71 is a globular cluster on its way to becoming a galactic cluster or a set of galactic clusters; *differential gravitational forces* strip away the loosely attached stars and matter, thus reducing the size and mass of the cluster--the Galaxy obtains this mass; because of this loss of dust and gas, the stars of the cluster *stop accreting mass* and so move *across the CM diagram* at *constant mass.*

6. M 67 shows an *upward displacement* of 2.6 mag on the Main Sequence, on a line parallel to B-C; this is called a "gravitational shift" by Larson; in the Galaxy, *part of the gravitational effect* of the Galaxy as a whole is used to *balance* the *rotation*; a *second gravitational component* is the Main Sequence *force equilibrium.* The contraction of the star *stops at a lower density or expands back to that density*--this puts the Main Sequence of the galactic stars somewhat *higher* on the C-M diagram than the Main Sequence of the globular cluster stars--this is about .8 mag. The metal-poor sub-dwarfs do *not* rotate with the Galaxy--hence they are *newly captured stars* and are so a bit to the *left* of the Main Sequence. The gravitational shift becomes *less as the stars expand out to that of the Galaxy's field stars.*

7. In the Hyades, which is somewhat older than M 67, the majority of stars have reached the Main Sequence; the Pleiades is even more advanced--its stars have moved *up* the Main Sequence; the Pleiades is located within a nebulosity--and so its evolution may have speeded up by *greater accretion*; Praesepe and the Hyades may have come from the same globular cluster; some Perseus stars have reached the explosion limit of the Main Sequence and so are beginning to produce 2C stars (Red Giants, second cycle).

8. Stellar rotational speed is a function of age; stars, star clusters, and galaxies all originate with *little or no rotation*, and *acquire rotational velocities* by the *acquisition of matter* which imparts an *angular momentum* to the bodies.

9. Small Elliptical Galaxies are comprised of Class 1A and 1B stars; early Spiral stage galaxies are comprised mostly of Classes 2C and 2D--which eventually both become 2B (like the Sun); the 2B stars eventually explode as Supernovae Type I and then become 3C and 3D; the cycle continues until the *upper age limit* is reached, when the stars go through a *Supernova Type II*; the limit is probably 4 or 5 cycles; central regions of galaxies become *denser and denser*, because galaxies have the *character of viscous liquids* such that the denser stars *spiral in*; Class 1A stars are distributed throughout the *outer Galactic structure*; Class 1B are common in the *spiral arms*; Class 2C and later are confined to the G*alactic disk and nucleus*; in the larger Elliptical Galaxies, the spectral type G (yellow) dominates; in the early Spirals the emission rises to type F (yellow-white) or even type A (white); in the older Spirals, the light shifts back toward the red, indicating the beginning of *another cycle*; the centers of the Galaxy and M 31are dominated by *old metal-rich stars.*

C. The First Giant Star Cycle

1. The pre-stellar dust and gas cloud contains some *heavy elements* plus *fragments* of heavy matter in the surroundings (from previous events); production of heavier elements *continues* with the *absorption of neutrinos from the large scale neutrino flux*; the structure of the cloud is "fluid"--so the heavy elements make their way to the *center*; there is a long period of *gradual warming*.

2. The lower limit for *visible radiation* is a surface temperature of 1000 K; IR (Infrared) stars have surface temperatures in the low 300 to 700 K range (due to very limited fission); IR stars are *predessors* to Red Giants.

3. Gravity causes the pre-stellar gas and dust cloud to *contract*: gravitational energy converts to *thermal energy*.

4. Eventually the thermal destructive limit of the *heaviest element in the core* is reached, converting mass to energy and causing a sudden jump in temperature.

5. Globular clusters have mostly "new" stars (1A); the mass, density, and central temperature of globular cluster stars can be obtained approximately by theory--as shown in Part I.

6. See Figure 52, for what follows

7. Stars on the Main Sequence are at approximately *equal density* (they are in *gravitational equilibrium*); the density at C on the C-M diagram is only 3 to 4 times that at B. The line B-C is only approximately iso-density; A-B is iso-mass line and iso-temperature; AC is iso-growth.

8. The conditions surrounding each star of a globular cluster are *somewhat different*, so different stars in a globular cluster may be at different positions in the C-M diagram

9. From point O to A to B: these are the *coolest* stars, but they have *large surfaces*, hence they have a relatively *high luminosity.* There are two types of these stars: 1) stars with approx. solar mass, and 2) more massive stars based on *fragments of pre-existing stars as a nucleus*; first group stops growing at point A (having run out of the supply of dust and gas); they make a *sharp turn down the line A-B*, where B is on the Main Sequence; this results in *decrease in size of radiating surface*, but an *increase in density* and a *small increase in surface temperature*--this causes a rapid *decrease in luminosity.*

10. From point O to A to C: the second group has an *increase in temperature*, along with *mass*; the decrease in luminosity is thus *smaller* than that of the first group; gravitational contraction ends at the Main Sequence for both groups. Then both groups move *up or down Main Sequence to establish thermal equilibrium* (dissipation of energy by radiation equals atomic energy generated). Note: 95% of stars are on the Main Sequence.

11. Class A stars are those on the paths OAB and OAC. Class B stars are those on the Main Sequence. Conventional theorists use the terms "Population I" and "Population II." In the Reciprocal System, Population II = Class A; Population I = Class B. *Younger stars* are currently called *Population II, whereas older stars* are currently called *Population I,* because the astronomers have the evolutionary sequence *backwards.*

12. Evolution on the Main Sequence depends on the *environment*; if there is not enough gas and dust the star might move down; but *usually there is enough, and so the star moves up*. The heavy element content is continually replenished by the *atomic build-up process*.

13. As mass builds up, the *central temperature increases*, making *successively higher elements* available for *fission* (as shown in Part I); *each new element causes the spectral class to move up one notch.*

14. *Gravitational segregation* occurs, so heavy elements are in the *core*; present day theory says that H is in the core, which is utter nonsense.

15. Fe represents an appreciable amount of a star's mass, so when its temperature limit is reached, the *star blows up in a Supernova Type I*; this occurs when the first portion of Fe converts to energy, the rest is *dispersed*. The predessors stars are *all hot and massive and very nearly alike*. Limit to mass: approximately 100 solar masses, but the normal star only reaches approximately 14 solar masses. Type I Supernovae occur in *all* galaxies.

D. Results of Supernovae Type I; the Second and Later Giant Star Cycles; Cepheids

1. Whereas in Type II Supernovae the outward moving cloud *escapes gravitational control of the mass of the former star*, the outward moving cloud for Type I does *not*; the visible portions of the Type I supernova remnants are mainly the fastest particles; for an O or B star mass of 14.9 solar masses, the gravitational limit is 4.27 pc, so it's unlikely that much of the mass of such a star would escape beyond the gravitational limit. Ultimately the constituent particles are *pulled back* to where the internal temperature can rise enough to *reactivate the energy generation process*, and the star is reborn.

2. The new star is IR; it is back at O in the CM diagram; then it becomes a Red Giant (2C); a second cycle star like this has a *gravitationally stable core*; the gravitational contraction proceeds *much more rapidly than in that of the first cycle*; surface temperature of the early second cycle star is similar to that of an early first cycle star, while the luminosity is similar to that of a Main Sequence star; these 2C stars occupy a *band along the right of the diagram* similar to the upper part of the Main Sequence on the left.

3. 2C stars move almost *horizontally* across the diagram, if accretion is minimal; any further accretion that takes place puts the terminal point, the location where it reaches *gravitational equilibrium* higher on the diagram; this is *quite above* Class A stars.

4. O associations are *O and B stars within accreting dust clouds*; T associations are *T Tauri stars*--much *smaller and cooler than O and B stars* and *not* having much accretion; stars of all classes have to grow at the expense of their environment to reach the O status.

5. Intrinsic variable stars: a newly formed star of either Class A or C is *variable* in the amount of its radiation because the *additional energy released causes an expansion, which then cools the core, shutting off the power supply*, until contraction causes it to *resume*; bright IR stars are the newly formed variables; *long-period variables* swing from *on to off positions* over a relatively *long period*; period and magnitude *decrease* as the fluctuations in the rate of energy production represent a *constantly decreasing proportion of the total energy of the star;* eventually there is an alteration between the *thermal destruction of two adjacent heavy elements.*

6. Stellar output varies, of course, by reason of the nature and amount of material *accreted* from the environment.

7. Pb is probably the element fissioning in the C*epheid zone*; long-period variables preceding the cepheid zone must be fissioning *higher atomic number elements*; RV Tauri variables lie between the Red, Mira type, long-period variables and the Cepheids. RR Lyrae stars are Class A stars which cross the Cepheid zone; W Virginis stars are *Class 1* Cepheids--these have a *lower temperature and surface density* (referred to as Type II in conventional theory); Class 2 Cepheids--*Classical Cepheids*--are *larger and more massive* because they are based on giant *Class C* stars; RV Tauri variables are similarly separable into two distinct groups.

8. Past the Cepheid zone, the pulsations *smooth out*.

9. The heavy element content (up to Fe) of Class 2C stars and above is *substantially above* that of the Class 1A stars, because the concentrations *build-up with each cycle.*

10. Interstellar matter remains approximately at the *same composition* because the *increase in mass with age* (by means of neutrino absorption) is offset by *influx of new, young matter* from the cosmic sector.

E. The Dwarf Star Cycles; Planetary Nebulae; Cataclysmic Variables--Novae

1. A Type I Supernova causes the velocity of some of the atoms in the *core* of the star to *rise above unity* (the speed c). The status of unit speed as the *true physical datum* means that the correct *speed addition sequence* is -1, 0, +1. The *second unit* is *not* the spatial motion (speed) of the first unit plus a unit of motion in time (energy), but the unit of motion in *time* only--hence the *velocity equation flips to v = t/s*. Thus, *intermediate speed* particles (between 1 and 2 natural units or 1c to 2c) are blown *outward in time*, which is the *inverse of outward in space*; the atoms are thus separated by *empty time*, not *empty space*; so here we have a White Dwarf!

2. Densities of White Dwarfs: Sirius B = 130000 g/cm^3; Procyon B = 900000 g/cm^3; thus the White Dwarfs are the *inverse* of the Red Giants; the density is *not* due to the "collapse" of the star to "degenerate matter"; removing the heat and pressure would *eliminate the space* between atoms, but would *not* cause the atom *itself* to be compressed; also, *gravitation reverses at unit distance*, becoming a *repulsive* force; so if heat and pressure are removed, the most that will happen is that the atoms will compact themselves in the *solid state*--and the more the compaction, the greater the *resistance* to it.

3. White Dwarfs have *inverse density gradients* (see Part I); the *center of the star* is where the compression in time is the greatest, and since compression in time is equivalent to expansion in space, the center of the a White Dwarf is the region of *lowest* density.

4. It is known that "the more *massive* a White Dwarf is, the *smaller* its radius"; this is because *more time is equivalent to less space*.

5. Stage 1 White Dwarf: this is the *immediate post-ejection* period; the White Dwarf is *expanding in time* (contracting in equivalent space). It emits only *radio frequency,* and so it is only identifiable as a "radio source" (these are called "blank fields" by the observers--it *cannot be seen* at all until the temperature is decreased significantly; energy is lost to the environment by this radiation. The White Dwarf has *no source of internal energy generation*.

6. Stage 2 White Dwarf: the expansion in time (contraction in space) *halts*, and a process of *re-expansion occurs*; energy loss continues, so the *temperature falls*; there is still not much energy production, because the supply of elements heavier than iron was reduced to near zero by the time of the supernova (and besides they're at the surface, which may be only at 70000 K on average); *eventually the atoms drop below unit velocity*, and then the star comes back to the Main Sequence.

7. Objects moving in time cannot be detected, except during an extremely *short interval* while they pass through the *reference system*, and then only *atom by atom.* If the the *net total three-dimensional scalar speed* is *below* the point of equal division between motion in space and motion in time, any *time motion component included in the total* acts as a *modifer of the spatial motion*--as a motion in *equivalent space*--rather an as an *independent motion in actual time*. A unit of motion in time, *intermediate speed*, is *one-dimensional* and the successive spatial positions of an object moving freely at an intermediate speed do *not lie on a straight line* in our reference system--motion in time has *no direction in space*; the spatial direction of *each successive unit of the time component is therefore determined by chance*, but the *average* position of the freely moving object follows the *straight line of the purely spatial motion,* because the total motion is still on the *spatial side of the sector boundary*. Therefore, the radiation from a White Dwarf in Stage 1 is not perceived from the *surface* of the star itself, but from a *much larger area centered* on the *average stellar location*; it is thus reduced *below the observable level*; but at Stage 2, when the White Dwarf is *expanding back toward the material sector*, the radiation remains unobservable as long as its *surface temperature is above the level corresponding to the unit speed boundary*; but *at and below the unit speed level*, the White Dwarf becomes *visible*; this temperature level is then approximately 100000 K.

8. Stage 3 White Dwarf: this star is in the *gaseous state in time.* The color index is -0.3; the radiation consists of *three independent components* = 1/3 of total rate of emission of thermal energy; these are "Hot Subdwarfs"--the *central stars of the planetary nebulae*.

9. The spatially outward moving result (component A) of the supernova explosion eventually comes to a halt, and *gravitation begins to bring this matter back together.* This matter eventually comes into *contact with the subdwarf* and is *heated up and becomes ionized and immediately is ejected back*; this results in a *sphere of ionized matter* centered on a newly observed White Dwarf--a *planetary nebula*; all Hot Subdwarfs are central stars of planetary nebulae; the ionization sphere does not actually expand much; actually, it eventually *contracts*; meanwhile, the White Dwarf keeps lowering its temperature, so the *repetitions--successive expansions and contractions*--do not regain their former level; thus *older* nebulae have *relatively filled centers* and are thus relatively *small*; the luminosity, though, *increases* because most of it is now being received by us directly from the White Dwarf rather than being absorbed by the nebula and re-radiated;

10. At 30000 K, the *transition from motion in time to space takes place*; a further cooling to the *equilibrium* temperature than takes place; all White Dwarfs or Subdwarfs (Class D stars) fall within the 100000 K and 30000 K lines on a C-M diagram.

11. The line A'-B' (from 3D motion in time to 1D motion in time and then to 1D motion in space) (see Fig. 52) is the equivalent of the Main Sequence line B-C but for motion in time; a White Dwarf of 1.1 solar mass is the *smallest star that has sufficient total thermal energy to maintain the 100000 K surface temperature in the gaseous type* of gravitational equilibrium; an *upward gravitational shift of .8 mag* occurs as the star reaches *equilibrium with the Galaxy and field stars*;

12 Note: the central star of a planetary nebula is *not* a Wolf-Rayet--this is a late *pre*-explosion star, whereas the central star is *post*-explosion star.

13. Because of the *inverse density gradient* in White Dwarfs, and because the speeds of the matter are dropping below unity, *gas bubbles form and descend to the center*, where they *accumulate*; a *gas pressure* than *builds up in the center,* and when this is *high enough*, the *compressed gas breaks through the overlying material* and the very hot mass from interior is exposed *briefly* at the surface of the star, increasing the luminosity by a factor as high as 50000--a Nova. It also becomes an *x-ray emitter*; then this matter *cools*, and the star *reverts back to its original status*; this process is *periodic*; the decreasing resistance with time *shortens the time interval between outbursts*, eventually approaching only 100 years; after the motion in time is fully converted to motion in space, the Nova explosions cease.

14. T Coronae Borealis and RS Ophiuchi are known as *Recurrent Novae*--but actually all Novae are; ordinary White Dwarfs produce less powerful Novae than the White Dwarfs of Planetary Nebulae (which are larger); Novae are classified as slow, fast, or very fast; see the RS Database for data on the major Novae; the number of days to decline by seven magnitudes is tabulated (Table III, Ref. [1], p. 185); *earliest outbursts are the fastest* and have the *maximum magnitude* range; as the White Dwarf ages, the luminosity and the magnitude range should *decrease* (the mass doesn't change significantly); the slowest Novae with the smallest magnitude are at the low end of the Novae size range and near the end of the Nova stage; the small scale Novae are called SS Cygni and U Geminorum type stars;

15. As state above, there are 8 speed levels in the *intermediate speed range*; four are on the spatial side, four are on the temporal side. In the white dwarf, which is on the *spatial side*, there are thus *four speed levels*; we could still, however, divide the shells by element, from the highest (Fe) at the surface to the lowest in the core (H); White Dwarfs with masses above 0.65 solar mass follow the *Classical Nova pattern* in its conversion to motion in space; White Dwarfs with masses below *0.65 solar masses follow the SS Cygni type variables (which have only 3 of the 4 possible speed levels).* Flare Stars (UV Ceti stars) have *only two speed levels* and have a mass from 0.40 solar masses to .25--their light curves rise to a maximum in a few seconds or minutes and then decline to normal in about a half hours; there are also stars which have only one speed level.

16. Novae come from the White Dwarf component of a binary system resulting from a Type I Supernova; the cataclysmic stage takes place during the *upward movement toward* the Main Sequence; the Novae from White Dwarfs (Pulsars which have not escaped the Galaxy) which are created from a Type II supernova would not have a partner (the surrounding nebula will disperse, not reform into a star).

F. Binary and Multiple Star Systems; Planet Formation; Moons; Meteorites; Asteroids; Comets

1. So the result of a Supernova Type I is a *Red Giant-White Dwarf pair.* Binary and Multiple (later than cycle 2) star systems are very *common,* but *cannot be* explained by conventional theories; Red Giant and White Dwarf pairs must thus have a *common origin*; the *A* component of the supernova explosion is *above and to the right* on the C-M diagram, whereas the *B* component is *below and to the left;* both components stay *within* the *gravitational limit* of the star that exploded.

2. Early stage: the A component is a *nebulosity of gas and dust* surrounding the White Dwarf B component. Later stage: the A component develops into a *pre-stellar aggregate*, then into a *giant IR star*--at this point the White Dwarf appears to be *alone*; when the Giant star gets into the *high luminosity* range, this situation *reverses*, and the bright star *overshadows* its fainter companion; eventually the Red Giant reaches the Main Sequence; the White Dwarf evolution is *slower*, so quite often we have a pairing of a Main Sequence star with a White dwarf, as in Sirius and Procyon.

3. Eventually, the White Dwarf reaches the Main Sequence, so now we have a pairing of two Main Sequence stars; but the former White Dwarf has a greater share of the *heavy elements*; e.g., the Wolf-Rayets (W-R) are *former* White Dwarfs who have reached the top portion of the Main Sequence and are fissioning Ni; *no* W-R have been found in the Orion Nebula, where O and B stars of Class 1 are plentiful, so W-R are Class 2 or later; component B, the White Dwarf, is almost always *less massive* than the component A (Sirius has 2x the mass of the White Dwarf).

4. Example pairs: two recurrent *Novae*, T Coronae Borealis and RS Ophiuchi, have Red Giant stars for companions; Capella is supposedly a pair of giant stars, but it must actually be a *multiple system* rather than a double star according to the RS--it must have two *unseen* White Dwarfs! Algol type stars are similar; in Algol itself, there is at least one, and possibly two small B components;

5. As long as there is adequate material for accretion, the stars keep *moving around the cycle* until their life span is terminated by a *Supernova Type II* explosion; the number of stars at each cycle is 1 --> 2 -->4 -->8. Our *local star group* is comprised mostly of type 2B--second cycle stars on the Main Sequence--so these will be *binary systems* or *star-planet systems;* but stars in globular clusters and early elliptical galaxies are almost entirely Type 1A or 1B, and hence there are few if any binaries there.

6. In the Reciprocal System, there is *no mass interaction* between the two components, unlike in conventional theory.

7. Component A contains mostly *light elements*; component B contains mostly *heavy elements.* Smaller White Dwarfs will be composed almost entirely of the *iron group*, whereas larger ones might have a greater proportion of lighter materials.

8. For both components, the gravitational forces are directed *radially inward* toward the center of mass of the dispersed material; the greater amount of matter, component A--that which is dispersed in space--thus dominates the smaller amount of matter, component B--that which is dispersed in time--and this latter than acquires *orbital* motion around the A component. If the B component is of stellar size, the result is a binary system; if not, the result is a *planetary system*; some of the *unconsolidated fragments* (mostly A, some B) may take up independent orbital positions, becoming *planetary satellites*; Ref. [1], p. 93: "We may therefore deduce that during the latter part of the formative period all of the larger members of the system increased their masses substantially by accretion of fragments of Substance A in various sizes from planetesimals down to atoms and sub-atomic particles. Some smaller amounts of Substance B, in assorted sizes, were also accreted. After the situation had stabilized, the central star, our Sun, for instance, consisted primarily of Substance A, with a small amount of Substance B derived from the heavy portions of the original Substance B mix, and the accretions of Substance B. Each planet consisted of a core of Substance B and an outer zone of Substance A, the surface layer, of which contained some minor amount of Substance B acquired by capture of small fragments. The planetary satellites, which had comparatively little opportunity to capture material from the surroundings because of their small masses and the proximity of their neighbors, were composed of Substance A with only a small dilution of Substance B."

9. Proof of segregation: the *iron, stony, and stony-iron meteorites.* The *iron meteorites* are obviously Substance B; the *stony meteorites* are Substance A; and the *stony-iron meteorites* are a combination; the differentiation of the light and heavy elements occurred *before* the supernova

10. Pre-Nova and post-Nova White Dwarfs *accrete matter* from the surroundings (mostly A); but during a Nova, some of the Substance B near the *surface* comes out (as entrained in the matter coming from the *interior*), and so the explosion spectra often show highly ionized iron.

11 Asteroids are *unconsolidated aggregates of Substance B*; new *cometary matter* is being made from incoming cosmic rays; the *rate of formation and rate of loss of comets* are about *equal* now.

G. Planet Spacing

In the Reciprocal System, planetary systems result from the supernova explosion of the parent star. The outward moving (in space), relatively slow speed, matter eventually reconstitutes as a Red Giant and then contracts to the Main Sequence. The inward moving (outward in time, inward in space), intermediate speed matter constitutes the eventual *planets*. There are three speed ranges in the Reciprocal System: 0-1c; 1c-2c; 2c-3c. The intermediate speed range is 1c-2c. 2c is the dividing point between one-dimensional motion in space (less than c) and one-dimensional motion in time (more than 3c from our perspective). Between 1c and 2c, the velocity equation flips, and the motion is in *time* for the individual particles, but the object as a *whole* remains in 3-D space (and may still have *spatial motion*, so that the *total motion is two-dimensional*). The speeds are *discretized* into units above 1c of 0, 1, 1.5, 2, 3, and 4. This brings the motion to the *neutral point* within the range of 1c to 2c. Beyond that the motion is (4), (3), (2), (1.5), and (1). There are thus a total of *11 speeds n* between 1c and 2c and thus *11 possible planets maximum*, in which the matter *aggregates* because the *units are at the same relative speed*. For the first half, the *inner planets*, the *semi-major axis* is proportional to n^2, whereas for the second half, the *outer planets*, the semi-major axis is proportional to $1/(n^2)$. Let a0 = the semi-major axis of the orbit of the closest planet to the star, in units of AU, and let k = an empirical coefficient--this value depends on the *amount of Substance B* and the *violence* of the supernova and so has to be determined by *iteration or least squares*. For the *inner* planets,

$$an := a0 + k \cdot n^2 \quad \text{AU} \qquad (92)$$

For the *outer* planets:

$$an := a0 + k \cdot \frac{288}{n^2} \quad \text{AU} \qquad (93)$$

where n = the *speed number given above* and the constant 288 comes from *matching* Eqs. (92) and (93) at the *neutral point* between 1c and 2c. See Ref. [1] for a more detailed derivation.

Seven stellar systems have been studied, and are in the *Macrocosmos* Module of the *Reciprocal System Database*, with the following results:

Solar System: a0 = 0.3871 AU, k = 0.276 (Ref. [1] gives a0 = 0.4 and k = 0.267, but the values given here give a slightly better fit)
Gliese 581: a0 = 0.0282 AU, k = 0.0124
HD 10180: a0 = 0.02 AU, k = 0.1100
HD 40307: a0 = 0.0468 AU, k = 0.033
Kepler 11: a0 = 0.0910, k = 0.0013
Kepler 186: a0 = 0.0378, k = 0.02
Tau Cet: a0 = 0.1050, k = 0.1

Please note that these values are for the *initial positions* of the planets after formation and do not take into account the *migration* of the planets over time, due to *clearing of the orbits of debris* and the *gravitational pull of the other planets.* Perhaps the empirical k values could be further tweaked, but the agreement is quite stunning nonetheless. Eqs. (92) and (93) can even be used to *predict* the existence of other planets for these systems. For example, for Kepler 11, planet positions 3 and 4 are *missing*: there should be a planet (or debris) at 0.2178 AU and one at 0.3578 AU!

H. Gravitational Limits

1. Limits in the Reciprocal System: *gravitational limit*, *mass limit*, *age limit.* Everything is *finite*, and there will be no "heat death"; the *cyclical universe* is similar to a *pendulum*: *gravitation concentrates motionless matter into stars*, attaining *high heat*--and then these stars eventually *explode*, bringing the *matter back to the cold dispersed state.*

2. First Gravitational Limit: the immense region within a star's gravitational limit is *reserved for that star alone*; however, the *central regions* of globular clusters and galaxies are *denser*--but this is because they are in *equivalent space* rather than in *actual space*--therefore there is no contradiction here. The Magellanic Clouds are on their way to capture by the Galaxy (they are *within the first gravitational limit* of the Galaxy); *tidal forces* are causing their original spiral structures to become *irregular*.

3. Andromeda and the Milky Way will combine, and then be joined by the remainder of the members of the Local Group to form a *large Spheroidal Galaxy.*

4. Second Gravitational Limit: this is reached when the *opposing force of gravity (to the space-time progression) drops below its lowest level*--the object then moves out at the *speed of light.*

I. Radiation Limits

1. From Ref. [1], p. 202: 11.67 microns is the *dividing line between motion in space and in time.* "Inasmuch as the natural unit in vibrational motion is a half cycle, the cycle is a double unit. The wavelength corresponding to unit speed is therefore two natural units of distance, or 9.118×10^{-6} cm. The distribution over 128 positions increases the effective distance to 1.167×10^{-3} cm (11.67 microns). This, then, is the *effective boundary* between motion in space and motion in time, as observed in the material sector. On the high frequency (short wavelength) side of the boundary there is first the *near infrared*, from 1.167×10^{-3} cm to 7×10^{-5} cm, next the *optical* region from the infrared boundary to 4×10^{-5} cm, and finally the *x-ray* and *gamma-ray* regions at the highest frequencies. Because of the *reciprocal relation* between space and time these high frequency regions are *duplicated on the low frequency (long wavelength) side of the neutral level.*"

2. At normal speeds, the radiation is *high frequency*; at intermediate speeds, the radiation is *low frequency* (like near IR); at upper speeds the radiation is in *radio and far IR frequencies;* upper speeds are produced by direct addition of speed units not by addition of energy units; analogous to the photoelectric effect--where the frequency of the incoming light must be high enough to create the charge on the massless, chargeless electron and sufficient kinetic energy to eject the now charged electron.

J. Type II Supernovae; Intermediate and Upper Range Speeds; Pulsars and Jets

1. Attaining the *age limit* of a star results in a Type II Supernova. *Neutrino magnetic ionization* increases as the star *ages*; when either the temperature or age limit is reached, part of the matter is *dispersed in space and part is dispersed in time.* The *oldest matter* in the star is at the *highest* magnetic ionization level and thus this is the part of the star that comes *first* to the age limit; whereas all Type I supernovae are *alike*, because they all stem from *hot, massive stars*,Type II supernovae can occur in *any* size star.

2. Whereas in Type I Supernovae, a White Dwarf results, in Type II Supernovae Pulsars result. Conventional astronomers believe that Supernova 1987A resulted from a hot, massive star, and so they classify it as Type II; however, it is more likely that it is Type I--*no Pulsar has been found*. The White Dwarf is probably in the *invisible stage* (Stage 1) and so cannot be observed currently.

3. A Type I event (if it occurs at all) must o*ccur before* a Type II event; Type II events should be in *older* galaxies (with the usual exception of *captured strays* in younger objects); thus, Type II events are not normally observed in the Magellanic Clouds, which are younger objects.

4. A Type II Supernova converts a *larger proportion* of mass into energy, but the *total mass* participating in the explosion is usually *less* than that of a Type I explosion; the optical radiation (coming from the spatially outward moving material) is less than that of the Type I because *less is moving in this manner--more is moving outward in time (inward in space)*; so observed average magnitude is -18.6 for Type I and -16.5 for Type II. Also that from Type II drops off more rapidly at first than that from Type I, so the *light curves* are quite different; the remnant moving at *upper range speeds* (which will eventually become a *Pulsar*) is *not optically visible at first*; the *long-continued radio emission* of the remnants of the Type II Supernovae is due to the *presence of the upper range products*; the *continued supply of energy* is due to *radioactivit*y in the *local concentration of upper range matter in the remnants*; whereas Type I events have observable radio emission for about 3000 years, those from Type II events have observable radio emission fro about 100000 years. Example: Cygnus Loop is Type II and is estimated to be 60000 years old; the historical Type I Supernovae (1572, 1604) have much less. The Crab Nebula (from 1006) is Type II--this has a *chaotic network of filaments*--which are *jets of matter* moving at *ultra-high speed* (i.e., with space and time *opposite* to that of Pulsars); the radiation is *continuous in the intermediate range speed*, almost like it's a *solid*--but it's really in the *temporal equivalent* of the *solid state* (the *space region*).

5. The particles in the jets are in the *ultra-high speed range* and thus radiate as *line spectra*; all radiation as emitted is *polarized and non-thermal*; the *intermediate* and *ultra-high speed zone* is by nature *two-dimensional.*

6. The stationary filaments of Cassiopeia A *substitute* for a Pulsar (meaning that the *Type II explosion was not sufficiently energetic to produce one*; the *pre-explosion star* was probably a *red or orange giant*; the intermediate speed products are *entrained in the outgoing low speed matter.*

7. The *intermediate speed zone* (between 1 to 2 natural units of speed) *cannot* be represented properly by either a *3-D spatial (time-space) or temporal (space-time) reference frame*; inside unit space = time region; inside unit time = space region; the intermediate zone is *purely scalar*; in the time-space region a *minor motion in time modifies* a motion in space, and *vice versa* (the *Lorentz transformations*). Doppler shifts *do* measure the *total motion* in the *upper range speed* of 2-3 units; here the *total equivalent spatial speed* is $z + 3.5 \times z^{1/2}$ where z = normal redshift; radiation from an object moving faster than c comes to us *through time*, rather than space! These speeds are *two-dimensional* by nature.

8. The *ultra-high speed zone* is between 2-3 units natural units of speed. A Type II Supernova adds an *outward translational motion* in a *second dimension of space* to the *outward* (inward from the spatial standpoint) *expansion in time*; this is a White Dwarf, but one that a*lso has an outward translational motion*--a Pulsar.

9. But there is an *opposing gravitational motion* to this explosion which *reduces* the speed below 2--so the object is *observable*; eventually the gravitational force is reduced so much so that the speed *goes above 2* and the object *disappears into the cosmic region* (space-time region), where gravitation then moves the object's particles *together in time--away from each other in space.*

10. Some of these objects may encounter material bodies on their way out and so may *return* to *below* the 2 unit limit--these are the incoming X-ray Pulsars. These then evolve like normal White Dwarfs.

11. An *alternate* result of this explosion is possible: *expansion in space* (contraction in time) and *translation in time.* The observed form is a *greatly extended jet*, divided into *oppositely directed streams of ultra-high speed particles*--a *dumbbell shape*; sometimes one of the directions is *impeded*, in which case the result is a (mostly) *one-sided jet.*

K. Pulsars--Ages, Pulsation Periods, Radii

1. *Weaker* Type II Supernovae result in White Dwarfs, like those from Type I, but usually *smaller*; stronger Type II Supernovae result in Pulsars, which have a *translational motion added* (in the range of 100 km/sec). In the *usual* case, these Pulsars reach the *two-unit net explosion speed* and *disappear into the cosmic sector*; those that do not, *stay* in the material sector; the radiation is *intermittent,* hence the name "Pulsar."

2. All known Pulsars are associated with Type II events. The variation of the *gravitational restraint* explains the difference in observed Pulsar speeds; the *inward gravitational motion is visible*, but the Pulsar *outward translational motion is not visible* in our reference system; hence, for instance the Crab Pulsar is moving very slowly because the gravitational retardation is *weak.* Pulsars produced by *isolated stars* in the outer regions of the Galaxy appear to move quite *slowly* and are located *in or near the remnants*, whereas those produced in *central locations* will be moving more *rapidly,* and most of them are found *well away from the Galactic plane.* The *absence* of most Pulsars from the observable remnants is due to *their movement away.*

3. The great majority of Type II explosions take place in the *central region* of the Galaxy (the *oldest* part of the Galaxy), and so may be *unobservable.*

4. The Pulsars do *not* supply the energy observed coming from the *outward moving remnants*; this comes from the *radioactivity* in these remnants.

5. Radiation from a Pulsar *beyond* the gravitational limit is received at the *same strength* as that from one at the *gravitational limit*, but only during a *constantly decreasing proportion* of the *total time.* Therefore all Pulsar periods are *lengthening* (except, of course, from the pulsating x-ray emitters which are coming back down).

6. Now let's calculate Pulsar ages and periods. Empirically, the Pulsar age unit is

$$age_{Pulsar\_unit} := 3.25 \cdot 10^{5} \quad yr \qquad (94)$$

Then, with n = the distribution speed factor (1, 1.5, 2, 3, 4, 5, 6, 7, or 8) and P = Pulsar period (sec),

$$age_{Pulsar} := age_{Pulsar\_unit} \cdot n \cdot \left(\frac{P}{.62}\right)^{2} \quad yr \qquad (95a)$$

(The .62 value for the longest period is also empirical.)

For the Vela Pulsar, n = 1.5 and P = .089 sec. So

$$age_{Pulsar\_Vela} := age_{Pulsar\_unit} \cdot 1.5 \cdot \left(\frac{.089}{.62}\right)^{2} \qquad age_{Pulsar\_Vela} = 10045.49 \quad yr$$

For the Crab Pulsar, n = 1.0 and P = .033 sec. So

$$age_{Pulsar\_Crab} := age_{Pulsar\_unit} \cdot 1.0 \cdot \left(\frac{.033}{.62}\right)^{2} \qquad age_{Pulsar\_Crab} = 920.72 \quad yr$$

If we know the age of the Pulsar, we can calculate P for each possible value of n.

$$P := \left(\frac{age_{Pulsar}}{age_{Pulsar\_unit} \cdot n}\right)^{.5} \cdot .62 \quad sec \qquad (95b)$$

The millisecond Pulsars must be very, very young.

7. Type S Pulsars have a *globular structure*, whereas Type C pulsars (which are older) have a *double structure.*

8. All radiation from *upper speed objects* is *polarized (two-dimensional)*; but may be *depolarized en route* to us.

9. The space equivalent of the maximum Pulsar period of .62 seconds is

$$s_{Pulsar\_equiv} := .62 \cdot c_{cgs} \qquad s_{Pulsar\_equiv} = 1.858713 \times 10^{10} \quad \text{cm/sec} \qquad (96)$$

Empirically, the *width* of an average Pulsar pulse is 3% of this, or

$$s_{pulse\_width} := .03 \cdot s_{Pulsar\_equiv} \qquad s_{pulse\_width} = 5.576141 \times 10^{8} \qquad (97)$$

This implies that the radius of a Pulsar (maximum) is approximately

$$R_{Pulsar} := \frac{1}{2} \cdot s_{pulse\_width} \qquad R_{Pulsar} = 2.78807 \times 10^{8} \quad \text{cm} \qquad (98)$$

So the ratio of this to that of our Sun is:

$$\frac{R_{Pulsar}}{R_{S\_cgs}} = 0.004008$$

This is within the White Dwarf size range. (Of course, depending on the explosion, a Pulsar could be much smaller.)

10. The equatorial rotational speed of a Pulsar is, by inspection,

$$v_{Pulsar\_equat} := 2\cdot\pi\cdot\frac{1}{P}\cdot R_{Pulsar} \quad \text{cm/sec} \qquad (100)$$

So, for example:

$P := .62 \quad R_{Pulsar} := 2.278807\cdot10^{8} \quad \text{cm}$

$v_{Pulsar\_equat} := 2\cdot\pi\cdot\frac{1}{P}\cdot R_{Pulsar} \quad v_{Pulsar\_equat} = 2.309382\times10^{9} \text{ cm/sec}$

11. Extreme *high density aggregates* are *not* neutron stars or black holes. They are very dense because their *component speeds exceed the speed of light--and the velocity equation flips to v =t/s--the motion is in time.* From Ref. [1], p. 240: "All of the stars with extremely high density, regardless of whether we observe them as White Dwarfs, Novae, Pulsars, X-ray emitters, or unidentified sources of radio emission are *identically the same kind of object*, differing only in their speeds and in the current *stage* of their *radioactivity. Quasars* are objects of the same nature, in which the *extremely fast-moving components are stars*, rather than atoms and particles."

L. Stars in General--the Four Forms of Radiation

1. The radiation from Quasars (compact groups of stars ejected from exploding galaxies) is *continuous*. The only solution conventional astronomers can think of is "synchrotron radiation" and "inverse Compton" processes--but these are simply *incidental* processes; besides, large regions of space are *electrically neutral,* and so a different theory of *non-thermal energy* is required; *all* of this radiation comes from objects at speeds *above* that of light!

2. Four major *forms* of radiation exist: 1) thermal; 2) non-thermal--radio waves, x-ray, gamma rays; 3) inverse thermal; and 4) inverse non-thermal.

3. *Thermal radiation* is produced by *ordinary matter moving at less than speed c*; its wavelength is shorter than 11.67 microns; *matter at temperatures above c* produces *inverse thermal radiation*--its wavelength is greater than 11.67 microns; *strong* inverse thermal radiation at wavelengths greater than 11.67 microns identifies the emitter as one whose components are moving at *upper range speeds*; anything additional will be in the *radio range*.

4. A change in the *atomic structure* produces *x-rays* and *gamma rays--non-thermal radiation*; *inverse non-thermal radiation* produces *radio waves.*

5. Neutrino magnetic ionization is the *two-dimensional analog* to *electric ionization*; when matter is accelerated to the *intermediate speed range* (1-2 units), the motion is in *time*, and so the neutrinos are *no longer trapped* in the atoms--they can *move out to the coordinate time* separating the atoms. The atoms then become *unstable*, emitting *radio waves* to eliminate some of their mass.

6. As the speed of such intermediate speed objects decreases, the matter *returns to the ordinary matter* of the material sector, and the *neutrinos come back in.* Now the *isotopic adjustments* take place *inversely* by *x-rays* and *gamma rays*. Of course, the half-lives of the radioactive atoms vary from seconds to billions of years. Note: local temporary reversals back to intermediate speed cause radio waves.

7. Sources of radio waves: ordinary White Dwarfs, Pulsars, central stars of Planetary Nebulae.

8. The evolutionary direction of *gravitational potential energy* is opposite to that of *kinetic energy*; thus there is no "heat death." The Second Law of thermodynamics applies *only to kinetic energy*; there is thus a *cyclic movement* between kinetic energy and potential energy!

9. Stage 1 White Dwarf: immediate post-ejection stage; *radio frequency* radiation is due to *isotopic readjustments* required to bring some of the components of the star back to the atomic zone of stability (because the neutrinos harbored by the atoms have moved out). Unit neutrino magnetization level is normal in the outer regions of the Galaxy.

10. Stage 2 White Dwarf: end of the *outward travel in time* of the White Dwarfs; isotopic transformations have completed and so there is *little or no radio emission* at this stage.

11. Stage 3 White Dwarf: White Dwarfs (usually in Planetary Nebulae) are now *observable*; accretion of some of the debris now occurs--observable radiation comes mainly from the low speed matter accreted.

12. Stage 4 White Dwarf: return to the speed range *below unity.* The *volumetric change* now increases the neutrino concentration, *restoring* the unit neutrino magnetization level--so now *isotopic readjustments* occur--radioactivity. Radiation is now in the *x-ray (both hard and soft) range*; the Stage 4 White Dwarfs--the Cataclysmic Variables--are therefore *x-ray emitters*; all x-ray emitters are members of *binary systems* or *supernova remnants.*

13. Eventually the White Dwarfs reach the Main Sequence; x-ray radiation ends.

14. A *small portion* of Pulsars may *not* reach the net 2 unit speed limit and disappear into the cosmic sector. They might, therefore, drop *below unit speed* and start *emitting x-rays, pulsed*, which is the *opposite of when they crossed the unit speed boundary* and *started emitting radio waves.* Eventually they revert to normal White Dwarfs.

15. HZ Herculis is a Pulsar now coming back down to the Galactic plane; this motion is in a *second scalar dimension* of motion, which is *normally not observable*--but is because of the effect of *gravity.* These incoming pulsars are *older* than outgoing ones.

16. Cygnus X-1 is an x-ray star pf 6-10 solar masses, whereas optically it is 10 solar masses. There is *no mass limit* of White Dwarfs in the RS unlike in conventional theory. Cygnus X-1 is certainly *not* a "black hole"!

17. The periods of the *incoming* Pulsars are *decreasing*; the periods of the *outgoing* Pulsars are *increasing.*

18. The x-rays in the space around the giant galaxies *cannot be thermally produced* (radiation does *not increase* the temperature of the surrounding gas *if that gas is free to expand*); the x-rays must be generated by a *non-thermal* process--here from *leakage of ultra-high speed or intermediate speed matter* from the *high pressure region* in the *interiors of the giant galaxies.* When this matter falls back *below unit speed*, *x-rays* are emitted;

19. Leakage of *intermediate speed matter* from a star's *core* causes the star's *corona*; when this matter comes back *below unit speed*, x-rays are emitted. Leakage is *easier in smaller M-class stars*, which have *less overlying material.*

Conclusion

This paper has presented a *computational* version of the Reciprocal System theory of stellar structure and evolution. This new paradigm is very different from that of conventional theory. *Gravitational segregation* exists in all classes of stars, rather than thorough *mixing*, as commonly assumed. Stellar energy generation is due to *fission* of *heavy elements* in the core, not *fusion* of *light elements*. Stars move *up* the Main Sequence, rather than staying put once formed; successively less heavy elements are made available for fission. At the *temperature* or *mass limit*, when Fe becomes the fission element, a Supernova Type I occurs, creating a White Dwarf-Red Giant pair (or possibly a planetary system). These stars then eventually *return* to the Main Sequence to begin another *cycle*. After several cycles, the *age limit* is reached, resulting in a Supernova Type II and creating a nebula (which disperses) and a Pulsar--which usually leaves our material sector for the cosmic (inverse) sector. Whereas Main Sequence stars and Red Giants have a *linear density gradient* (decreasing from core to surface), White Dwarfs and Pulsars have an *inverse linear density gradient* (decreasing from surface to core); at present, observation *cannot verify* these density gradients, but they are *consistent* with the Postulates of the Reciprocal System. Plots of density, pressure, and temperature for the major classes of stars, from the surface to the core or vice versa, are exhibited. The theory is in agreement with all *known* facts of the macrocosmos.

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Appendix A: The Postulates of the Reciprocal System

FIRST FUNDAMENTAL POSTULATE: The physical universe is composed entirely of one component, *space-time* or *motion in the most general sense*, existing in three dimensions, in discrete units, and with two reciprocal aspects, space and time.

SECOND FUNDAMENTAL POSTULATE: The physical universe conforms to the relations of ordinary commutative mathematics, its magnitudes are absolute, and its geometry is Euclidean.

THIRD FUNDAMENTAL POSTULATE (IMPLIED): The physical universe is logical, orderly, and rational.

These Postulates give rise to a *complete theoretical universe*, which is then compared item by item with the *real physical universe*. The Reciprocal System Database has *thousands* of such comparisons--and these *verify* the theory!

Appendix B: Stellar Mass Accretion Rate and Time Scale

The flow of matter--dust and gas--into a star within its gravitational limit is *spherically symmetric*. Let

c_s = speed of sound in the dust and gas surrounding the star, cm/sec

γ = adiabatic index or ratio of specific heats

ρ_{surr} = average density of dust and gas surrounding the star, g/cm^3

By definition:

$$c_s := \left(\gamma \cdot \frac{R_{gas} \cdot conv_{atmtodynescm2}}{w} \cdot T\right)^{.5} \qquad \text{cm/sec} \qquad \text{(B-1)}$$

Then, adapting Eq. (8.133) of Ref. [14] (Vol. 1),

$$\frac{d}{dt}M_{star\_cgs} := \pi \cdot G_{cgs}{}^2 \cdot M_{star\_cgs} \cdot \frac{\rho_{surr}}{c_s{}^3} \cdot \left(\frac{2}{5 - 3 \cdot \gamma}\right)^{\frac{(5-3\cdot\gamma)}{2\cdot(\gamma-1)}} \qquad \text{g/sec} \qquad \text{(B-2a)}$$

The constants can all be combined into k_{accr}:

$$k_{accr} := \pi \cdot G_{cgs}^{2} \cdot \frac{\rho_{surr}}{c_s^{3}} \cdot \left(\frac{2}{5-3\cdot\gamma}\right)^{\frac{(5-3\cdot\gamma)}{2\cdot(\gamma-1)}} \qquad \text{(B-3)}$$

So Eq. (B-2a) can be expressed as

$$\frac{d}{dt}M_{star\_cgs} := k_{accr}\cdot M_{star\_cgs}^{2} \quad \text{g/sec} \qquad \text{(B-2b)}$$

Separating variables and integrating:

$$\int_{M_i}^{M_f} \frac{1}{M_{star\_cgs}^{2}}\, dM_{star\_cgs} = k_{accr}\cdot\int_{t_i}^{t_f} 1\, dt \qquad \text{(B-4)}$$

Solving:

$$\frac{M_f - M_i}{M_f\cdot M_i} = k_{accr}\cdot\left(t_f - t_i\right) \qquad \text{(B-5a)}$$

Or

$$M_f := \left| \frac{M_i}{M_i \cdot k_{accr} \cdot (t_f - t_i) - 1} \right| \quad g \qquad \text{(B-5b)}$$

if t_f is known. Or, if M_f is known or assumed, then

$$t_f := \frac{k_{accr} \cdot t_i + \frac{M_f - M_i}{M_f \cdot M_i}}{k_{accr}} \quad sec \qquad \text{(B-5c)}$$

worked example--the Sun

$\rho_{surr} := 10^{-22}$ g/cm^3 (average for interplanetary space)

$w := 1.205$ (average over all elements)

$T := 150$ K (very approx. average for interplanetary space)

$\gamma := 1.333$ (for polyatomic particles)

$$c_s := \left(\gamma \cdot \frac{R_{gas} \cdot conv_{atmtodynescm2}}{w} \cdot T \right)^{.5} \qquad c_s = 1.174437 \times 10^5 \quad \text{cm/sec}$$

$$k_{accr} := \pi \cdot G_{cgs}{}^2 \cdot \frac{\rho_{surr}}{c_s{}^3} \cdot \left(\frac{2}{5-3\cdot\gamma}\right)^{\frac{(5-3\cdot\gamma)}{2\cdot(\gamma-1)}} \qquad k_{accr} = 2.44369 \times 10^{-51}$$

$$t_i := \frac{4.5 \cdot 10^9}{conv_{sec\_to\_yr}} \quad t_i = 1.420006 \times 10^{17} \text{ sec}$$ (current age of Sun)

$$M_i := M_{S\_cgs} \quad M_i = 1.9884 \times 10^{33} \text{ g}$$ (current mass of Sun)

$$M_f := 31.89164 \cdot M_i \qquad M_f = 6.341334 \times 10^{34} \text{ g}$$ (assumed mass just prior to Supernova Type I)

$$t_f := \frac{k_{accr} \cdot t_i + \frac{M_f - M_i}{M_f \cdot M_i}}{k_{accr}} \qquad t_f = 3.413497 \times 10^{17} \text{ sec} \qquad t_f \cdot conv_{sec\_to\_yr} = 1.081737 \times 10^{10} \text{ yr}$$

(which seems to be about right)

Or, solving for mass M_f instead, with t_f known:

$$M_f := \left| \frac{M_i}{M_i \cdot k_{accr} \cdot (t_f - t_i) - 1} \right| \qquad M_f = 6.341334 \times 10^{34} \text{ g}$$ (checks)

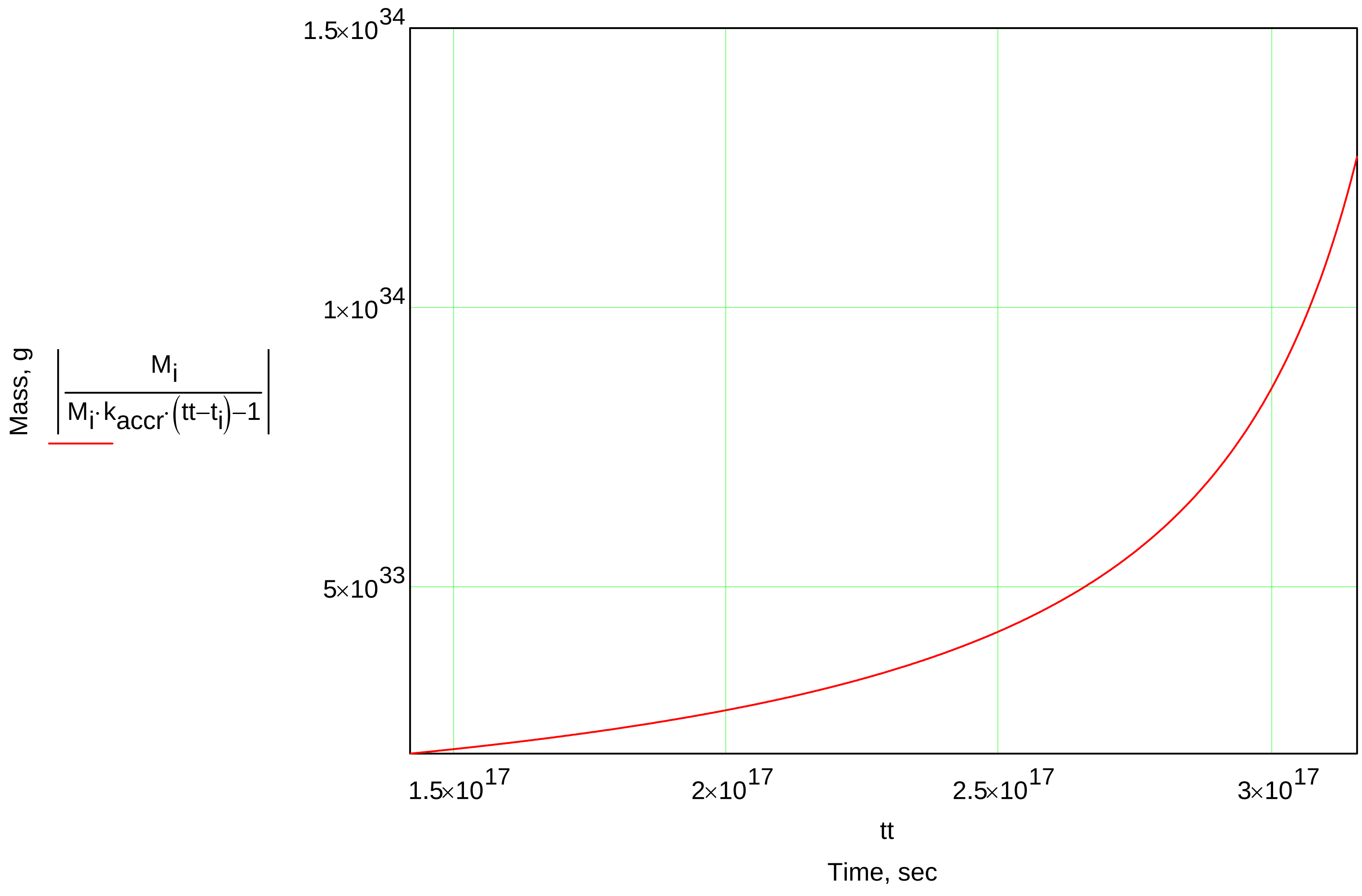

Figure B-1. Mass Increase of the Sun from the Present to Its End

Appendix C: Review of J. Pasachoff, A. Filippenko, *The Cosmos: Astronomy in the New Millennium*, 4th ed.

This work is an up-to-date discursive treatment of contemporary astronomy. It can serve as a supplementary text for scientists and engineers, because it contains much new observational information from recent space missions and from telescopes using adaptive optics.

The chapter titles are as follows:

1. A Grand Tour of the Heavens
2. Light, Matter, and Energy: Powering the Universe
3. Light and Telescopes: Extending Our Senses
4. Observing the Stars and Planets: Clockwork of the Universe
5. Gravitation and Motion: The Early History of Astronomy
6. The Terrestrial Planets: Earth, Moon, and Their Relatives
7. The Jovian Planets: Windswept Planets
8. Pluto, Comets, and Space Debris
9. Our Solar System and Others
10. Our Star: The Sun
11. Stars: Distant Suns
12. How the Stars Shine: Cosmic Furnaces
13. The Death of Stars: Recycling
14. Black Holes: The End of Space and Time
15. The Milky Way: Our Home in the Universe
16. A Universe of Galaxies
17. Quasars and Active Galaxies
18. Cosmology: The Birth and Life of the Cosmos
19. In the Beginning
20. Life in the Universe

There are 7 Appendixes, a Bibliography, a fine glossary, and a very detailed index.

The authors are excellent observers and educators. However, they are not theoreticians--nor do they claim to be. They present the conventional, consensus theories, with little in the way of critical analysis. I'm not going to "beat up" the authors because of this--you'll find the same nonsense in most other books in astronomy and astrophysics.

To help the reader of this review understand that the conventional theories are not on very solid ground, I'm going to give a brief critique of them here.

1. Space is flat, Euclidean; there is no sign of curved space, or even how matter could curve space. But the authors tell us that, nonetheless, General Relativity is correct or mostly correct. They do, however, mention that Quantum Mechanics and General Relativity are in conflict, so one or both are wrong.

2. The so-called Big Bang is a singularity, in which–allegedly--all the matter and energy of the universe are concentrated into a dot smaller than an atom. This is utter nonsense of course; singularities cannot exist in the real world. This concentration of matter and energy breaks every known physical law. But with the discovery of the acceleration of the universe, even the Big Bang is not enough for the theorists--they now postulate the existence of "dark energy" to explain this. The Reciprocal System of theory, not discussed by the authors, states that the space-time progression is the cause of the expansion; the acceleration is caused by the reduction of the opposing gravitational force by the inverse square law.

3. The cosmic microwave background radiation is, supposedly, the afterglow of the Big Bang. This is cited by the authors as the main support for this theory. But this radiation could equally well be the diffuse, isotropic radiation coming from another region of the universe. In the Reciprocal System of theory, this region is the cosmic sector--it is localized in time, not space.

4. Radiation is stated as being "electromagnetic." But light does not cause any changes in other, known, electric or magnetic fields. Also, conventional theory mixes up waves and particles.

5. Atoms are allegedly composed of subatoms, and subatoms are allegedly composed of quarks--having fractional charges. But no one has seen a fractional charge! Also, if I put a negatively-charged subatom next to a positively-charged subatom, the one doesn't orbit the other; the charges neutralize themselves! Also, if I put two positively-charged subatoms next to one another, they don't attract one another -they repel one another! The nuclear atom is therefore a bizarre mythology which has been foisted on us for over a hundred years. The reality is that atoms and subatoms are rotating photons, with quantized spins. Spectra result from changes in the discrete translational motions of atoms, not from electrons "jumping" from one orbit to another. Heck, the spectroscopists do not even accept the "cloud" theory of the electrons, touted by other scientists. How many more decades is this nonsensical theory going to be taught?

6. The authors assert, as in other books, that fusion must be the source of energy in stars. However, the supposed temperature in stellar cores is not high enough to overcome the Coulombic barrier. More advanced texts assert that the protons must "tunnel" through the barrier to fuse--but this is just another Quantum Mechanics myth. The reality is that stars are gravitationally-segregated; the more massive atoms are in the core. The actual source of energy in stars is fission of these heavy atoms!

7. The authors also assert that heavy elements result from fusion in the core or from supernovae, not from forming outside the core. However, there is a very dense neutrino flux throughout the universe. Neutrinos usually pass through matter--but sometimes they are absorbed, causing an increase in atomic number of the absorbing element. Heavy elements are therefore created in intergalactic and interstellar space--not in stellar cores, where are they destroyed to create the energy we all see with. Iron has been found in intergalactic space!

8. Globular clusters continue to be formed in intergalactic space, contrary to the assertions of conventional theorists. They are therefore "young"--not "old"! They are eventually merged into galaxies, changing into galactic clusters--and these are therefore "old"--not "young." Proto-globular clusters are the source of the alleged "dark matter" in the universe (and specifically in the halo around the Galaxy).

9. Globular clusters sometimes combine to form small elliptical galaxies. These can combine to form spiral or larger elliptical galaxies. Present day theory doesn't distinguish very well between small and large elliptical galaxies--which should really be called "spheroidal" galaxies.

10. The spiral arms of spiral galaxies do wind up over time; there is no "winding" problem. When two spiral galaxies combine, the result is an elliptical galaxy. Incidentally, Kepler's Laws do not apply to galaxies; the spirals clearly indicate that we are dealing with the equivalent of heterogenous rotating fluid structures.

11. Supernovae Type I are not from exploding white dwarfs (which have only novas). They are exploding O and B and Wolf-Rayet stars. The remnant is a white dwarf. Supernovae Type II result from explosions of all kinds of stars when they reach their age limit; the bolometric magnitude is larger than that for Type I stars. The remnant is a pulsar; often the pulsar moves out of the explosion zone. Pulsars are not neutron stars; neutrons are not stable--they decay in 12-15 minutes! Pulsars are simply white dwarf stars with an additional translational motion.

12. Solar systems and binary stars (red giant plus white dwarf) are formed from supernovae, contrary to the conventional theory. If the explosion is powerful enough, a binary star system results, otherwise a solar system. The planets have most of the angular momentum because they are formed from the spinning iron-nickel core of the parent star. The new star comes from the gas and dust which exploded outward, rather than inward.

13. Olbers's Paradox is resolved by there being a limit to the age of galaxies; at the limit, they explode, creating quasars--which eventually leave our sector for the cosmic sector. The quasars are not at "cosmological" distances. Their redshift is a function of the redshift of their associated peculiar galaxy; they cannot therefore be blueshifted. Quasars and pulsars and white dwarfs are compact because they are expanding in coordinate time, rather than in coordinate space. Eventually white dwarfs come back to the main sequence; eventually red giants come back to the main sequence (this explains why Betelgeuse is shrinking); the so-called "turn-off" point is actually a "turn-in" point on the H-R diagram. Pulsars, if ejected powerfully enough, leave our sector; otherwise they come back to the plane of the Galaxy, generating x-rays.

14. The authors admit that the one way path to the heat death of the universe is "depressing." Fortunately, this is not the reality. The universe is cyclic. There is no heat death. The white dwarfs don't become black dwarfs; they come back to the main sequence. Our sector loses matter to the cosmic sector; the cosmic sector loses matter to our sector, as indicated by the cosmic rays. There is complete balance. There is no net change in entropy!

In conclusion, you can purchase the authors' book for an up-to-date treatment of current observations and the conventional theories. Just keep in mind that the Reciprocal System is vastly superior.

References: Dewey B. Larson, *The Universe of Motion*; Ronald W. Satz, The Unmysterious Universe; Ronald W. Satz, Transpower Corporation, *The Reciprocal System: Microcosmos Database*, www.reciprocalsystem.guru; Ronald W. Satz, "Theory of Stellar Structure and Evolution"--(this paper)

$$e^2 \cdot k_{B\_MeV}^2 - 6 \cdot T_{destructive} \cdot k_{B\_MeV} \cdot m_{MeV}$$

$$\cdots \frac{\cdots \dfrac{\cdots \dfrac{4\cdot R\cdot r_1^{\,3}}{3} + r_1^{\,4} + \dfrac{R\cdot \Delta M}{\pi\cdot \rho_c}\Big)}{243} + 256\cdot\left(\dfrac{R^4}{27} - \dfrac{4\cdot R\cdot r_1^{\,3}}{3} + r_1^{\,4} + \dfrac{R\cdot \Delta M}{\pi\cdot \rho_c}\right)^3}{\cdots} + \frac{8\cdot R^6}{243} - \frac{8\cdot R^2\cdot\left(\dfrac{R^4}{27} - \dfrac{4\cdot R\cdot r_1^{\,3}}{3} + r_1^{\,4} + \dfrac{R\cdot \Delta M}{\pi\cdot \rho_c}\right)}{9}\Bigg]^{\frac{2}{3}} - 12\cdot r_1^{\,4} + 4\cdot R^2\cdot\left[\frac{\sqrt{3}\cdots}{\cdots}\right.$$

$$6\cdot\left[\frac{\sqrt{3}\cdot\sqrt{\dfrac{512\cdot R^4\cdot\left(\dfrac{R^4}{27} - \dfrac{4\cdot R\cdot r_1^{\,3}}{3} + r_1^{\,4} + \dfrac{R\cdot \Delta M}{\pi\cdot \rho_c}\right)^2}{9} + \dfrac{2048\cdot R^{12}}{19683} - \dfrac{1280\cdot R^8\cdot\left(\dfrac{R^4}{27} - \dfrac{4\cdot R\cdot r_1^{\,3}}{3} + r_1^{\,4} + \dfrac{R\cdot \Delta M}{\pi\cdot \rho_c}\right)}{243} + 256\cdots}}{18}\right.$$

*

^{nv}dynescm2toatm

$$\frac{}{10}\cdot(w)^{\frac{1}{3}}$$

ynescm2toatm

$(w)^{\frac{1}{3}}$

*

3

$$\left.\frac{\text{ean}^{4}}{\text{R}^{4}}\right)\cdot conv_{dynescm2toatm}$$

$$\left.\begin{matrix} 1 \\ - \end{matrix}\right) \cdot conv_{dynescm2toatm}$$

*

if $i > 3$

$$\cdot 10^{27} \cdot T \cdot xlelem2_{i,10} \Big)$$

$${}^{27} \cdot T \cdot xlelem2_{j,10} \Big)$$

$$\cdot\sqrt{\frac{512\cdot R^4\cdot\left(\frac{R^4}{27}-\frac{4\cdot R\cdot r_1^{\,3}}{3}+r_1^{\,4}+\frac{R\cdot\Delta M}{\pi\cdot\rho_c}\right)^2}{9}+\frac{2048\cdot R^{12}}{19683}-\frac{1280\cdot R^8\cdot\left(\frac{R^4}{27}-\frac{4\cdot R\cdot r_1^{\,3}}{3}+r_1^{\,4}+\frac{R\cdot\Delta M}{\pi\cdot\rho_c}\right)}{243}+256\cdot\left(\frac{R^4}{27}-\frac{4\cdot R\cdot r_1^{\,3}}{3}+r_1^{\,4}+\frac{R\cdot\Delta}{\pi\cdot\rho_c}\right.}$$

$$\overline{\qquad 18 \qquad}$$

$$\left.\cdot\frac{\left(\frac{R^4}{27}-\frac{4\cdot R\cdot r_1^{\,3}}{3}+r_1^{\,4}+\frac{R\cdot\Delta M}{\pi\cdot\rho_c}\right)^3}{}+\frac{8\cdot R^6}{243}-\frac{8\cdot R^2\cdot\left(\frac{R^4}{27}-\frac{4\cdot R\cdot r_1^{\,3}}{3}+r_1^{\,4}+\frac{R\cdot\Delta M}{\pi\cdot\rho_c}\right)}{9}\right]^{\frac{1}{6}}$$

$$\cdots \left(\cdots \frac{\cdots M}{\cdots c} \right)^3 + \frac{8 \cdot R^6}{243} - \frac{8 \cdot R^2 \cdot \left(\frac{R^4}{27} - \frac{4 \cdot R \cdot r_1^{\,3}}{3} + r_1^{\,4} + \frac{R \cdot \Delta M}{\pi \cdot \rho_c} \right)}{9} \Bigg]^{\frac{1}{3}} - \frac{12 \cdot R \cdot \Delta M}{\pi \cdot \rho_c} \cdots$$

$$\cdots - \sqrt{12 \cdot \left(\frac{R^4}{27} - \frac{4 \cdot R \cdot r_1^{\,3}}{3} + r_1^{\,4} + \frac{R \cdot \Delta M}{\pi \cdot \rho_c} \right) \cdot \sqrt{16 \cdot R \cdot r_1^{\,3} + 9 \cdot \left[\frac{\sqrt{3} \cdot \sqrt{\frac{512 \cdot R \cdots}{\cdots}}}{\cdots} \cdots \right.}}$$

$$\frac{\frac{R^4\cdot\left(\frac{R^4}{27}-\frac{4\cdot R\cdot {r_1}^3}{3}+{r_1}^4+\frac{R\cdot\Delta M}{\pi\cdot\rho_c}\right)^2}{9}+\frac{2048\cdot R^{12}}{19683}-\frac{1280\cdot R^8\cdot\left(\frac{R^4}{27}-\frac{4\cdot R\cdot {r_1}^3}{3}+{r_1}^4+\frac{R\cdot\Delta M}{\pi\cdot\rho_c}\right)}{243}+256\cdot\left(\frac{R^4}{27}-\frac{4\cdot R\cdot {r_1}^3}{3}+{r_1}^4+\frac{R\cdot\Delta M}{\pi\cdot\rho_c}\right)^3}{18}+\frac{8}{2}$$

$$\cdot R^6 \over 243 - \frac{8\cdot R^2\cdot\left(\frac{R^4}{27} - \frac{4\cdot R\cdot r_1^{\,3}}{3} + r_1^{\,4} + \frac{R\cdot \Delta M}{\pi\cdot\rho_c}\right)}{9}\Bigg]^{\frac{2}{3}} - 12\cdot r_1^{\,4} + 4\cdot R^2\cdot\left[\frac{\sqrt{3}\cdot\sqrt{\frac{512\cdot R^4\cdot\left(\frac{R^4}{27} - \frac{4\cdot R\cdot r_1^{\,3}}{3} + r_1^{\,4} + \frac{R\cdot\Delta M}{\pi\cdot\rho_c}\right)^2}{9} + \frac{2048\cdot R^{12}}{19683} - \frac{1280\cdot R^8}{}}}{}\right.$$

$$\cdots 3\cdot\frac{\left(\frac{R^4}{27}-\frac{4\cdot R\cdot r_1^{\,3}}{3}+r_1^{\,4}+\frac{R\cdot\Delta M}{\pi\cdot\rho_c}\right)}{243}+256\cdot\left(\frac{R^4}{27}-\frac{4\cdot R\cdot r_1^{\,3}}{3}+r_1^{\,4}+\frac{R\cdot\Delta M}{\pi\cdot\rho_c}\right)^3 \Big/ \cdots 18+\frac{8\cdot R^6}{243}-\frac{8\cdot R^2\cdot\left(\frac{R^4}{27}-\frac{4\cdot R\cdot r_1^{\,3}}{3}+r_1^{\,4}+\frac{R\cdot\Delta M}{\pi\cdot\rho_c}\right)}{9}\Bigg]^{\frac{1}{3}}-\frac{12\cdot R\cdot\Delta M}{\pi\cdot\rho_c}$$

$$-9\cdot\left[\frac{\sqrt{3}\cdot\sqrt{\frac{512\cdot R^4\cdot\left(\frac{R^4}{27}-\frac{4\cdot R\cdot r_1^{\,3}}{3}+r_1^{\,4}+\frac{R\cdot\Delta M}{\pi\cdot\rho_c}\right)^2}{9}+\frac{2048\cdot R^{12}}{19683}-\frac{1280\cdot R^8\cdot\left(\frac{R^4}{27}-\frac{4\cdot R\cdot r_1^{\,3}}{3}+r_1^{\,4}+\frac{R\cdot\Delta M}{\pi\cdot\rho_c}\right)}{243}+256\cdot\left(\frac{R^4}{27}-\frac{4\cdot R\cdot r_1^{\,3}}{3}+r_1\right.}}{18}\right.$$

$$\ldots{}^{4} + \frac{R\cdot\Delta M}{\pi\cdot\rho_c}\Bigg)^3 \ldots + \frac{8\cdot R^6}{243} - \frac{8\cdot R^2\cdot\left(\frac{R^4}{27} - \frac{4\cdot R\cdot r_1^{\,3}}{3} + r_1^{\,4} + \frac{R\cdot\Delta M}{\pi\cdot\rho_c}\right)}{9}\Bigg]^{\frac{2}{3}} \cdot \sqrt{16\cdot R\cdot r_1^{\,3} + 9\cdot\left[\frac{\sqrt{3}\cdot\sqrt{\frac{512\cdot R^4\cdot\left(\frac{R^4}{27} - \frac{4\cdot R\cdot r_1^{\,3}}{3} + r_1^{\,4} + \frac{R\cdot\Delta M}{\pi\cdot\rho_c}\right)^2}{9} + \frac{2048\ldots}{19\ldots}\ldots}}{\ldots}\right.\ldots}$$

$$\frac{\cdots R^{12}}{\cdots 683} - \frac{1280 \cdot R^{8} \cdot \left(\frac{R^{4}}{27} - \frac{4 \cdot R \cdot r_{1}^{3}}{3} + r_{1}^{4} + \frac{R \cdot \Delta M}{\pi \cdot \rho_{c}}\right)}{243} + 256 \cdot \left(\frac{R^{4}}{27} - \frac{4 \cdot R \cdot r_{1}^{3}}{3} + r_{1}^{4} + \frac{R \cdot \Delta M}{\pi \cdot \rho_{c}}\right)^{3}$$

$$\cdots \frac{\cdots}{18} + \frac{8 \cdot R^{6}}{243} - \frac{8 \cdot R^{2} \cdot \left(\frac{R^{4}}{27} - \frac{4 \cdot R \cdot r_{1}^{3}}{3} + r_{1}^{4} + \frac{R \cdot \Delta M}{\pi \cdot \rho_{c}} \cdots\right.}{9} \cdots$$

$$\left.\right)\right]^{\frac{2}{3}} - 12\cdot r_1^{\,4} + 4\cdot R^2\cdot\left[\frac{\sqrt{3}\cdot\sqrt{\dfrac{512\cdot R^4\cdot\left(\dfrac{R^4}{27}-\dfrac{4\cdot R\cdot r_1^{\,3}}{3}+r_1^{\,4}+\dfrac{R\cdot\Delta M}{\pi\cdot\rho_c}\right)^2}{9}+\dfrac{2048\cdot R^{12}}{19683}-\dfrac{1280\cdot R^8\cdot\left(\dfrac{R^4}{27}-\dfrac{4\cdot R\cdot r_1^{\,3}}{3}+r_1^{\,4}+\dfrac{R\cdot\Delta M}{\pi\cdot\rho_c}\right)}{243}+256\cdot\left(\dfrac{R}{2}\right.}}{18}\right.$$

$$6\cdot\left[\frac{\sqrt{3}\cdot\sqrt{\dfrac{512\cdot R^4\cdot\left(\dfrac{R^4}{27}-\dfrac{4\cdot R\cdot r_1^{\,3}}{3}+r_1^{\,4}+\dfrac{R\cdot\Delta M}{\pi\cdot\rho_c}\right)^2}{9}+\dfrac{2048\cdot R^{12}}{19683}-\dfrac{1280\cdot R^8\cdot\left(\dfrac{R^4}{27}-\dfrac{4\cdot R\cdot r_1^{\,3}}{3}+r_1^{\,4}+\dfrac{R\cdot\Delta M}{\pi\cdot\rho_c}\right)}{243}+256\cdot\left(\dfrac{R^4}{27}-\dfrac{4\cdot R\cdot r_1^{\,3}}{3}+r\right.}}{18}\right.$$

$$\cdots \frac{\cdots^{4}}{\cdots 7} - \frac{4\cdot R\cdot {r_1}^{3}}{3} + {r_1}^{4} + \frac{R\cdot \Delta M}{\pi\cdot \rho_c}\Bigg)^{3} \Bigg/ \cdots + \frac{8\cdot R^{6}}{243} - \frac{8\cdot R^{2}\cdot\left(\frac{R^{4}}{27} - \frac{4\cdot R\cdot {r_1}^{3}}{3} + {r_1}^{4} + \frac{R\cdot \Delta M}{\pi\cdot \rho_c}\right)}{9}\Bigg]^{\frac{1}{3}} - \frac{12\cdot R\cdot \Delta M}{\pi\cdot \rho_c} - \frac{4\cdot R^{4}\cdot\sqrt{16\cdot R\cdot {r_1}^{3} + 9\cdot\left[\frac{\sqrt{3}\cdot\sqrt{\frac{512\cdot R^{4}\cdot\left(\frac{R^{\cdots}}{27}\cdots\right.}{\cdots}}}{\cdots}\right.}}{\cdots}$$

$$\cdots {}_{1}^{4} + \frac{R\cdot \Delta M}{\pi\cdot \rho_c}\Bigg)^{3} \Bigg/ \cdots + \frac{8\cdot R^{6}}{243} - \frac{8\cdot R^{2}\cdot\left(\frac{R^{4}}{27} - \frac{4\cdot R\cdot {r_1}^{3}}{3} + {r_1}^{4} + \frac{R\cdot \Delta M}{\pi\cdot \rho_c}\right)}{9}\Bigg]^{\frac{1}{6}} \cdot \left[16\cdot R\cdot {r_1}^{3} + 9\cdot\left[\sqrt{3}\cdot\sqrt{\frac{512\cdot R^{4}\cdot\left(\frac{R^{4}}{27} - \frac{4\cdot R\cdot {r_1}^{3}}{3} + {r_1}^{4} + \frac{R\cdot \Delta M}{\pi\cdot \rho_c}\right)^{2}}{9} + \frac{2048\cdots}{19\cdots}\cdots}\right.\right.$$

$$\frac{\frac{\left(\cdots - \frac{4\cdot R\cdot r_1^{\,3}}{3} + r_1^{\,4} + \frac{R\cdot\Delta M}{\pi\cdot\rho_c}\right)^2}{9} + \frac{2048\cdot R^{12}}{19683} - \frac{1280\cdot R^8\cdot\left(\frac{R^4}{27} - \frac{4\cdot R\cdot r_1^{\,3}}{3} + r_1^{\,4} + \frac{R\cdot\Delta M}{\pi\cdot\rho_c}\right)}{243} + 256\cdot\left(\frac{R^4}{27} - \frac{4\cdot R\cdot r_1^{\,3}}{3} + r_1^{\,4} + \frac{R\cdot\Delta M}{\pi\cdot\rho_c}\right)^3}{18} + \frac{8\cdot R^6}{243} - \cdots$$

$$\frac{\cdots\frac{\cdot R^{12}}{\cdots 683} - \frac{1280\cdot R^8\cdot\left(\frac{R^4}{27} - \frac{4\cdot R\cdot r_1^{\,3}}{3} + r_1^{\,4} + \frac{R\cdot\Delta M}{\pi\cdot\rho_c}\right)}{243} + 256\cdot\left(\frac{R^4}{27} - \frac{4\cdot R\cdot r_1^{\,3}}{3} + r_1^{\,4} + \frac{R\cdot\Delta M}{\pi\cdot\rho_c}\right)^3}{18} + \frac{8\cdot R^6}{243} - \frac{8\cdot R^2\cdot\left(\frac{R^4}{27} - \frac{4\cdot R\cdot r_1^{\,3}}{3} + r_1^{\,4} + \frac{R\cdot\Delta M}{\pi\cdot\rho_c}\cdots\right.}{9}$$

$$\frac{\left[\dfrac{8\cdot R^{2}\cdot\left(\dfrac{R^{4}}{27}-\dfrac{4\cdot R\cdot r_{1}^{3}}{3}+r_{1}^{4}+\dfrac{R\cdot\Delta M}{\pi\cdot\rho_{c}}\right)}{9}\right]^{\frac{2}{3}}-12\cdot r_{1}^{4}+4\cdot R^{2}\cdot\left[\dfrac{\sqrt{3}\cdot\sqrt{\dfrac{512\cdot R^{4}\cdot\left(\dfrac{R^{4}}{27}-\dfrac{4\cdot R\cdot r_{1}^{3}}{3}+r_{1}^{4}+\dfrac{R\cdot\Delta M}{\pi\cdot\rho_{c}}\right)^{2}}{9}+\dfrac{2048\cdot R^{12}}{19683}-\dfrac{1280\cdot R^{8}\cdot\left(\dfrac{R^{4}}{27}\right.}{}}}{18}\right.}{9}$$

$$\left.\right)\Bigg]^{\frac{2}{3}}-12\cdot r_{1}^{4}+4\cdot R^{2}\cdot\left[\dfrac{\sqrt{3}\cdot\sqrt{\dfrac{512\cdot R^{4}\cdot\left(\dfrac{R^{4}}{27}-\dfrac{4\cdot R\cdot r_{1}^{3}}{3}+r_{1}^{4}+\dfrac{R\cdot\Delta M}{\pi\cdot\rho_{c}}\right)^{2}}{9}+\dfrac{2048\cdot R^{12}}{19683}-\dfrac{1280\cdot R^{8}\cdot\left(\dfrac{R^{4}}{27}-\dfrac{4\cdot R\cdot r_{1}^{3}}{3}+r_{1}^{4}+\dfrac{R\cdot\Delta M}{\pi\cdot\rho_{c}}\right)}{243}+256\cdot\left(\dfrac{R}{2}\right.}}{18}\right.$$

$$\frac{\dfrac{\dfrac{\cdots - \dfrac{4\cdot R\cdot r_1^{\,3}}{3} + r_1^{\,4} + \dfrac{R\cdot\Delta M}{\pi\cdot\rho_c}\Big)}{243} + 256\cdot\left(\dfrac{R^4}{27} - \dfrac{4\cdot R\cdot r_1^{\,3}}{3} + r_1^{\,4} + \dfrac{R\cdot\Delta M}{\pi\cdot\rho_c}\right)^3}{\cdots} + \dfrac{8\cdot R^6}{243} - \dfrac{8\cdot R^2\cdot\left(\dfrac{R^4}{27} - \dfrac{4\cdot R\cdot r_1^{\,3}}{3} + r_1^{\,4} + \dfrac{R\cdot\Delta M}{\pi\cdot\rho_c}\right)}{9}\Bigg]^{\frac{1}{3}} - \dfrac{12\cdot R\cdot\Delta M}{\pi\cdot\rho_c}}{\cdots} + \frac{8\cdot\sqrt{\cdots}}{\cdots}$$

$$\frac{\cdots\dfrac{\cdots^4}{\cdots 7} - \dfrac{4\cdot R\cdot r_1^{\,3}}{3} + r_1^{\,4} + \dfrac{R\cdot\Delta M}{\pi\cdot\rho_c}\Big)^3}{\cdots} + \frac{8\cdot R^6}{243} - \frac{8\cdot R^2\cdot\left(\dfrac{R^4}{27} - \dfrac{4\cdot R\cdot r_1^{\,3}}{3} + r_1^{\,4} + \dfrac{R\cdot\Delta M}{\pi\cdot\rho_c}\right)}{9}\Bigg]^{\frac{1}{3}} - \frac{12\cdot R\cdot\Delta M}{\pi\cdot\rho_c}\Bigg]^{\frac{1}{4}}$$

$$\frac{\sqrt{6}\cdot R^3\cdot\sqrt{3\cdot\sqrt{3}\cdot\sqrt{\frac{512\cdot R^4\cdot\left(\frac{R^4}{27}-\frac{4\cdot R\cdot {r_1}^3}{3}+{r_1}^4+\frac{R\cdot\Delta M}{\pi\cdot\rho_c}\right)^2}{9}+\frac{2048\cdot R^{12}}{19683}-\frac{1280\cdot R^8\cdot\left(\frac{R^4}{27}-\frac{4\cdot R\cdot {r_1}^3}{3}+{r_1}^4+\frac{R\cdot\Delta M}{\pi\cdot\rho_c}\right)}{243}+256\cdot\left(\frac{R^4}{27}-\frac{4\cdot R\cdot {r_1}^3}{3}\right.}}}{9}$$

$$\cdots + r_1^4 + \frac{R\cdot\Delta M}{\pi\cdot\rho_c}\Bigg)^3 + \frac{16\cdot R^6}{9} - 48\cdot R^2\cdot\left(\frac{R^4}{27} - \frac{4\cdot R\cdot r_1^{\,3}}{3} + r_1^{\,4} + \frac{R\cdot\Delta M}{\pi\cdot\rho_c}\right) + 8\cdot R^2\cdot\left[\sqrt{3}\cdot\sqrt{\frac{512\cdot R^4\cdot\left(\frac{R^4}{27} - \frac{4\cdot R\cdot r_1^{\,3}}{3} + r_1^{\,4} + \frac{R\cdot\Delta M}{\pi\cdot\rho_c}\right)^2}{9} + \frac{2048\cdot R^{1\cdots}}{19683}\cdots}\right.$$

$$\cdots^{2} - \frac{\;\cdots - \dfrac{1280 \cdot R^{8} \cdot \left(\dfrac{R^{4}}{27} - \dfrac{4 \cdot R \cdot r_{1}^{3}}{3} + r_{1}^{4} + \dfrac{R \cdot \Delta M}{\pi \cdot \rho_{c}}\right)}{243} + 256 \cdot \left(\dfrac{R^{4}}{27} - \dfrac{4 \cdot R \cdot r_{1}^{3}}{3} + r_{1}^{4} + \dfrac{R \cdot \Delta M}{\pi \cdot \rho_{c}}\right)^{3}}{18} + \frac{8 \cdot R^{6}}{243} - \frac{8 \cdot R^{2} \cdot \left(\dfrac{R^{4}}{27} - \dfrac{4 \cdot R \cdot r_{1}^{3}}{3} + r_{1}^{4} + \dfrac{R \cdot \Delta M}{\pi \cdot \rho_{c}}\right)}{9} \Bigg]^{\frac{1}{3}}$$

$$\cdot\sqrt{16\cdot R\cdot r_1^{\,3}+9\cdot\left[\frac{\sqrt{3}\cdot\sqrt{\dfrac{512\cdot R^4\cdot\left(\dfrac{R^4}{27}-\dfrac{4\cdot R\cdot r_1^{\,3}}{3}+r_1^{\,4}+\dfrac{R\cdot\Delta M}{\pi\cdot\rho_c}\right)^2}{9}+\dfrac{2048\cdot R^{12}}{19683}-\dfrac{1280\cdot R^8\cdot\left(\dfrac{R^4}{27}-\dfrac{4\cdot R\cdot r_1^{\,3}}{3}+r_1^{\,4}+\dfrac{R\cdot\Delta M}{\pi\cdot\rho_c}\right)}{243}+256\cdot\left(\dfrac{R^4}{27}-4\right.}}{18}\right.}$$

$$\frac{\left(\frac{\cdot R\cdot r_1^3}{3}+r_1^4+\frac{R\cdot\Delta M}{\pi\cdot\rho_c}\right)^3}{}+\frac{8\cdot R^6}{243}-\frac{8\cdot R^2\cdot\left(\frac{R^4}{27}-\frac{4\cdot R\cdot r_1^3}{3}+r_1^4+\frac{R\cdot\Delta M}{\pi\cdot\rho_c}\right)}{9}\Bigg]^{\frac{2}{3}}-12\cdot r_1^4+4\cdot R^2\cdot\left[\frac{\sqrt{3}\cdot\sqrt{\frac{512\cdot R^4\cdot\left(\frac{R^4}{27}-\frac{4\cdot R\cdot r_1^3}{3}+r_1^4+\frac{R\cdot\Delta}{\pi\cdot\rho}\right.}{9}}}{}\right.$$

$$\frac{\frac{\left(\frac{\ldots M}{\ldots c}\right)^2 + \frac{2048 \cdot R^{12}}{19683} - \frac{1280 \cdot R^8 \cdot \left(\frac{R^4}{27} - \frac{4 \cdot R \cdot r_1{}^3}{3} + r_1{}^4 + \frac{R \cdot \Delta M}{\pi \cdot \rho_c}\right)}{243} + 256 \cdot \left(\frac{R^4}{27} - \frac{4 \cdot R \cdot r_1{}^3}{3} + r_1{}^4 + \frac{R \cdot \Delta M}{\pi \cdot \rho_c}\right)^3}{18} + \frac{8 \cdot R^6}{243} - \frac{8 \cdot R^2 \cdot \left(\frac{R^4}{27} - \frac{4 \cdot R \cdot r_1{}^3}{3} + r \ldots\right.}{9}}{\ldots}$$

$$\ldots 1^{4} + \frac{R \cdot \Delta M}{\pi \cdot \rho_c}\Bigg) \ldots \Bigg]^{\frac{1}{3}} - \frac{12 \cdot R \cdot \Delta M}{\pi \cdot \rho_c}$$