Theory of Microeconomics and Macroeconomics: Larsonian Econophysics
version 1.0

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Abstract

This paper presents a *computational* version of the theory of microeconomics and macroeconomics developed by scientist-engineer D. B. Larson. Unlike most economic theories, Larson's theory is based on *scientific* methodology, not sociology or "social science", and thus can be considered *econophysics*. It identifies the *fundamental component* of the economic *mechanism* and the *fundamental equation* underlying all economic transactions. The theory posits seventeen principles, from which all theoretical deductions are made; thus the theory is *axiomatic, unified,* and *general*. This paper derives detailed equations to solve the maximization problems of the representative producer-retailer, the representative worker-consumer, and the representative country as a whole: including the GDP, the growth rate, and the inflation rate. The paper shows how the *aggregate* price level is affected by numerous factors and shows the means by which it can be kept *constant*, while letting the free market set interest rates. The calculations are verified by comparison with actual economic data.

keywords: microeconomics, macroeconomics, econophysics, purchasing power theory, fundamental equation of economics

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Introduction and Literature Review

Dewey B. Larson is best known for his work on the Reciprocal System of theoretical physics; what's mostly unknown is that he was also a fine theoretical economist. Ref. [1] is his work on microeconomics; Ref. [2] is is his work on macroeconomics. Both also contain recommendations for government policy: Ref. [1] gives policy recommendations to maximize employment; Ref. [2] gives policy recommendations to eliminate the booms and busts of the business cycle so as to create permanent prosperity--a consistently growing economy. As with his work in theoretical physics, Larson begins his economics theory with a set of fundamental principles and states the fundamental component of the economic mechanism: *purchasing power*. He then gives the fundamental equation of economics: $p = B/V$, where p is the unit price, B is the nominal (money) purchasing power of the good or service, and V is the number of units. This paper will apply different forms of this equation to cover all of the major concerns of economics.

Conventional economic theory does not have a fundamental equation or a fundamental set of principles. Many different models of theoretical economics exist: classical, neo-classical, Keynesian, neo-Keynesian, monetarist, rational expectations, market-clearing, representative agent, dynamic stochastic general equilibrium, real business cycle, overlapping generations, and others. The author has reviewed selected works by the most prominent current economists: Mankiw, Abel, Bernamke, Fischer, Auerbach, Kotlikoff, Blanchard, Barro, Williamson, Nellis, Parker, Carlberg, Gartner, Turnovsky, Wessels, Miao, Romer, and Intriligator. The economics literature is vast, but the references at the end of this paper provide a representative sample.

Mankiw's work, Ref. [3], discusses the economy in the long run, in the very long run, and in the short run. At the end of the book, p. 600, he says, "The current state of macroeconomics offers many insights, but it also leaves many questions open." Mankiw is a neo-Keynesian.

The work by Abel, Bernanke, and Croushore, Ref. [4], focuses mostly on the conventional IS-LM/AD-AS model, based on the work of Keynes and Hicks. The empirical Cobb-Douglas production function is explained in detail and applied to the US from 1991 to 2010.

The work by Dornbusch, Fischer, and Startz, Ref. [5], is nicely organized and has numerous graphs. It focuses more on government policy than most of the other references.

Auerbach and Kotlikoff, Ref. [6], develop a two period overlapping generations model, mostly neo-classical: in the first period (lasting about 30 years) the worker saves for retirement and then in the second period (lasting about another 30 years) lives off the income from the capital invested in the first period. This is an interesting approach and is not inconsistent with the representative agent model utilized in our formulation of Larsonian econophysics.

Blanchard's and Johnson's work, Ref. [7], focuses on the IS/LM model and Keynesian multipliers and rational expectations--it is very much at odds with what we develop in this paper. Larsonian econophysics completely rejects all of the so-called multipliers.

Barro, Ref. [8], uses his market-clearing model and real business cycle theory to explain macroeconomics. To his credit, he discusses the Robinson Crusoe model (an isolated individual on an island), which is a good place to start in analyzing economic issues. But his equations and graphs are very abstract; there are no numerical calculations in this text at all.

Williamson's work, Ref. [9], is mostly text, but it has a fine mathematical appendix.

Nellis and Parker's textbook, Ref. [10] is non-mathematical and elementary, but it does have numerous figures and tables.

Carlberg's work, Ref. [11], provides useful, specific functions for closed and open economies, and with fixed or flexible exchange rates. The numerical examples are fictitious, though, and there's no comparison with actual economic data. Nonetheless, this work is consistent with this paper, although we won't be discussing exchange rates. Carlberg demonstrates that government fiscal policy will not work properly with flexible exchange rates, and monetary policy will not work properly with fixed exchange rates.

Gartner, Ref. [12], covers in great detail various models of macroeconomics under flexible exchange rates. Exchange rates will not be discussed in this paper, but the effects of imports and exports on the aggregate price level will be.

Turnovsky, in Ref. [13] and Ref. [22], develops a representative agent model for both the domestic economy and for international trade, stated using continuous time. The work is very abstract, but it's not necessarily incompatible with what we show in this paper.

In recent years there has been a remarkable proliferation of economic data freely available on the Web. Ref. [15], [16], [17], [18], [19], and [27] provide the data we will use in this paper, together with a few company annual reports.

Ref. [20], by Reisman, provides a modern Austrian view of microeconomics and macroeconomics and a solid critique of Keynesian economics. It is mostly non-mathematical, but he does give an equation which is quite close to the fundamental equation of Larsonian econophysics; see the note given.

Ref. [23], Miao's book, is mostly microeconomics, but it does have a little macroeconomics. The equations are based on discrete time, rather than continuous time. Our equations are stated in discrete time, likewise, so it's useful to look at the equations given in this book.

Ref. [24], Romer's work, is the only advanced comprehensive work studied for this paper. The equations are very detailed and useful, and the solutions manual is excellent. Romer presents numerous models and critiques *all* of them.

Ref. [25], by Intriligator, is still the best organized, clearest work in economic optimization. It's very helpful in understanding the general equilibrium of the economy at any point in time.

Ref. [27], Bastiat's most well-known work, is still the best statement of minarchy and laissez-faire capitalism; Pareto later showed that this is the system of maximum production.

Ref. [28] is the author's doctoral dissertation. It includes a thermodynamic First Law Energy Analysis and a thermodynamic Second Law Available Energy analysis for a regenerative Brayton-cycle engine. As will be shown in the next section of this paper, purchasing power in economics is analogous to available energy in thermodynamics!

Nomenclature

a_{prod1} = change in productivity (output per worker) in Year 1, decimal

a_{vol1} = change in volume of production due to change in number of employees or work-hours, decimal

B = general expression for purchasing power

B_{aggr} = general expression for aggregate purchasing power created/expended/consumed in a year, $

B_{aggr_0} = aggregate purchasing power created/expended/consumed in Year 0 (the base year), $

B_{aggr_1} = aggregate purchasing power created/expended/consumed in Year 1 (the following year), $

B_{cons_nom} = purchasing power consumed, $ (often the subscript *nom* will be left off)

B_{cons_real} = purchasing power consumed, in goods

B_{nom} = purchasing power, $ (often the subscript *nom* will be left off)

B_{prod_nom} = purchasing power produced, $ (often the subscript *nom* will be left off)

B_{prod_real} = purchasing power produced, in goods

B_{real} = purchasing power, in goods

b_t = investment income (interest, dividends, rents, etc.) of worker-consumer in time period t, $

C_1 = nominal aggregate consumption in Year 1, $

c_{res1} = change in the consumer reservoirs (negative of private sector savings rate), decimal

c_t = costs (consumption) of worker-consumer in time-period t, $

CPI_0 = consumer price index for Year 0 (the base year) = 100

CPI_1 = consumer price index for Year 1 (the following year)

def_1 = nominal deficit of federal government in Year 1, $

e_{i0} = fraction of volume in Year 0 (the base year) from imports, decimal

e_{i1} = fraction of volume in Year 1 (the following year) from imports, decimal

FV = future value of stream of payments or savings

f_{w1} = change in average wage rate in Year 1, decimal

G_1 = nominal total government spending in Year 1, $

G_{1check} = check calculation for nominal total government spending in Year 1, $

GDP_0 = gross domestic product in Year 0 (the base year), $

GDP_1 = nominal gross domestic product in Year 1 (the following year), $

GDP_{1obs} = observed gross domestic product in Year 1, stated in base year $

GDP_{1true} = gross domest product in Year 1 stated in base year currency, $

g_1 = change in FOMC purchases or sales of government bonds to public in Year 1, decimal (same as change in M0)

g_{1opt} = change in FOMC purchases or sales of government bonds to public in Year 1 so as keep inflation exactly 0, decimal

g_{1optm1} = change in FOMC purchases or sales of government bonds to public in Year 1 so as produce a deflation of 1%, decimal

gr_1 = growth of GDP in nominal terms, %

gr_{1true} = growth of GDP in terms of base year currency, %

I_0 = aggregate private investment in Year 0, $

I_1 = nominal aggregate private investment in Year 1, $

I_{1true} = aggregate investment in Year 1, stated in base year $

Imp_0 = nominal aggregate imports in Year 0, $

Imp_1 = nominal aggregate imports in Year 1, $

$Infl_1$ = inflation rate in Year 1, %

$Infl_T$ = inflation over T years

K_0 = capital stock in Year 0, $

K_1 = nominal capital stock in Year 1, $

K_{1true} = capital stock in Year 1, stated in base year $

k_{C_t} = coefficient of cost of producer-retailer relative to that of the *average* firm in the sector (normalized to 1 for average firm)

k_{R_t} = coefficient of revenue productivity of producer-retailer relative to that of the *average* firm in the sector (normalized to 1 for average firm) for time period t

N_0 = aggregate number of employees in Year 0 (full- or part-time)

N_1 = aggregate number of employees in Year 1 (full- or part-time)

NX_1 = net exports minus imports in Year 1, $

N_{p_r} = number of aggregate representative producer-retailers

n_e = number of employees of particular producer-retailer

PV_{p_r} = (net) present value (i.e., reflected back to time zero) of a particular producer-retailer in a particular industry sector, $

PV_{w_c} = (net) present value of worker_consumer, $

p = general expression for unit or average price

p_{aggr_0} = aggregate unit price level for Year 0 (the base year), $/unit

p_{aggr_1} = aggregate unit price leve for Year 1 (the following year), $/unit

p_{nom} = unit or average price, $ (often the subscript *nom* will be left off)

p_{real} = price, in goods

Res_{p_r} = current equivalent monetary reserve of producer_retailer, $

Res_{w_c} = current equivalent monetary reserve of worker-consumer, $

r_1 = effective rental price of capital (interest rate) for Year 1, decimal

r_{1true} = effective rental price of capital (interest rate) after depreciation and taxes, decimal

r_{p_r} = nominal discount rate for time period representing perceived risks of the firm and the interest cost of borrowed funds (assumed not to change for periods t = 1 to T)

S_1 = aggregate nominal public *and* private savings in Year 1, $

S_{1true} = aggregate public and private savings in Year 1, stated in base year $

s_1 = nominal savings of worker-consumer in Year 1, $

s_t = savings of worker-consumer in time period t, $

TC_1 = nominal aggregate total cost of producer-retailers in Year 1, $

TC_t = total cost or expense of *average* producer-retailer in *sector* for time period t, $

TCw_c_t = total cost or expense of worker-consumer in time period t, $

TR_1 = nominal aggregate total revenues of producer-retailers in Year 1, $

TR_t = total revenue of *average* producer-retailer in *sector* for time period t, $

TRw_c_t = total revenue of worker-consumer in time period t, $

T_{p_r} = total number of time periods considered (time horizon for the producer_retailer), years

T_{w_c} = total number of time periods considered (time horizon for the worker-consumer), years

TXP_1 = nominal total taxes paid out by federal government in transfers to worker-consumers in Year 1, $

TXR_1 = nominal total taxes received by federal government in Year 1, $

t = time period (subscript), years

tr_1 = transfer payments made by federal government in Year 1, $

tr_{1true} = transfer payments made by federal government in Year 1, stated in base year $

V = volume of goods and services, in number of units

V_{aggr_0} = aggregate volume of goods and services in Year 0 (the base year), in number of units

V_{aggr_1} = aggregate volume of goods and services in Year 1 (the following year), in number of units

v_1 = change in the velocity of circulation of the monetary base, decimal

w_1 = nominal wage or salary per year of worker-consumer in Year 1, $

wb_{1disp} = disposable income of worker-consumer in Year 1, $

w_{1true} = wager or salary per year of worker-consumer in Year 1, in terms of base year $

w_t = wage or salary of worker-consumer in time period t, $

X_0 = aggregate total of exports in Year 0, $

X_1 = nominal aggregate total of exports in Year 1, $

x_0 = fraction of aggregate volume in Year 0 of exports, decimal

x_1 = fraction of aggregate volume in Year 1 of exports, decimal

y_{vol1} = change in capacity utilization for the production of both goods *and* services, decimal

z_{p1} = change in price level due to change in capacity utilization and savings, decimal

β = fraction of aggregate income going to the suppliers of capital

δ_1 = depreciation rate of capital stock, decimal

τ_{B1} = tax rate on business earnings for Year 1, decimal

τ_{K1} = tax rate on interest or rental income for Year 1, decimal

τ_{w1} = tax rate on wages or salaries for Year 1, decimal

Note: A black square in the upper right of an equation means that the equation is disabled from running in *Mathcad*. This is done because not all variables in the equation have, as yet, been given numerical values. Currency is given in US dollars; if you are not an American, then simply substitute your own currency. For the graphs, some variables have a doubled first letter.

$ORIGIN := 1$ (so that matrices and vectors start at index 1)

1. Principles

a. fundamental component of the economic mechanism

Energy conversion plays a major role in physical science, biological science, and engineering science. In mechanical engineering, for instance, *available energy* is defined as the maximum amount of the total energy of a process or a set of processes that can be *converted* to *mechanical work*; the remainder is *unavailable energy*. Whereas, according to thermodynamics *First Law* accounting, *energy input equals energy output*, in thermodynamics *Second Law* accounting, *available energy gain equals available energy loss*. Table VII of Ref. [28], the author's doctoral dissertation, shows that for a regenerative Brayton cycle engine, energy *input* is comprised solely of the fuel; the energy *output* is then the net indicated mechanical work, the coolant energy, the duct heat losses, and the exhaust cooling. Table VIII of Ref. [28] follows with the components of available energy *gain*--intake (compression side), compression, regeneration, and combustion--and with the components of available energy *loss*--air-filter silencer, throttle, the various ports, intake (expansion side), expansion, duct heat losses, regeneration, exhaust silencer, cooling, and release.

Now the question is: what is the *fundamental component* for the *economic mechanism* which is analogous to available energy in mechanical engineering? Answer: *purchasing power*. None of the conventional economics textbooks listed in the References contain any mention of a fundamental component of economics. If they discuss purchasing power at all, it is only considered to be an *antecedent* to *demand* or as part of *purchasing power parity* in discussions of international trade. However, by inspection, one can see that in the *production process*, purchasing power is *gained* at each step, whereas in the *consumption process*, purchasing power is *lost*--whether quickly or over a long period of time. And, of course, purchasing power *produced* equals purchasing power *consumed!* This is a *new* expression of *Say's Law*.

b. fundamental equation of the economic mechanism

Variables in economics can be expressed in terms of *goods*, in which case they are referred to as *real* quantities; or they can be expressed in terms of *currency*, in which case they are referred to as *nominal* or *money* quantities. Purchasing power, B, can be represented in real or nominal (money) terms of unit price, p, and volume (number of units), V, as follows:

$$B_{real} := p_{real} \cdot V \qquad \text{goods} \qquad (1\text{-}1a)$$

$$B_{nom} := p_{nom} \cdot V \qquad \$ \qquad (1\text{-}1b)$$

Ordinarily, nominal variables are assumed, so if the subscript *nom* is absent, the variable is nominal:

$$B := p \cdot V \qquad \$ \qquad (1\text{-}1c)$$

If we let the subscript *prod* mean *produced* and the subscript *cons* mean *consumed*, then our form of Say's Law can be expressed as:

$$B_{cons} := B_{prod} \qquad \text{Say's Law, in analogy with available energy loss = available energy gain} \qquad (1\text{-}2)$$

Of course, a small quantity of goods may be stored in inventory, either by the producer or by the consumer, for a short or long period of time, but eventually the goods or services are consumed, in one way or another.

c. reservoirs between the producer and the consumer

The goods or money purchasing power *stream* moves from the producer to the consumer. In between there may be various *reservoirs*, from which money (or goods) may be put *into* the stream or into which money (or goods) may be taken *out* of the stream. This means that the consumer price is *not* necessarily the same as the producer price, as we will see. This has a detrimental effect on the operation of the economic mechanism, causing either *inflation* or *deflation*, but a method to correct the problem will be presented in the macroeconomics section.

d. seventeen fundamental principles

Now that we have identified the fundamental component of economics and the fundamental equation of economics, the principles governing the economic mechanism naturally follow. Larson, in Ref. [2], pp. 235-237, sets forth these seventeen principles (with comments added by the present author in square brackets):

PRINCIPLE I: Purchasing power is created solely by the production of *transferable utilities*, and it is not extinguished until those utilities are destroyed by *consumption* or otherwise.

PRINCIPLE II: Only goods can pay for goods. [Money is an intermediary.]

PRINCIPLE III: Purchasing power and goods are simply two aspects of the same thing, and they are produced at the same time, by the same act, and in the same quantity.

PRINCIPLE IV: Exchanges between individuals or agencies at the same economic location (the same location with respect to the economic streams) have no effect on the general economic situation. [This is because the goods have already been produced and paid for originally; transfers between consumers do not alter the economic situation.]

PRINCIPLE V: The income to the producer from goods produced is exactly equal to the expenditures for labor and the services of capital. The net result to the producer is zero. [Nothing is left over--unless one wishes to count reserves--but these actually belong to the suppliers of capital.]

PRINCIPLE VI: The circulating purchasing power arriving at any point in the stream is equal to that leaving the last previous processing point, plus or minus net reservoir transactions. [Simple algebra.]

PRINCIPLE VII: Except as modified by reservoir transactions, the purchasing power (money or real) available in the goods market is equal to the purchasing power expended in the production market. [Purchasing power *expended* in the production market = purchasing power *available* in the goods market for consumers to *consume*--this is another form of Say's Law.]

PRINCIPLE VIII: Any net change in the levels of the consumer purchasing power reservoirs results in a corresponding change in the money price level in the goods market, except insofar as it may be counterbalanced by a net change in the levels of the goods reservoirs. [Levels of consumer purchasing power reservoirs influence money price level in the goods market. The goods reservoirs can usually be neglected.]

PRINCIPLE IX: The market price levels are independent of the volume of production. [Volume of production does not influence money price levels.]

PRINCIPLE X: Any net flow of money from the consumer reservoirs to the purchasing power stream, or vice versa, causes a corresponding change either in production volume, production price, or both. [Money flow from consumer reservoirs may increase or decrease production volume, price, or both.]

PRINCIPLE XI: Arbitrary increases or decreases in wage rates have no effect on the volume of production or the ability of *consumers as a whole* to buy goods. [Attempts by labor unions to increase wage rates do not change ability of consumers as a whole to buy goods.]

PRINCIPLE XII: Voluntary market price changes by producers have no effect on the volume of production or the ability of *consumers as a whole* to buy goods.

PRINCIPLE XIII: All consumer purchasing power must be used for the purchase of goods from producers; it cannot be used for the purchase of goods already in the hands of consumers, or for raising the prices of such goods.

PRINCIPLE XIV: The quantity of money existing within an economic system has no effect on prices or on the general operation of the system, except insofar as the method by which money is introduced into or withdrawn from the system may constitute a purchasing power *reservoir transaction*. [*Velocity of money* is as important as *quantity* of money; money is *neutral* in the *long run*, but *not* in the *short run*.]

PRINCIPLE XV: Credit can make goods available to one individual or group of individuals only by diverting them from other individuals.

PRINCIPLE XVI: The cost of the services of capital is fixed by competitive conditions *independently* of productivity. [Cost of capital has not varied by much over the centuries. But, unfortunately, central banks have a tendency to fix interest rates, which then cause distortions in the market.]

PRINCIPLE XVII: Average real wages are determined by productivity, and are equal to total production per worker less the items of cost that are determined independently of productivity: taxes and capital costs. [*Productivity* is what counts.]

Larson continues:

"Here in these seventeen basic principles and the General Economic Equation [which Larson expresses as p = B/V] are the teachings of *economic science* as they apply to the subject matter under consideration. These principles rest firmly on solid facts, not on assumption, speculation, or guesswork, and they have been derived from those facts by processes which are logically and mathematically exact, even though extremely simple. Because of their factual nature they are specific.

"In addition to being specific, these principles are universal. Unlike many of the conclusions of conventional economics, they are not limited to any particular economic system or to any special set of conditions. They governed the Cave Dwellers in their strenuous efforts to earn their living at the dawn of history, and they will apply with equal force to the streamlined multi-cylinder economic machine of the far distant future. They govern economic processes, not merely the systems of which these processes are constituent parts, and they are applicable to the processes wherever and under whatever system they may appear. The familiar contention that a socialistic economy is subject to a set of principles that differ from those which rule our individual enterprise system is as absurd as if we were to contend that the laws of physics applicable to a concrete bridge are not the same as those which apply to a steel structure."

2. Microeconomics

a. stages of economic development

The human species has existed for approximately 250000 years. In that long period of time, our species has gone through these four economic stages (Ref. [2], pp. 25-26, p. 44, p. 65, p. 73):

(1) *Each* economic unit, whether an individual, family, or tribe, obtains and consumes its *own* products. This is the *amoeboid* stage.

(2) *Trade* develops between individuals, families, and tribes; goods are *exchanged* for other goods. This is the *barter* stage, and it illustrates Principle II in action. Barter is a single step process.

(3) *Money* is introduced. Ref. [2], p. 73: "This activates the circulating purchasing power circuit and also results in a separation between producer and consumer in the goods market. The producer exchanges goods (purchasing power) for money, which passes through the inoperative production market into the hands of the same individual in his capacity as a consumer. He then completes the cycle by exchanging the money for goods (articles of consumption) in the consumer section of the goods market." Ref. [2], p. 78: "...the producer starts from zero [and] makes a certain advance outlay for labor, capital, and materials, and ... he endeavors to sell his finished products for a price which will reimburse him for his actual expenditures and in addition will give him a satisfactory rate of return on the capital that has been invested." This is the *medium of exchange* stage, or sometimes referred to as the *small farmer* or *sole proprietor* stage; money makes for a two-step process.

[4] Production and consumption are handled *separately*. Ref. [2], p. 65: "By this innovation, the original single step barter transaction, which was expanded to a two-step process through the use of a medium of exchange, has now become a four-step process. The cobbler, who is still in the third economic stage so far as his own personal productive efforts are concerned, exchanges the goods (purchasing power) that he produces for money and then completes the transaction by exchanging this money for goods (articles of consumption). The new helper [which the cobbler hires] participates in a cycle of an entirely different character. He never handles goods as purchasing power at any time. He exchanges his labor for money and then exchanges money for goods (articles of consumption). But this is only half of the full exchange cycle. The money the helper receives comes from the cobbler, not from the ultimate consumer. To complete the transaction it is necessary for the cobbler to step into a new role. Here he is no longer a combination producer and consumer, but merely a producer. As such, he first exchanges goods (purchasing power) for money and then completes the cycle by exchanging money for labor.

"It should be noted that in his capacity as a producer the cobbler puts nothing into the economic process and takes nothing out. In his capacity as a supplier of labor he gets a portion of the proceeds that can be classified as wages, and in his capacity as a supplier of tools and equipment he gets another portion as compensation for the use of that capital. In his capacity as a consumer he exchanges the purchasing power thus obtained for consumer goods, perhaps putting some of it back into the business, retaining the ownership thereof. When each of these actions is viewed in its economic significance, rather than in its social significance (as actions of a single individual), it can be seen that all of the proceeds of the business are paid out, actually or constructively, to the suppliers of labor and the suppliers of capital. All of the net production of goods goes to consumers." This is Principle V.

b. stage 1--Robinson Crusoe

It's helpful in analyzing complex economic issues to first consider the isolated producer-consumer. Imagine that Robinson Crusoe is stranded on an otherwise uninhabited island in the middle of nowhere. Whatever he consumes he must produce himself! There is no one to trade with, borrow from, steal from, or mooch from. Therefore, from Eq. (1-2),

$$B_{real\_cons\_Robinson\_Crusoe} := B_{real\_prod\_Robinson\_Crusoe} \qquad (2\text{-}1)$$

In the later stages, many individuals try to live off of others by means of the State. But if someone gets *something for nothing* then *someone else gets nothing for something* (Ref. [2], p. 119). As Bastiat says in the note in Ref. [26], "The State is that great fiction by which everyone tries to live at the expense of everyone else."

c. stage 2--Robinson Crusoe and Robin Crusoe

Now suppose a young woman, named Robin, washes up on shore and meets Robinson. Now they have each other and so trade can be made in (real) goods and services. Consumption and production are now

$$B_{real\_cons\_Robinson\_Crusoe} + B_{real\_cons\_Robin\_Crusoe} := B_{real\_prod\_Robinson\_Crusoe} + B_{real\_prod\_Robin\_Crusoe} \qquad (2\text{-}2)$$

d. stage 3--small farm

Eventually, Robinson Crusoe and Robin Crusoe are rescued and come back to civilization; they become small farmers, with no employees. They sell the farm's output for *money*, and with the proceeds pay their expenses with *money*. So here:

$$B_{nom\_cons\_small\_farm} := B_{nom\_prod\_small\_farm} \quad (2\text{-}3)$$

e. stage 4--representative producer-retailer and representative worker-consumer

The descendents of Robinson and Robin Crusoe live in a large nation-state; the economic system is now in stage 4. In this stage (the modern stage) we have *producer-retailers* and *worker-consumers*. We will consider a *representative* producer-retailer in a particular sector, and a *representative* worker-consumer. Assuming that both are rational and optimizing, they wish to *maximize benefits minus costs* or to *maximize the ratio of benefits to costs*.

(1) the representative producer-retailer

Because of the wide variation in the capital/labor ratio across business sectors, we have to consider individual sectors in order to properly *compare* companies. The representative producer-retailer (in a particular sector, producing and selling final *consumer* goods) wishes to *maximize total revenue*, to *minimize total costs*, and thus to *maximize profit* over time. Let

PV_{p_r} = present value (i.e., reflected back to time zero) of a particular producer-retailer in a particular business sector, \$
Res_{p_r} = current equivalent monetary reserve of producer_retailer, \$
t = time period (subscript for the variables listed here), years
T_{p_r} = total number of time periods considered (time horizon for the producer_retailer), years
r_{p_r} = nominal discount rate for time period representing perceived risks of the firm and the interest cost of borrowed funds (assumed not to change for periods $t = 1$ to T_{p_r})

TR = total revenue of *average* producer-retailer in *sector* for time period t, $
TC = total cost or expense of *average* producer-retailer in *sector* for time period t (including any taxes), $
n_e = number of employees of particular producer-retailer
k_R = coefficient of revenue productivity of producer-retailer relative to that of the *average* firm in the sector (normalized to 1 for average firm) (may change for each period t)
k_C = coefficient of cost of producer-retailer relative to that of the *average* firm in the sector (normalized to 1 for average firm) (may change for each period t)

Then:

problem of representative producer-retailer:

maximize

$$PV_{p\_r} := \left[\sum_{t=1}^{T_{p\_r}} \left[n_{e_t} \frac{\left[\left(\frac{k_{R_t}}{n_{e_t}} \cdot TR_t\right) - \left(\frac{k_{C_t}}{n_{e_t}} \cdot TC_t\right)\right]}{\left(1 + r_{p\_r}\right)^t} \right]\right] + Res_{p\_r_0}$$

(2-4)

The total revenue equals the price per unit times the number of units sold for each product line plus any investment income. The total cost include materials and energy purchases, wages paid to employees, interest paid for the services of capital (and principal paid back to lenders), and any taxes.The values of the parameters are calculated by *extrapolation* from past periods, if available, or by *projection*, if not. See the worked example below.

For each period, by inspection:

$$B_{prod\_nom\_firm} := k_{R_t} \cdot TR_t \qquad (2\text{-}5)$$

$$B_{cons\_nom\_firm} := k_{C_t} \cdot TC_t \qquad (2\text{-}6)$$

The difference, the *profit* or *net earnings*, in each period, goes into the company's reserves or is "plowed back" or is paid out in *dividends* to shareholders.

$$Res_{p\_r_t} := B_{prod\_nom\_firm} - B_{cons\_nom\_firm} \qquad (2\text{-}7)$$

The reserves are actually owned by the *suppliers of capital*, and so the firm actually gets *nothing*--Principle V. If total revenue is less than total cost for a period, then the reserves will have to be *drawn down*.

For long-term survival of the firm, the managers must try (each period) to

1. increase k_R/n_e (i.e., increase relative revenue productivity per employee)
2. decrease k_C/n_e (i.e., reduce relative costs per employee)
3. increase Res (i.e., increase reserves to get through tough times)

The author has devoted much of his career to *applied optimization* for managers, engineers, and scientists; see Ref. [29]-Ref. [33].

The coefficients k_R and k_C are normalized so that for the average firm in the particular industry sector the values are equal to 1. A more successful firm will have a higher value of k_R and a lower value of k_C, and vice versa for a less successful firm. Many phenomena (including the distribution of individual IQ's) follow the Bell curve or Gaussian probability distribution, so we can assume that this distribution applies to the coefficients. The ratio of k_R to k_C represents a single *figure of merit* or *benefit to cost ratio* of the firm in the sector and is itself a probability distribution.

Of interest here is this question: what is the ratio of k_R/k_C below which will put the business in jeopardy? If PV goes to 0, the firm goes out of business, and all involved become *unemployed*. For algebraic simplicity, let's consider just one period for Eq. (2-4) and solve for the ratio of k_R/k_C in order for PV to be greater than 0, with no help from reserves, so that the firm survives. It's clear from inspection that $k_R TR_t > k_C TC_t$ for positive PV, so therefore

$$\frac{k_{R_t}}{k_{C_t}} > \frac{TC_t}{TR_t} \tag{2-8}$$

The ratio of total cost to total revenue of the *average* firm in the sector then represents the survival limit for *all* firms in the sector: the lower this ratio, the lower the survival limit for all firms in the sector. Over many periods of time, not just the one period we just considered, a particular firm's figure of merit (k_R/k_C) will fluctuate, but clearly it must *usually* be above the average TC/TR. When it's not, the firm's reserves will decrease. When it's way above the average TC/TR, the firm may increase its reserves.

If there is unemployment in a particular sector, the survival limit must be *too high* in that sector. If there is *general* unemployment, the *general survival limit must be too high*. The *surplus labor* is “substandard” in that its *ratio of relative revenue production to relative cost is too low for it to be utilized under current conditions*. Of course, for a governmental agency or for a socialist firm the survival limit is nearly zero; they are *subsidized,* and *very little of value is produced*. The correct general survival limit is that which provides *full employment* and *not a whit lower, because otherwise less will be produced*.

So, what is it that causes the survival limit of firms to be so high that we have unemployment? It is *government taxes and laws*, such as minimum wage laws and prevailing wage laws (e.g., the Davis-Bacon Act), and *over-regulation* of business that cause unemployment. It is also the *union scale*, which mandates the same labor rates across industry, regardless of *differences in productivity*. Thus *unemployment is not the result of “market failure.”* To have full employment, we must *reduce the survival limit* of businesses by repealing minimum wage laws and prevailing wage laws, by de-regulating business and discontinuing union scale, and by cutting taxes, particularly property taxes (especially in business down-turns).

A worked example utilizing Eq. (2-4) will be considered after we analyze the representative worker-consumer.

(2) the representative worker-consumer

The economics literature is full of "utility functions" for consumers, but in reality the maximization problem of a worker-consumer is not all that dissimilar from that of the producer-retailer. The worker-consumer wishes to save for retirement or to make a bequest to the children or to a charitable foundation, etc. In analogy with Eq. (2-4), we have

maximize $$PV_{w\_c} := \left[\sum_{t=1}^{T_{w\_c}} \left[\frac{\left[\left(TR_{w\_c_t}\right) - \left(TC_{w\_c_t}\right)\right]}{\left(1 + r_{w\_c}\right)^t}\right]\right] + Res_{w\_c_0} \qquad (2\text{-}9a)$$

where T_{w_c} is the time horizon for the worker_producer and r_{w_c} is the discount rate. If, per period, we let w = wage, b = investment income, tr = government transfer receipts, and c = consumption (including any taxes), then

maximize $$PV_{w\_c} := \left[\sum_{t=1}^{T_{w\_c}} \left[\frac{\left[\left(w_t + b_t + tr_t\right) - \left(c_t\right)\right]}{\left(1 + r_{w\_c}\right)^t}\right]\right] + Res_{w\_c_0} \qquad (2\text{-}9b)$$

Worker-consumers are not sector-bound, unlike business firms, so we do not need the k_R and k_C coefficients here. One could use the two-period overlapping generations model, with only w_t and c_t in the first period and only b_t and c_t in the second period, but in the *general* situation, each worker-consumer has *both* w_t and b_t, and it should be possible to deduce the values for the *median* or *mean* individual--this will be done later in the macroeconomics section. The values of tr should drop out on net.

The numerator in the expression is equal to savings, s_t.

$$s_t := \left(w_t + b_t + tr_t\right) - c_t \qquad (2\text{-}10)$$

If s is constant over the T years, then an alternative equation for (2-9b) is

$$PV_{w\_c} := s \cdot \frac{\left(1 + r_{w\_c}\right)^{T_{w\_c}} - 1}{r_{w\_c} \cdot \left(1 + r_{w\_c}\right)^{T_{w\_c}}} + Res_{w\_c_0} \qquad (2\text{-}11)$$

Engineering economics textbooks, like Ref. [35], have many more of these types of calculations. The future value of this stream of constant savings amounts to

$$FV_{w\_c} := s \cdot \frac{\left(1 + r_{w\_c}\right)^{T_{w\_c}} - 1}{r_{w\_c}} \qquad (2\text{-}12a)$$

But this needs to be reduced by the expected cumulative inflation over those T_{w_c} years, $Infl_T$.

$$FV_{w\_c} := s \cdot \frac{\left(1 + r_{w\_c}\right)^{T_{w\_c}} - 1}{r_{w\_c} \cdot Infl_T} \qquad (2\text{-}12b)$$

If the worker-consumer is just starting his or her career, the reserves could be negative due to college loans. On the other hand, if the worker-consumer is near retirement, the reserves could include assets which could be sold for cash, like one's house or stock holdings. But this has no effect on the general economic situation (Principle IV).

f. example calculations for the representative producer-retailer and the representative worker-consumer

1) representative producer-retailer

Let's consider three restaurant chains in the fast-food sector: McDonald's, Ruby Tuesday's, and Wendy's. (Originally we were going to use Burger King, instead of Ruby Tuesday's, but Burger Kings is a Canadian company and does not have 10-K reports.) Ref. [27] has the 10-K reports from 1993 to 2015 for the three companies, and with this data a spreadsheet can be made to perform the calculations so as to determine the average values for TR and TC for the three restaurants--and all the other parameters. The *Excel* spreadsheet is imported into *Mathcad* worksheet:

$xl :=$ ...\company_comparison.xls

To view columns 1 and 2 of the spreadsheet, we set:

$xl1 := xl^{\langle 1 \rangle}$ $\quad$ $xl2 := xl^{\langle 2 \rangle}$ $\quad$ $xl1\_2 := \mathrm{augment}(xl1, xl2)$

xl1_2 =

	1	2
1	"Period Range"	"1993-2015"
2	"Numer of Companies:"	3
3	"Name of Company 1"	"McDonald's"
4	"Name of Company 2"	"Ruby Tuesday's"
5	"Name of Company 3"	"Wendy's"
6	"TR Company 1"	0
7	"Net Earnings Company 1"	0
8	"TC Company 1"	0
9	"Employees Company 1"	0
10	"TR Company 1 / employees"	$4.991503 \cdot 10^4$
11	"TC Company 1 / employees"	$4.210218 \cdot 10^4$
12	"Net Earnings / TR Company 1"	0.150795
13	let Earnings / employees Company 1"	7812.844243
14	"TR Company 2"	0
15	"Net Earnings Company 2"	0
16	"TC Company 2"	0
17	"Employees Company 2"	0
18	"TR Company 2 / employees"	$3.090935 \cdot 10^4$
19	"TC Company 2 / employees"	$2.986477 \cdot 10^4$
20	"Net Earnings / TR Company 2"	0.036492
21	let Earnings / employees Company 2"	1044.578372
22	"TR Company 3"	0
23	"Net Earnings Company 3"	0
24	"TC Company 3"	0
25	"Employees Company 3"	0
26	"TR Company 3 / employees"	$9.132731 \cdot 10^4$
27	"TC Company 3 / employees"	$9.116867 \cdot 10^4$
28	"Net Earnings / TR Company 3"	0.002082
29	let Earnings / employees Company 3"	158.633986
30	"TR Average"	0
31	"TC Average"	0
32	"TC/TR Average"	0.864155

(Note the zeros are actually blank values in the actual spread sheet.)

33	"TR/TC Average"	1.158665
34	"kR Company 1"	2.672776
35	"kC Company 1"	2.627134
36	"kR/kC Company1"	1.018056
37	"kR Company 2"	0.193533
38	"kC Company 2"	0.185966
39	"kR/kC Company 2"	1.039748
40	"kR Company 3"	0.157432
41	"kC Company 3"	0.1869
42	"kR/kC Company 3"	0.871473
43	0	0
44	nd 2001--these have been taken out."	0
45		
46		

From these values, one can see that for the *average of the three firms* (the *representative firm*, so to speak), we have TC/TR = 0.864155. The three companies have average values over the time period for k_R/k_C of 1.018056, 1.039748, and 0.871473, all above 0.864155, but Wendy's is close to the danger zone. The net earnings to total revenue, on average, for the three firms are 0.150795, 0.036492, and 0.22434. The average earnings per employees at the three companies are 7812.84, 1044.58, and 158.63. (Note that Wendy's had extraordinary earnings in 2000 and 2001, but these have been taken out of the computations to avoid distortions.) Clearly, McDonald's is the most stable of these three companies, followed by Ruby Tuesday's. Wendy's is the least stable.

In order to *extrapolate* the data to the next ten years in order to get *present values*, we need to extract the relevant data from the tables and then use *Mathcad*'s predict function. To keep thinks simple, we'll extrapolate just the net earnings values (the k_R, k_C, and n_e values drop out of Eq. (2-4) and do not matter for calculating present values of the individual companies).

net_earnings_company_1 := $(\text{submatrix}(xl, 7, 7, 3, 25))^T$

net_earnings_company_2 := $(\text{submatrix}(xl, 15, 15, 3, 25))^T$

net_earnings_company_3 := $(\text{submatrix}(xl, 23, 23, 3, 25))^T$

Mathcad's predict function uses Burg's method. See Ref. [34] for a description of this method.

net_earnings_company_1_extrap := predict(net_earnings_company_1, 22, 10)

net_earnings_company_2_extrap := predict(net_earnings_company_2, 22, 10)

net_earnings_company_3_extrap := predict(net_earnings_company_3, 22, 10)

Now the plots: $t := 1 .. 10$

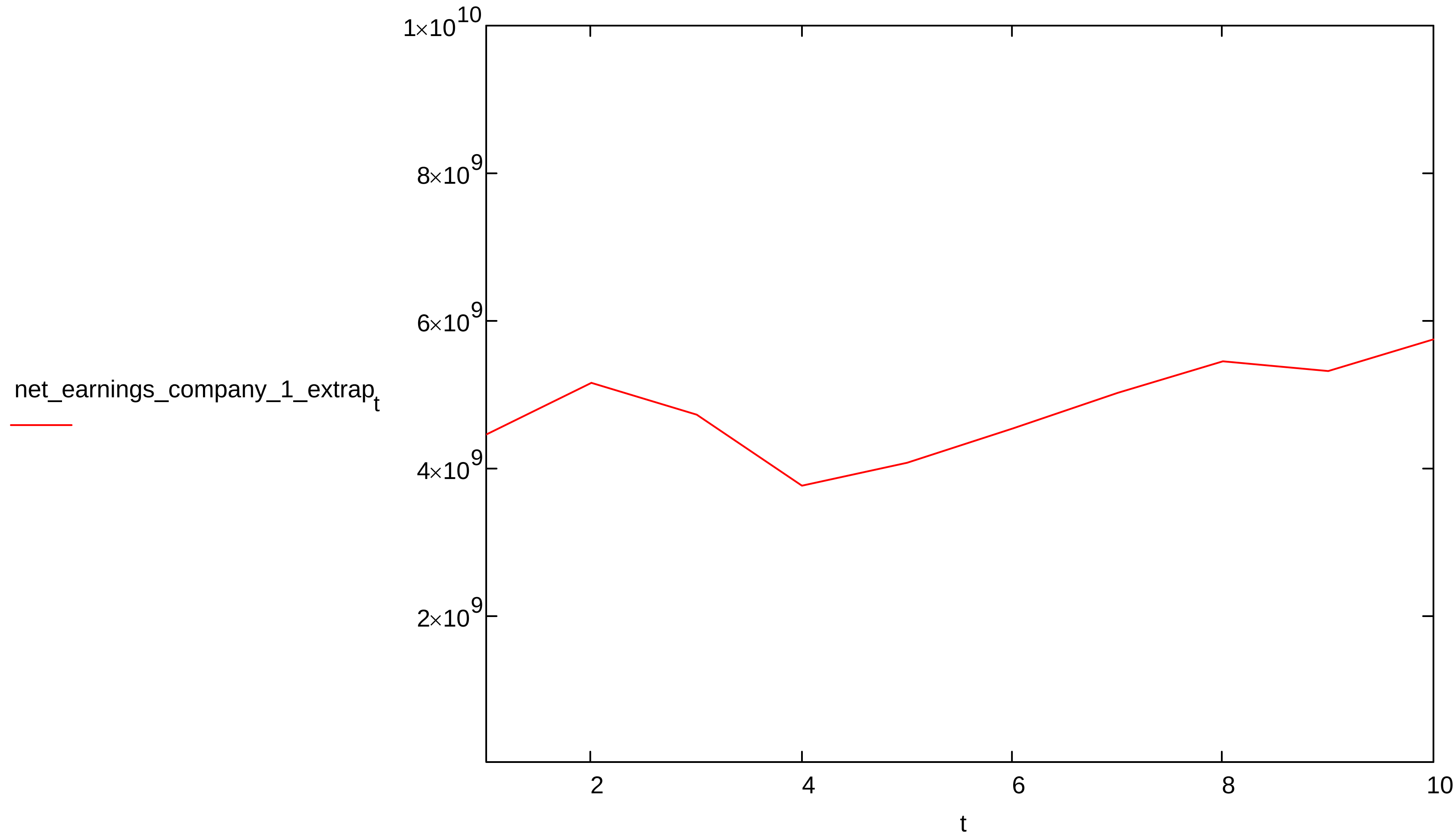


Figure 2-1. McDonald's

Company 1 looks fairly steady going forward.

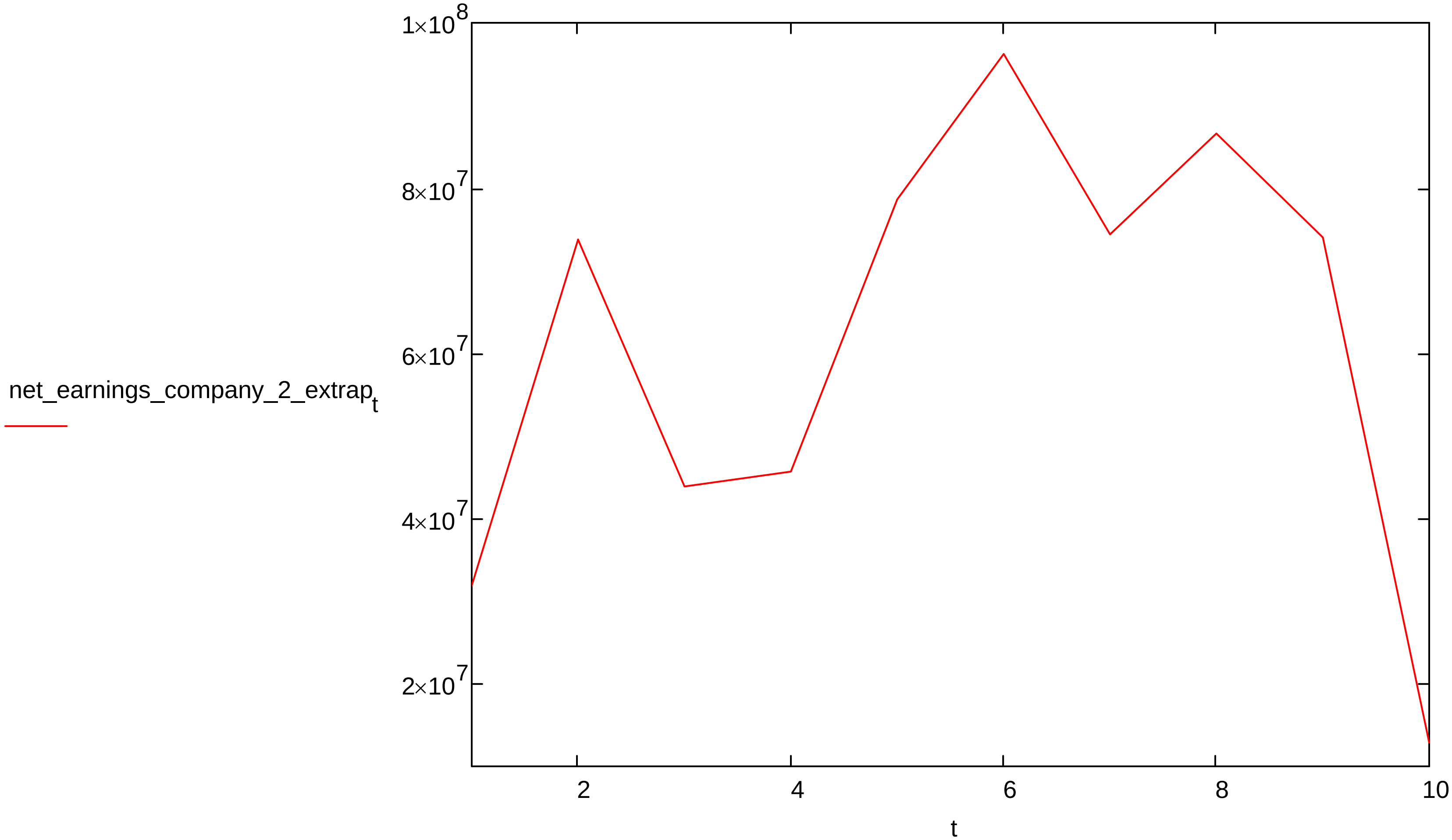


Figure 2-2. Ruby Tuesday's

Company 2 looks like it may have a somewhat rocky road ahead (and as of 2016 that seems to be the case).

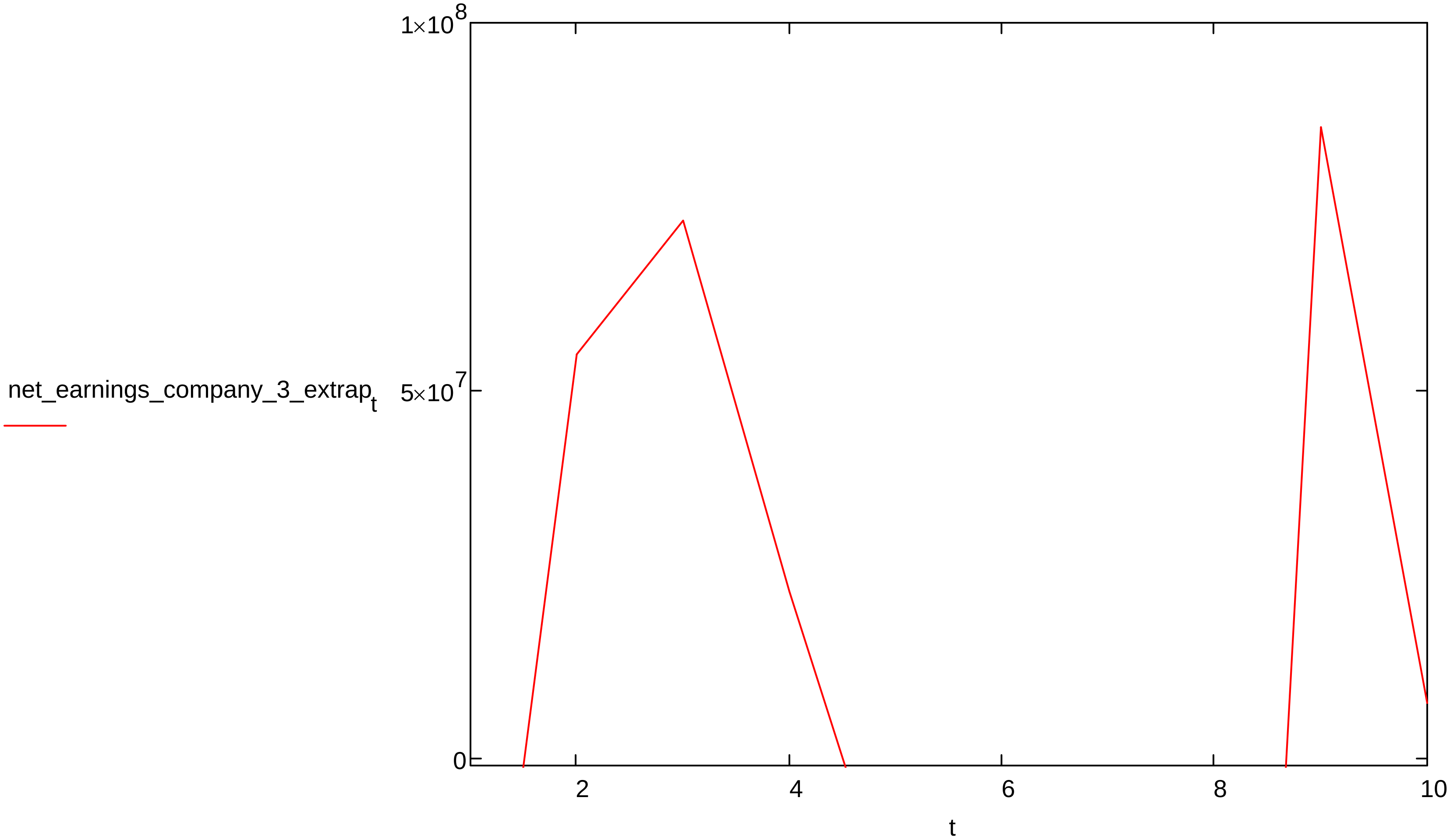


Figure 2-3. Wendy's

Company 3 will continue to have ups and downs; it may or may not survive.

Now we can calculate the present values of the three firms:

$r_{p\_r} := .065$ (assumed discount factor for each of the 10 years) $T_{p\_r} := 10$

$Res_{company\_1} := 44594500000$ (retained earnings as of end of 2015)

$Res_{company\_2} := 392032000$

$Res_{company\_3} := -356632000$ (accumulated deficit!)

$$PV_{company\_1} := \left[\sum_{t=1}^{T_{p\_r}} \left[\frac{net\_earnings\_company\_1\_extrap_t}{\left(1+r_{p\_r}\right)^t}\right]\right] + Res_{company\_1}$$

$PV_{company\_1} = 7.878905 \times 10^{10}$

$$PV_{company\_2} := \left[\sum_{t=1}^{T_{p\_r}} \left[\frac{net\_earnings\_company\_2\_extrap_t}{\left(1+r_{p\_r}\right)^t}\right]\right] + Res_{company\_2}$$

$PV_{company\_2} = 8.312051 \times 10^{8}$

$$PV_{company\_3} := \left[\sum_{t=1}^{T_{p\_r}} \left[\frac{net\_earnings\_company\_3\_extrap_t}{\left(1 + r_{p\_r}\right)^t}\right]\right] + Res_{company\_3}$$

$PV_{company\_3} = -5.294444 \times 10^8$ (In some years the earnings are positive; in others, they're negative.)

This doesn't look so good for Wendy's! Management must take action now to ensure the survival of the company!

2) representative worker-consumer

Let's consider a recent college graduate, age 22, expecting to work for 43 years and to save $20000/year, every year, through thick and thin. The *projected* present value of this individual is then

$r_{w\_c} := .065$ (assumed discount rate)

$Res_{w\_c} := -75000$ (college debt)

$T_{w\_c} := 43$ $s := 20000$

$$PV_{w\_c} := \left[\sum_{t=1}^{T_{w\_c}} \left[\frac{s}{\left(1 + r_{w\_c}\right)^t}\right]\right] + Res_{w\_c}$$

$PV_{w\_c} = 2.121767 \times 10^5$

Let's use the alternative equation for PV, as a check:

$$PV_{w\_c} := s \cdot \frac{\left(1 + r_{w\_c}\right)^{T_{w\_c}} - 1}{r_{w\_c} \cdot \left(1 + r_{w\_c}\right)^{T_{w\_c}}} + Res_{w\_c}$$

$$PV_{w\_c} = 2.121767 \times 10^{5} \qquad \text{(checks)}$$

The future value of this stream of savings, for the worker-consumer at age 65, is

$$FV := s \cdot \frac{\left(1 + r_{w\_c}\right)^{T_{w\_c}} - 1}{r_{w\_c}}$$

$$FV = 4.307075 \times 10^{6}$$

But in the 43-year period from 1972 to 2015, there was a cumulative inflation of 467% (Ref. [37])! Therefore, if that holds for the next 43 years (which hopefully it won't!), the actual value of FV in *current* dollars would be

$$Infl_{T} := 4.67$$

$$\frac{FV}{Infl_{T}} = 9.222858 \times 10^{5}$$

or about a million dollars worth of today's purchasing power.

According to Ref. [36], the median retirement savings for those between 65 and 74 as of the writing of this paper amount to a mere $148,900--hardly enough. They should have been more rational and optimizing when younger!

3. Macroeconomics

Macroeconomics deals with the *aggregate* quantities of production, consumption, and prices in a given country or region having its own currency. Here, the Fundamental Equation of Economics, Eq. (1-1c), becomes

$$B_{aggr} := p_{aggr} \cdot V_{aggr} \qquad \$ \qquad (3\text{-}1a)$$

p_{aggr} is, of course, the "general price level" of the country or region. Economists choose a certain base year in which to express currency values in subsequent years in order to get the true values for economic growth and inflation. Signifying the base year with the subscript 0, we have

$$B_{aggr\_0} := p_{aggr\_0} \cdot V_{aggr\_0} \qquad \$ \qquad \text{(base year)} \qquad (3\text{-}1b)$$

The total purchasing power created in the base year, B_{aggr_0}, is, obviously the gross domestic product, GDP_0:

$$B_{aggr\_0} := GDP_0 \qquad \$ \qquad (3\text{-}2)$$

It will be convenient in what follows to solve for p_{aggr_0} in Eq. (3-1b):

$$p_{aggr\_0} := \frac{GDP_0}{V_{aggr\_0}} \qquad \$/\text{unit} \qquad (3\text{-}3)$$

For the base year, we know the aggregate price level--it's set to an index of 100 (the CPI). We also know GDP_0, of course. And so we know V_{aggr_0} by Eq. (3-3). To calculate the values of these quantities for the *following year* (designated Year 1), we have to modify each quantity of Eq. (3-3) by a *series of factors* which represent the *changes* of in the economic conditions.

a. net consumer reservoir extraction or injection

Let c_{res1} = the rate of change in the consumer reservoirs, considered either positive (an extraction from the reservoirs to put into the purchasing power stream) or negative (an injection into the reservoirs). By Principle VIII, this change modifies *both* sides of Eq. (3-3).

$$c_{res1} := \frac{-(I_1 - I_0)}{GDP_0} \qquad (3\text{-}4)$$

$$p_{aggr\_0} \cdot (1 + c_{res1}) := \frac{GDP_0 \cdot (1 + c_{res1})}{V_{aggr\_0}} \qquad \text{\$/unit} \qquad (3\text{-}5)$$

We can identify c_{res1} as the *negative of the change in the gross investment rate* of the *private sector*. If more is saved in Year 1, then consumption will be relatively less and so p_{aggr_1} and GDP_1 will be *lower* than those of Year 0. But the savings go into investments, which will then increase the GDP over the *long term*.

b. increase in volume of production due to change in the number of workers (or worker-hours)

The number of workers (or worker-hours) of a country can increase or decrease, causing a change in the volume of production. Let a_{vol1} = the ratio of the number of Year 1 workers (N_1) to those of Year 0 (N_0).

$$a_{vol1} := \frac{N_1}{N_0} \qquad (3\text{-}6)$$

By Principle IX:

$$p_{aggr\_0} \cdot \left(1 + c_{res1}\right) := \frac{GDP_0 \cdot \left(1 + c_{res1}\right) \cdot \left(a_{vol1}\right)}{V_{aggr\_0} \cdot \left(a_{vol1}\right)} \qquad \text{\$/unit} \qquad (3\text{-}7)$$

Therefore the aggregate price level is *not* affected by this factor.

c. producer-retailer response to change in consumer reservoir levels

Producer-retailers may respond to a change in the consumer reservoir levels either by *changing volume* or *changing price* or *both*. Let y_{vol1} = decimal change in volume due to change in consumer reservoirs (by means, say, of a change in *capacity utilization*), and let z_{p1} = change in price level. Then

$$p_{aggr\_0} \cdot \left(1 + z_{p1}\right) := \frac{GDP_0 \cdot \left(1 + c_{res1}\right) \cdot \left(a_{vol1}\right)}{V_{aggr\_0} \cdot \left(a_{vol1}\right) \cdot \left(1 + y_{vol1}\right)} \qquad \text{\$/unit} \qquad (3\text{-}8)$$

Here z_{p1} has replaced c_{res1} on the LHS of Eq. (3-6) because of the possible change of volume.

$$\left(1 + z_{p1}\right) := \frac{\left(1 + c_{res1}\right)}{\left(1 + y_{vol1}\right)} \qquad (3\text{-}9)$$

d. change in average wage rate

Suppose the workers manage to get a pay raise in Year 1. Let f_{w1} = decimal change in average wage rate plus fringe benefits; one plus this factor multiplies both sides of Eq. (3-7), without changing the volume:

$$p_{aggr\_0}\cdot(1+z_{p1})\cdot(1+f_{w1}) := \frac{GDP_0\cdot(1+c_{res1})\cdot(a_{vol1})\cdot(1+f_{w1})}{V_{aggr\_0}\cdot(a_{vol1})\cdot(1+y_{vol1})} \qquad \$/\text{unit} \qquad (3\text{-}10)$$

e. change in average productivity

Continual technological improvements cause average productivity (output per worker) to rise (usually). Let a_{prod1} = decimal change in productivity. Volume increases, and the aggregate price level drops:

$$p_{aggr\_0}\cdot\frac{(1+z_{p1})\cdot(1+f_{w1})}{(1+a_{prod1})} := \frac{GDP_0\cdot(1+c_{res1})\cdot(a_{vol1})\cdot(1+f_{w1})}{V_{aggr\_0}\cdot(a_{vol1})\cdot(1+y_{vol1})\cdot(1+a_{prod1})} \qquad \$/\text{unit} \qquad (3\text{-}11)$$

Obviously, *if* wages increase at the *same rate* as productivity, there will be *no* inflation due to wage increases.

f. effects of exports and imports

Exports do *not* change the *domestic* volume quantity--but they do change the domestic *purchasing power* and the *price*. Imports change the *domestic volume* and the *price* , but *not* the *purchasing power*. Let X_0 = exports ($) in Year 0, and X_1 = exports ($) in Year 1; let Imp_0 = imports ($) in Year 0, and Imp_1 = imports ($) in Year 1. Let δX = decimal change in exports relative to GDP_0, and δImp = decimal volume change in imports then:

$$\delta X := \frac{X_1 - X_0}{GDP_0} \qquad (3\text{-}12)$$

$$\delta Imp := \frac{V_{Imp1} - V_{Imp0}}{V_{aggr\_0}} \quad \text{or} \quad \delta Imp := \frac{Imp_1 - Imp_0}{GDP_0} \qquad (3\text{-}13)$$

by proportionality.

$$p_{aggr\_0} \cdot \frac{(1 + z_{p1}) \cdot (1 + f_{w1}) \cdot (1 + \delta X)}{(1 + a_{prod1}) \cdot (1 + \delta Imp)} := \frac{GDP_0 \cdot (1 + c_{res1}) \cdot (a_{vol1}) \cdot (1 + f_{w1}) \cdot (1 + \delta X)}{V_{aggr\_0} \cdot (a_{vol1}) \cdot (1 + y_{vol1}) \cdot (1 + a_{prod1}) \cdot (1 + \delta Imp)} \qquad (3\text{-}14)$$

$/unit

The aggregate price goes up with an increase in exports; it comes down with an increase in imports. If there is no difference in imports or exports between Year 0 and Year 1, or if the change is the same for both, there is no effect on price or volume or GDP.

g. Federal Reserve Open Market operations and velocity of circulation

The Federal Reserve can *sell* government bonds to the public and then *retire the currency* in order to fight inflation, or it can *buy* government bonds from the public in order to fight deflation. Let g_1 = the decimal change in government credit goods (specifically treasury bonds), positive if sold to the public or negative if bought from the public; also let v_1 = change in velocity of circulation of monetary base. Then Eq. (3-9) becomes

$$p_{aggr\_0} \cdot \frac{(1+z_{p1})\cdot(1+f_{w1})\cdot(1+\delta X)}{(1+a_{prod1})\cdot(1+\delta Imp)\cdot(1+g_1)} := \frac{GDP_0\cdot(1+c_{res1})\cdot(a_{vol1})\cdot(1+f_{w1})\cdot(1+\delta X)}{V_{aggr\_0}\cdot(a_{vol1})\cdot(1+y_{vol1})\cdot(1+a_{prod1})\cdot(1+\delta Imp)\cdot(1+g_1)}$$

\$/unit (3-15)

If g_1 is negative, credit goods are purchased by the Fed from the pubic with newly "printed" money (which enters the purchasing power stream, raising prices). If g_1 is positive, credit goods are sold to the public by the Fed and an equivalent amount of currency is retired. As Larson says in Ref. [2], p. 174: "The outstanding advantage of this method of purchasing power control is that *no individual gains or loses by the transactions*. The exchanges that take place simply *substitute an asset in one form for an asset of equal value* in another form, and the desired effect on the economic system is accomplished without disturbing other economic relations." At first it might be thought that g_1 should be the decimal change in M1, but upon reflection it should be the decimal change in M0, so-called "high-powered" money or "monetary base."

Note that by the Fisher identity

$$GDP_0 := M0_0 \cdot Vel_0 \qquad (3\text{-}16a)$$

where Vel_0 is the velocity of circulation of M0. Likewise, for GDP_1:

$$GDP_1 := M0_1 \cdot Vel_1 \qquad (3\text{-}16b)$$

But $M0_1$ and Vel_1 can be written so that (with g_1 being *negative* for an injection)

$$GDP_1 := M0_0 \cdot (1 - g_1) \cdot Vel_0 \cdot (1 + v_1) \qquad (3\text{-}16c)$$

where v_1 is the change in the velocity of circulation of M0. Then, from Eq. (3-16a),

$$GDP_1 := GDP_0 \cdot (1 - g_1) \cdot (1 + v_1) \qquad (3\text{-}16d)$$

Thus from Eq. (3-15),

$$(1 + c_{res1}) \cdot (a_{vol1}) \cdot (1 + f_{w1}) \cdot (1 + \delta X) = (1 - g_1) \cdot (1 + v_1) \qquad (3\text{-}16e)$$

The value of g_1 is easily obtainable, whereas v_1 is not; but Eq. (3-16d) can be solved for v_1:

$$v_1 := -\frac{GDP_1 + GDP_0 \cdot (g_1 - 1)}{GDP_0 \cdot (g_1 - 1)} \qquad (3\text{-}16f)$$

Clearly, v_1 is *not* an independent variable.

h. Year 1 quantities

By inspection of Eq. (3-11), we can see that:

$$p_{aggr\_1} := p_{aggr\_0} \cdot \frac{\left(1 + z_{p1}\right) \cdot \left(1 + f_{w1}\right) \cdot (1 + \delta X)}{\left(1 + a_{prod1}\right) \cdot (1 + \delta Imp) \cdot \left(1 + g_1\right)} \qquad \$/unit \qquad (3\text{-}17)$$

$$GDP_1 := GDP_0 \cdot \left(1 + c_{res1}\right) \cdot \left(a_{vol1}\right) \cdot \left(1 + f_{w1}\right) \cdot (1 + \delta X) \qquad \$ \qquad (3\text{-}18)$$

$$V_{aggr\_1} := V_{aggr\_0} \cdot \left(a_{vol1}\right) \cdot \left(1 + y_{vol1}\right) \cdot \left(1 + a_{prod1}\right) \cdot (1 + \delta Imp) \cdot \left(1 + g_1\right) \qquad units \qquad (3\text{-}19)$$

i. taking the ratio of Year 1 to Year 0 values

From the Fundamental Equation:

$$p_{aggr\_0} := \frac{GDP_0}{V_{aggr\_0}} \qquad \$/unit$$

$$p_{aggr\_1} := \frac{GDP_1}{V_{aggr\_1}}$$ $/unit

Therefore:

$$\frac{p_{aggr\_1}}{p_{aggr\_0}} := \frac{\frac{GDP_1}{GDP_0}}{\frac{V_{aggr\_1}}{V_{aggr\_0}}} \qquad (3\text{-}20)$$

Using Eqs. (3-13), (3-14), and (3-15) in Eq. (3-19):

$$\frac{p_{aggr\_1}}{p_{aggr\_0}} := \frac{\left(1+z_{p1}\right)\cdot\left(a_{vol1}\right)\cdot\left(1+f_{w1}\right)\cdot(1+\delta X)}{\left(a_{vol1}\right)\cdot\left(1+y_{vol1}\right)\cdot\left(1+a_{prod1}\right)\cdot(1+\delta Imp)\cdot\left(1+g_1\right)} \qquad (3\text{-}21)$$

The factor (a_{vol1}) drops out from the numerator and the denominator, and the ratio of p_{aggr_1}/p_{aggr_0} can be recognized as the ratio of CPI_1 to CPI_0. So:

$$\frac{CPI_1}{CPI_0} := \frac{\left(1+z_{p1}\right)\cdot\left(1+f_{w1}\right)\cdot(1+\delta X)}{\left(1+y_{vol1}\right)\cdot\left(1+a_{prod1}\right)\cdot(1+\delta Imp)\cdot\left(1+g_1\right)} \qquad (3\text{-}22)$$

Of course,

$$CPI_0 := 100 \qquad (3\text{-}23)$$

$$CPI_1 := CPI_0 \cdot \frac{\left(1 + z_{p1}\right) \cdot \left(1 + f_{w1}\right) \cdot (1 + \delta X)}{\left(1 + y_{vol1}\right) \cdot \left(1 + a_{prod1}\right) \cdot (1 + \delta Imp) \cdot \left(1 + g_1\right)} \qquad \text{index} \qquad (3\text{-}24a)$$

The inflation rate is then:

$$Infl_1 := 100 \cdot \frac{CPI_1 - CPI_0}{CPI_0} \qquad \text{or} \qquad Infl_1 := \left(CPI_1 - CPI_0\right) \qquad \% \qquad (3\text{-}25)$$

If $Infl_1$ is known or CPI_1 is known, then y_{vol1} is determined:

$$y_{vol1} := \frac{\sqrt{CPI_0} \cdot \sqrt{c_{res1} + 1} \cdot \sqrt{f_{w1} + 1} \cdot \sqrt{\delta X + 1}}{\sqrt{CPI_1} \cdot \sqrt{a_{prod1} + 1} \cdot \sqrt{g_1 + 1} \cdot \sqrt{\delta Imp + 1}} - 1 \qquad (3\text{-}24b)$$

To get the true growth rate of the economy, we need to take into consideration the inflation. To get GDP_1 expressed *in the value of the currency in the base year*, we must multiply by the ratio of CPI_0 to CPI_1.

$$GDP_{1true} := GDP_1 \cdot \frac{CPI_0}{CPI_1} \qquad \$ \qquad (3\text{-}26)$$

Therefore, the true growth rate in GDP is:

$$gr_{1true} := 100 \cdot \frac{GDP_{1true} - GDP_0}{GDP_0} \qquad \% \qquad (3\text{-}27)$$

j. optimal Federal Open Market Operations

From Eq. (3-10), we can solve for g_1 in order to get precisely *zero change in aggregate price* from Year 0 to Year 1:

$$p_{aggr\_1} := p_{aggr\_0} \cdot \frac{(1 + z_{p1}) \cdot (1 + f_{w1}) \cdot (1 + \delta X)}{(1 + y_{vol1}) \cdot (1 + a_{prod1}) \cdot (1 + \delta Imp) \cdot (1 + g_1)} \qquad \$/unit$$

$$g_1 := \frac{p_{aggr\_0}}{p_{aggr\_1}} \cdot \frac{(1 + f_{w1}) \cdot (1 + \delta X) \cdot (1 + z_{p1})}{(1 + a_{prod1}) \cdot (1 + \delta Imp) \cdot (1 + y_{vol1})} - 1 \qquad decimal \qquad (3\text{-}28)$$

Setting $p_{aggr\_0} = p_{aggr\_1}$:

$$g_1 := \frac{\left(1+f_{w1}\right)\cdot(1+\delta X)\cdot\left(1+z_{p1}\right)}{\left(1+a_{prod1}\right)\cdot(1+\delta Imp)\cdot\left(1+y_{vol1}\right)} - 1 \qquad \text{decimal} \qquad (3\text{-}29)$$

(for zero inflation)

A gentle 1% deflation might actually be even more desirable. For this,

$$g_{1optm1} := \frac{100}{99}\cdot\left[\frac{\left(1+f_{w1}\right)\cdot(1+\delta X)\cdot\left(1+z_{p1}\right)}{\left(1+a_{prod1}\right)\cdot(1+\delta Imp)\cdot\left(1+y_{vol1}\right)}\right] - 1 \qquad \text{decimal} \qquad (3\text{-}30)$$

(for 1% deflation)

There is a *time lag* for FOMC operations. It would probably be best to use Eq. (3-25) *each quarter* based on the change in the values of the parameters *projected for the year* from measurements in the quarter and then dividing the value of g_1 by 4 to get the *quarterly value* to use in practice.

k. income share of capital and labor

From Principle V, *nominal* GDP_1 can be expressed as the *sum of the returns to capital and labor*:

$$GDP_1 := r_1 \cdot K_1 + w_1 \cdot N_1 \qquad \$ \qquad (3\text{-}31)$$

where r_1 = the effective (equivalent) rental price of capital in Year 1 (decimal), K_1 = total capital stock in Year 1 ($), w_1 = wage or salary per year ($), and N_1 = average number of workers in Year 1. If we let β = the share of income received by the suppliers of capital, then $(1 - \beta)$ must be the share of income received by the workers. This then means:

$$r_1 := \beta \cdot \frac{GDP_1}{K_1} \qquad \text{(decimal)} \qquad (3\text{-}32)$$

$$w_1 := \frac{(1-\beta) \cdot GDP_1}{N_1} \qquad \$/\text{worker} \qquad (3\text{-}33)$$

The value of β cannot be computed theoretically at this time. Empirically, Ref. [4] states that $\beta = 0.3$; Ref. [5] says that $\beta = 0.25$; Ref. [24] says that $\beta = 0.33$. In the worked example below, we will use $\beta = 0.3$ as the best estimate currently.

To get the true rate of return to capital, we must subtract the depreciation rate of capital, δ_1, the inflation rate, $Infl_{0\_1}/100$, and the effective tax rate on capital, τ_{K1}:

$$r_{1true} := \beta \cdot \frac{GDP_1}{K_1} \cdot \left(1 - \tau_{K1}\right) - \delta_1 - \frac{Infl_1}{100} \qquad \text{(decimal)} \qquad (3\text{-}34)$$

The ratio of GDP_1/K_1 would of course be the same as GDP_{1true}/K_{1true}.

To get the *disposable value* of the wage or salary we must subtract the effective wage tax (and neglecting any government transfers, which should be 0 on net):

$$wb_{1disp} := w_1 + b_1 - \frac{TXR_1}{N_1} \quad \$/worker \qquad (3\text{-}35)$$

I. national income accounting

The macroeconomics textbooks listed in the references all give the same standard identity equations for national income. Let NX_1 = net exports minus imports. Then, in nominal terms:

$$NX_1 := X_1 - Imp_1 \quad \$ \qquad (3\text{-}36)$$

$$GDP_1 := C_1 + G_1 + I_1 + NX_1 \quad \$ \qquad (3\text{-}37a)$$

where C_1 = national consumption, G_1 = government spending, and I_1 = gross private national investment. C_1 is given below. I_1 is

$$I_1 := I_0 - c_{res1} \cdot GDP_0 \qquad (3\text{-}38)$$

(c_{res1} is the negative of the private sector investment rate)

Then

$$G_1 := GDP_1 - C_1 - I_1 - NX_1 \qquad (3\text{-}37b)$$

National savings, private plus public, is

$$S_1 := I_1 + NX_1 \quad \$ \qquad (3\text{-}39)$$

Interestingly, the *government deficit equals the trade deficit* (the so-called "twin deficit hypothesis"):

$$def_1 := NX_1 \qquad (3\text{-}40)$$

m. capital stock

The nominal capital stock in Year 1 must equal that in Year 0 minus depreciation plus *private* investment. From Eq. (3-34):

$$K_1 := \left(1 - \delta_1\right) \cdot K_0 + I_1 \quad \$ \qquad (3\text{-}41)$$

n. general equilibrium

If there are N_{p_r} representative producer-retailers in the country having a representative total cost of TC_1 and if there are N_1 representative workers in the country having an income of w_1 and b_1, then for equilibrium the nominal values are:

$$b_1 := \frac{\beta \cdot GDP_1}{N_1} \qquad \$ \qquad (3\text{-}42)$$

$$s_1 := \frac{I_1}{N_1} \qquad (3\text{-}43)$$

$$c_1 := (1 - \tau_{w1}) \cdot w_1 + (1 - \tau_{K1}) \cdot b_1 + tr_1 - s_1 \qquad \$ \qquad (3\text{-}44)$$

However, for the representative worker, tr_1 should net out to zero.

$$N_{p\_r} \cdot TC_1 := N_1 \cdot (w_1 + b_1) \qquad \$ \qquad (3\text{-}45)$$

$$N_{p\_r} \cdot TR_1 - X_1 + Imp_1 := N_1 \cdot c_1 \qquad \$ \qquad (3\text{-}46)$$

Now consider the government. τ_{K1} is the *effective* tax rate on capital; τ_{w1} is the *effective* tax rate on labor. Then total taxes and transfers are

$$TXR_1 := \tau_{K1} \cdot r_1 \cdot K_1 + \tau_{W1} \cdot w_1 \cdot N_1 \qquad \$ \qquad (3\text{-}47)$$

$$TXP_1 := N_1 \cdot tr_1 \qquad \$ \qquad (3\text{-}48)$$

Transfers, such as social security, are supposed to net out to zero in a pay-as-you-go system such as that in America. The social security payments, etc., by young workers are supposed to be exactly equal to the payments sent to retirees. So, at least hypothethetically, TXP_1 should be zero.

Therefore, the net consumption of the workers must be:

$$C_1 := N_1 \cdot c_1 \qquad (3\text{-}49)$$

As a check on the calculation of G_1:

$$G_{1check} := TXR_1 - TXP_1 + |def| \qquad (3\text{-}50)$$

o. values reflected back to base year currency

$$GDP_{1true} := \frac{CPI_0}{CPI_1} \cdot GDP_0 \quad \$ \qquad (3\text{-}51)$$

$$gr_{1true} := 100 \frac{GDP_{1true} - GDP_0}{GDP_0} \quad \% \qquad (3\text{-}52)$$

$$I_{1true} := \frac{CPI_0}{CPI_1} \cdot I_1 \qquad (3\text{-}53)$$

$$NX_{1true} := \frac{CPI_0}{CPI_1} \cdot NX_1 \quad \text{(this should also be } def_{1true}\text{)} \qquad (3\text{-}54)$$

$$S_{1true} := I_{1true} + NX_{1true} \qquad (3\text{-}55)$$

$$C_{1true} := \frac{CPI_0}{CPI_1} \cdot C_1 \qquad (3\text{-}56)$$

$$G_{1true} := \frac{CPI_1}{CPI_0} \cdot G_1 \qquad (3\text{-}57)$$

$$K_{1true} := \frac{CPI_0}{CPI_1} \cdot K_1 \qquad (3\text{-}58)$$

$$b_{1true} := b_1 \cdot \frac{CPI_0}{CPI_1} \qquad (3\text{-}59)$$

$$c_{1true} := c_1 \cdot \frac{CPI_0}{CPI_1} \qquad (3\text{-}60)$$

$$s_{1true} := s_1 \cdot \frac{CPI_0}{CPI_1} \qquad (3\text{-}61)$$

$$tr_{1true} := tr_1 \cdot \frac{CPI_0}{CPI_1}$$ (but this should be zero for the representative worker-consumer) (3-62)

These values would then be used as the *next set of base year values* to use for the *succeeding year*.

p. functional form of macroeconomics equations

All of the above equations can be put into *Mathcad* functional form for ease of computation.

$$\mathrm{MacroEcon}\left(GDP_0, N_0, I_0, K_0, X_0, x_0, x_1, \delta X, Imp_0, e_{i0}, e_{i1}, \delta Imp, c_{res1}, a_{vol1}, y_{vol1}, f_{w1}, a_{prod1}, \delta_1, \beta, \tau_{K1}, \tau_{w1}, g_1, GDP_{1c}\right.$$

q. worked example 1: 2006

2005 will be Year 0 (the base year) and 2006 will be Year 1 for this example. The data are known, so no extrapolations need be done. Let

$GDP_0 := 12638.4 \times 10^9$ (Ref. [5], p. 591) $N_0 := 141.7 \cdot 10^6$ $I_0 := 2172.2 \cdot 10^9$ $K_0 := 13584 \cdot 10^9$ $X_0 := 1305.1 \cdot 10^9$ (Ref.[5], p. 591)

$X_1 := 1422.0 \cdot 10^9$ $\delta X := \dfrac{X_1 - X_0}{GDP_0}$ $\delta X = 0.00925$ $Imp_0 := 2027.8 \cdot 10^9$ $Imp_1 := 2151.2 \cdot 10^9$ $GDP_{1obs} := 12976.2 \cdot 10^9$

$\delta Imp := \dfrac{Imp_1 - Imp_0}{GDP_0}$ $\delta Imp = 0.009764$ $x_0 := \dfrac{X_0}{GDP_0}$ $x_0 = 0.103265$ $x_1 := \dfrac{X_1}{GDP_{1obs}}$ $I_1 := 2230.4 \cdot 10^9$ (Ref. [25], p. 591)

$c_{res1} := -\dfrac{I_1 - I_0}{GDP_0}$ $e_{i0} := \dfrac{Imp_0}{GDP_0}$ $e_{i0} = 0.160448$ $e_{i1} := \dfrac{Imp_1}{GDP_{1obs}}$ $e_{i1} = 0.16578$ $x_1 = 0.109585$

(e_{i0}, e_{i1}, x_0, and x_1 are misc. values)

$c_{res1} = -0.004605$ $Infl_1 := 3.2$

$y_{vol1} := 0.000$

$N_1 := 144.4 \cdot 10^6$ $a_{vol1} := \dfrac{N_1}{N_0}$ $a_{vol1} = 1.019054$ $y_{vol1} := 0.01511$ (more likely)

$f_{w1} := \dfrac{51666.2 - 49743.12}{49743.12}$ (from Ref. [38]) $a_{prod1} := .00753$ (change in output per worker from 2005)

$f_{w1} = 0.03866$ $g_1 := -.03554$

$\delta_1 := .066$ $\beta := .3$ $\tau_{K1} := 0.13$ $\tau_{w1} := 0.13$ (iterated to get G_{1true} and GDP_{1true} to equal that observed, since there are no extant data for effective capital and wage tax rates for the representative worker-consumer)

Let Y = the output vector. Then

$Y := MacroEcon(GDP_0, N_0, I_0, K_0, X_0, x_0, x_1, \delta X, Imp_0, e_{i0}, e_{i1}, \delta Imp, c_{res1}, a_{vol1}, y_{vol1}, f_{w1}, a_{prod1}, \delta_1, \beta, \tau_{K1}, \tau_{w1}, g_1, GD$

Results:

$GDP_1 := Y_1$ $N_1 := Y_3$ $s_1 := Y_4$ $b_1 := Y_5$

$GDP_1 = 1.343869 \times 10^{13}$ $N_1 = 1.444 \times 10^{8}$ $s_1 = 1.463989 \times 10^{4}$ $b_1 = 2.791972 \times 10^{4}$

$w_1 := Y_6$ $c_1 := Y_7$ $C_1 := Y_8$ $K_1 := Y_9$ $X_1 := Y_{10}$

$w_1 = 6.514601 \times 10^{4}$ $c_1 = 6.632729 \times 10^{4}$ $C_1 = 9.577661 \times 10^{12}$ $K_1 = 1.480146 \times 10^{13}$ $X_1 = 1.422 \times 10^{12}$

$Imp_1 := Y_{11}$ $NX_1 := Y_{12}$ $def_1 := Y_{13}$ $S_1 := Y_{14}$ $r_1 := Y_{15}$

$Imp_1 = 2.1512 \times 10^{12}$ $NX_1 = -7.292 \times 10^{11}$ $def_1 = -7.292 \times 10^{11}$ $S_1 = 1.3848 \times 10^{12}$ $r_1 = 0.272379$

$TXR_1 := Y_{16}$ $G_1 := Y_{17}$ $G_{1check} := Y_{18}$ $z_{p1} := Y_{19}$ $CPI_1 := Y_{20}$

$TXR_1 = 1.74703 \times 10^{12}$ $G_1 = 2.47623 \times 10^{12}$ $G_{1check} = 2.47623 \times 10^{12}$ $z_{p1} = -0.019422$ $CPI_1 = 103.199899$

$Infl_1 := Y_{21}$ $gr_1 := Y_{22}$ $gr_{1true} := Y_{23}$ $r_{1true} := Y_{24}$ $wb_{1disp} := Y_{25}$

$Infl_1 = 3.199899$ $gr_1 = 6.332218$ $gr_{1true} = 3.035196$ $r_{1true} = 0.138971$ $wb_{1disp} = 8.096718 \times 10^{4}$

$b_{1true} := Y_{26}$ $c_{1true} := Y_{27}$ $C_{1true} := Y_{28}$ $K_{1true} := Y_{30}$

$b_{1true} = 2.705402 \times 10^{4}$ $c_{1true} = 6.42707 \times 10^{4}$ $C_{1true} = 9.280689 \times 10^{12}$ $K_{1true} = 1.434251 \times 10^{13}$

$NX_{1true} := Y_{31}$ $S_{1true} := Y_{32}$ $G_{1true} := Y_{33}$ $G_{1checktrue} := Y_{34}$

$NX_{1true} = -7.065898 \times 10^{11}$ $S_{1true} = 1.341862 \times 10^{12}$ $G_{1true} = 2.39945 \times 10^{12}$ $G_{1checktrue} = 2.39945 \times 10^{12}$

$g_{1opt0} := Y_{35}$ $g_{1opt0m1} := Y_{36}$ $GDP_{1true} := Y_{37}$ $v_1 := Y_{38}$

$g_{1opt0} = -0.004678$ $g_{1opt0m1} = 0.005376$ $GDP_{1true} = 1.3022 \times 10^{13}$ $v_1 = -0.00851$

Here are some ratios: $\frac{GDP_{1true}}{GDP_{1obs}} = 1.00353$

$G_{1obs} := 2402.1 \cdot 10^{9}$ $\frac{G_{1true}}{G_{1obs}} = 0.998897$ $\frac{G_{1true}}{GDP_{1true}} = 0.184261$

Clearly, g_{1opt0} is very different from the actual g_1. Also note that the two methods of calculating G_1 check.

Now let's describe the *representative* American worker-consumer in 2006 nominal terms based on the above results:

wage or salary income:	$w_1 = 65146.01$	(this obviously includes fringe benefits!)
investment income:	$b_1 = 27919.72$	
total income:	$w_1 + b_1 = 93065.73$	
taxes:	$\frac{TXR_1}{N_1} = 12098.54$	
disposable income:	$wb_{1disp} = 80967.18$	
consumption:	$c_1 = 66327.29$	
savings:	$s_1 = 14639.89$	$\frac{s_1}{wb_{1disp}} = 0.180813$

Please note that these results are per worker, not per capita. Also, note that these results are *mean* values, not *median* values. Average depreciation rate is set at .066/year and assuming straight-line depreciation over 15 years.

r. worked example 2: 2007

2006 will be Year 0 (the base year) and 2007 will be Year 1 for this example. The *data calculated above are used*, and other data are known, so no extrapolations need be done. Let

$GDP_0 := GDP_{1obs}$ $N_0 := N_1$ $I_0 := I_1$ $K_0 := K_{1true}$ $X_0 := X_1$

$X_1 := 1546.1 \cdot 10^9$ $\delta X := \frac{X_1 - X_0}{GDP_0}$ $\delta X = 0.009564$ $Imp_0 := Imp_1$ $Imp_1 := 2193.8 \cdot 10^9$

$\delta Imp := \frac{Imp_1 - Imp_0}{GDP_0}$ $\delta Imp = 0.003283$ $x_0 := \frac{X_0}{GDP_0}$ $x_0 = 0.109585$ $GDP_{1obs} := 13254.1 \cdot 10^9$ (Ref. [25], p. 591) $x_1 := \frac{X_1}{GDP_{1obs}}$

$e_{i0} := \frac{Imp_0}{GDP_0}$ $e_{i0} = 0.16578$ $e_{i1} := \frac{Imp_1}{GDP_{1obs}}$ $e_{i1} = 0.165519$ $I_1 := 2146.2 \cdot 10^9$ $x_1 = 0.116651$

$c_{res1} := -\frac{I_1 - I_0}{GDP_0}$ $c_{res1} = 0.006489$ $Infl_1 := 2.8$ (e_{i0}, e_{i1}, x_0, and x_1 are misc. values)

$N_1 := 146.0 \cdot 10^6$ $a_{vol1} := \frac{N_1}{N_0}$ $a_{vol1} = 1.01108$ $y_{vol1} := 0.004969$ (considering goods only)

$y_{vol1} := 0.01646$ (considering both goods and services)

$f_{w1} := \frac{53694.52 - 51666.2}{51666.2}$ (from Ref. [16], using only base wage) $a_{prod1} := .010223$

$f_{w1} = 0.039258$ $g_1 := -.01904$

$\delta_1 := .066$ $\beta := .3$ $\tau_{K1} := 0.134$ $\tau_{w1} := 0.134$ (iterated to get G_{1true} and GDP_{1true} to equal that observed, since there are no extant data for effective capital and wage tax rates for the representative worker-consumer)

Let Y = the output vector. Then

$Y := MacroEcon(GDP_0, N_0, I_0, K_0, X_0, x_0, x_1, \delta X, Imp_0, e_{i0}, e_{i1}, \delta Imp, c_{res1}, a_{vol1}, y_{vol1}, f_{w1}, a_{prod1}, \delta_1, \beta, \tau_{K1}, \tau_{w1}, g_1, GD$

Results:

$GDP_1 := Y_1$ $N_1 := Y_3$ $s_1 := Y_4$ $b_1 := Y_5$

$GDP_1 = 1.385477 \times 10^{13}$ $N_1 = 1.46 \times 10^{8}$ $s_1 = 1.585342 \times 10^{4}$ $b_1 = 2.84687 \times 10^{4}$

$w_1 := Y_6$ $c_1 := Y_7$ $C_1 := Y_8$ $K_1 := Y_9$ $X_1 := Y_{10}$

$w_1 = 6.642698 \times 10^{4}$ $c_1 = 6.632623 \times 10^{4}$ $C_1 = 9.68363 \times 10^{12}$ $K_1 = 1.57105 \times 10^{13}$ $X_1 = 1.5461 \times 10^{12}$

$Imp_1 := Y_{11}$ $NX_1 := Y_{12}$ $def_1 := Y_{13}$ $S_1 := Y_{14}$ $r_1 := Y_{15}$

$Imp_1 = 2.1938 \times 10^{12}$ $NX_1 = -6.477 \times 10^{11}$ $def_1 = -6.477 \times 10^{11}$ $S_1 = 1.6669 \times 10^{12}$ $r_1 = 0.264564$

$TXR_1 := Y_{16}$ $G_1 := Y_{17}$ $G_{1check} := Y_{18}$ $z_{p1} := Y_{19}$ $CPI_1 := Y_{20}$

$TXR_1 = 1.856539 \times 10^{12}$ $G_1 = 2.504239 \times 10^{12}$ $G_{1check} = 2.504239 \times 10^{12}$ $z_{p1} = -0.00981$ $CPI_1 = 102.800101$

$Infl_1 := Y_{21}$ $gr_1 := Y_{22}$ $gr_{1true} := Y_{23}$ $r_{1true} := Y_{24}$ $wb_{1disp} := Y_{25}$

$Infl_1 = 2.800101$ $gr_1 = 6.77062$ $gr_{1true} = 3.862369$ $r_{1true} = 0.135111$ $wb_{1disp} = 8.217966 \times 10^{4}$

$b_{1true} := Y_{26}$ $c_{1true} := Y_{27}$ $C_{1true} := Y_{28}$ $K_{1true} := Y_{30}$

$b_{1true} = 2.769326 \times 10^{4}$ $c_{1true} = 6.451962 \times 10^{4}$ $C_{1true} = 9.419864 \times 10^{12}$ $K_{1true} = 1.528258 \times 10^{13}$

$NX_{1true} := Y_{31}$ $S_{1true} := Y_{32}$ $G_{1true} := Y_{33}$ $G_{1checktrue} := Y_{34}$

$NX_{1true} = -6.300577 \times 10^{11}$ $S_{1true} = 1.621496 \times 10^{12}$ $G_{1true} = 2.436028 \times 10^{12}$ $G_{1checktrue} = 2.436028 \times 10^{12}$

$g_{1opt0} := Y_{35}$ $g_{1opt0m1} := Y_{36}$ $GDP_{1true} := Y_{37}$ $v_1 := Y_{38}$

$g_{1opt0} = 0.008428$ $g_{1opt0m1} = 0.018614$ $GDP_{1true} = 1.347739 \times 10^{13}$ $v_1 = 0.002332$

Here are some ratios: $\frac{GDP_{1true}}{GDP_{1obs}} = 1.016847$

$G_{1obs} := 2443.1 \cdot 10^{9}$ $\frac{G_{1true}}{G_{1obs}} = 0.997105$ $\frac{G_{1true}}{GDP_{1true}} = 0.180749$

Note that the two methods of calculating G_1 check.

Now let's describe the *representative* American worker-consumer in 2007 nominal terms based on the above results:

wage or salary income: $w_1 = 66426.98$ (this obviously includes fringe benefits!)

investment income: $b_1 = 28468.7$

total income: $w_1 + b_1 = 94895.68$

taxes: $\frac{TXR_1}{N_1} = 12716.02$

disposable income: $wb_{1disp} = 82179.66$

consumption: $c_1 = 66326.23$

savings: $s_1 = 15853.42$ $\frac{s_1}{wb_{1disp}} = 0.192912$

Please note that these results are per worker, not per capita. Also, note that these results are *mean* values, not *median* values. Average depreciation rate is set at .066/year and assuming straight-line depreciation over 15 years.

s. worked example 3: 2008

2007 will be Year 0 (the base year) and 2008 will be Year 1 for this example. The *data calculated above are used*, and other data are known, so no extrapolations need be done. Let

$GDP_0 := GDP_{1obs}$ $N_0 := N_1$ $I_0 := I_1$ $K_0 := K_{1true}$ $X_0 := X_1$

$X_1 := 1629.3 \cdot 10^9$ $\delta X := \frac{X_1 - X_0}{GDP_0}$ $\delta X = 0.006277$ $Imp_0 := Imp_1$ $Imp_1 := 2123.5 \cdot 10^9$

$\delta Imp := \frac{Imp_1 - Imp_0}{GDP_0}$ $\delta Imp = -0.005304$ $x_0 := \frac{X_0}{GDP_0}$ $x_0 = 0.116651$ $GDP_{1obs} := 13312.2 \cdot 10^9$ $x_1 := \frac{X_1}{GDP_{1obs}}$

(Ref. [25], p. 591)

$e_{i0} := \frac{Imp_0}{GDP_0}$ $e_{i0} = 0.165519$ $e_{i1} := \frac{Imp_1}{GDP_{1obs}}$ $e_{i1} = 0.159515$ $I_1 := 1989.4 \cdot 10^9$ $x_1 = 0.122391$

$c_{res1} := -\frac{I_1 - I_0}{GDP_0}$ $c_{res1} = 0.01183$

$N_1 := 145.4 \cdot 10^6$ $a_{vol1} := \frac{N_1}{N_0}$ $a_{vol1} = 0.99589$ $y_{vol1} := -0.113721$ (change in capacity utilization)

$g_1 := -.181813$ $y_{vol1} := 0.110384$ (very high, so much of g_1 didn't really get into purchasing power stream)

$f_{w1} := \frac{55396.15 - 53694.52}{53694.52}$ (from Ref. [38])

$a_{prod1} := .008528$

$f_{w1} = 0.031691$

$\delta_1 := .066$ $\beta := .3$ $\tau_{K1} := .153$ $\tau_{w1} := .153$ (iterated to get G_{1true} and GDP_{1true} to equal that observed, since there are no extant data for effective capital and wage tax rates for the representative worker-consumer)

Let Y = the output vector. Then

$Y := MacroEcon(GDP_0, N_0, I_0, K_0, X_0, x_0, x_1, \delta X, Imp_0, e_{i0}, e_{i1}, \delta Imp, c_{res1}, a_{vol1}, y_{vol1}, f_{w1}, a_{prod1}, \delta_1, \beta, \tau_{K1}, \tau_{w1}, g_1, GD$

Results:

$GDP_1 := Y_1$ $N_1 := Y_3$ $s_1 := Y_4$ $b_1 := Y_5$

$GDP_1 = 1.386554 \times 10^{13}$ $N_1 = 1.454 \times 10^{8}$ $s_1 = 1.583906 \times 10^{4}$ $b_1 = 2.86084 \times 10^{4}$

$w_1 := Y_6$ $c_1 := Y_7$ $C_1 := Y_8$ $K_1 := Y_9$ $X_1 := Y_{10}$

$w_1 = 6.675294 \times 10^{4}$ $c_1 = 6.493199 \times 10^{4}$ $C_1 = 9.441112 \times 10^{12}$ $K_1 = 1.657693 \times 10^{13}$ $X_1 = 1.6293 \times 10^{12}$

$Imp_1 := Y_{11}$ $NX_1 := Y_{12}$ $def_1 := Y_{13}$ $S_1 := Y_{14}$ $r_1 := Y_{15}$

$Imp_1 = 2.1235 \times 10^{12}$ $NX_1 = -4.942 \times 10^{11}$ $def_1 = -4.942 \times 10^{11}$ $S_1 = 1.8088 \times 10^{12}$ $r_1 = 0.250931$

$TXR_1 := Y_{16}$ $G_1 := Y_{17}$ $G_{1check} := Y_{18}$ $z_{p1} := Y_{19}$ $CPI_1 := Y_{20}$

$TXR_1 = 2.121428 \times 10^{12}$ $G_1 = 2.615628 \times 10^{12}$ $G_{1check} = 2.615628 \times 10^{12}$ $z_{p1} = -0.088756$ $CPI_1 = 103.80008$

$Infl_1 := Y_{21}$ $gr_1 := Y_{22}$ $gr_{1true} := Y_{23}$ $r_{1true} := Y_{24}$ $wb_{1disp} := Y_{25}$

$Infl_1 = 3.80008$ $gr_1 = 4.61321$ $gr_{1true} = 0.783361$ $r_{1true} = 0.108538$ $wb_{1disp} = 8.077106 \times 10^{4}$

$b_{1true} := Y_{26}$ $c_{1true} := Y_{27}$ $C_{1true} := Y_{28}$ $K_{1true} := Y_{30}$

$b_{1true} = 2.756106 \times 10^{4}$ $c_{1true} = 6.255486 \times 10^{4}$ $C_{1true} = 9.095477 \times 10^{12}$ $K_{1true} = 1.597005 \times 10^{13}$

$NX_{1true} := Y_{31}$ $S_{1true} := Y_{32}$ $G_{1true} := Y_{33}$ $G_{1checktrue} := Y_{34}$

$NX_{1true} = -4.761075 \times 10^{11}$ $S_{1true} = 1.742581 \times 10^{12}$ $G_{1true} = 2.51987 \times 10^{12}$ $G_{1checktrue} = 2.51987 \times 10^{12}$

$g_{1opt0} := Y_{35}$ $g_{1opt0m1} := Y_{36}$ $GDP_{1true} := Y_{37}$ $v_1 := Y_{38}$

$g_{1opt0} = -0.150721$ $g_{1opt0m1} = -0.142143$ $GDP_{1true} = 1.335793 \times 10^{13}$ $v_1 = -0.150133$

Here are some ratios: $\frac{GDP_{1true}}{GDP_{1obs}} = 1.003435$

$G_{1obs} := 2518.11 \cdot 10^{9}$ $\frac{G_{1true}}{G_{1obs}} = 1.000699$ $\frac{G_{1true}}{GDP_{1true}} = 0.188642$

Now let's describe the *representative* American worker-consumer in 2008 nominal terms based on the above results:

wage or salary income: $w_1 = 66752.94$ (this obviously includes fringe benefits!)

investment income: $b_1 = 28608.4$

total income: $w_1 + b_1 = 95361.34$

taxes: $\frac{TXR_1}{N_1} = 14590.29$

disposable income: $wb_{1disp} = 80771.06$

consumption: $c_1 = 64931.99$

savings: $s_1 = 15839.06$ $\frac{s_1}{wb_{1disp}} = 0.196098$

Please note that these results are per worker, not per capita. Also, note that these results are *mean* values, not *median* values. Average depreciation rate is set at .066/year and assuming straight-line depreciation over 15 years.

t. worked example 4: 2009

2008 will be Year 0 (the base year) and 2009 will be Year 1 for this example. The *data calculated above are used*, and other data are known, so no extrapolations need be done. Let

$GDP_0 := GDP_{1obs}$ $N_0 := N_1$ $I_0 := I_1$ $K_0 := K_{1true}$ $X_0 := X_1$

$X_1 := 1472.4 \cdot 10^9$ $\delta X := \dfrac{X_1 - X_0}{GDP_0}$ $\delta X = -0.011786$ $Imp_0 := Imp_1$ $Imp_1 := 1828.0 \cdot 10^9$

$\delta Imp := \dfrac{Imp_1 - Imp_0}{GDP_0}$ $\delta Imp = -0.022198$ $x_0 := \dfrac{X_0}{GDP_0}$ $x_0 = 0.122391$ $GDP_{1obs} := 12987.4 \cdot 10^9$ (Ref. [25], p. 591) $x_1 := \dfrac{X_1}{GDP_{1obs}}$

$e_{i0} := \dfrac{Imp_0}{GDP_0}$ $e_{i0} = 0.159515$ $e_{i1} := \dfrac{Imp_1}{GDP_{1obs}}$ $e_{i1} = 0.140752$ $I_1 := 1527.6 \cdot 10^9$ $x_1 = 0.113371$

$c_{res1} := -\dfrac{I_1 - I_0}{GDP_0}$ $c_{res1} = 0.03469$ $Infl_1 := -0.4$

$N_1 := 139.9 \cdot 10^6$ $a_{vol1} := \dfrac{N_1}{N_0}$ $a_{vol1} = 0.962173$

$y_{vol1} := -0.0265$ (change in capacity utilization)

$y_{vol1} := 1.217015$ (compensates for huge g_1)

$f_{w1} := \dfrac{55615.44 - 55396.15}{55396.15}$ (from Ref. [38]) $a_{prod1} := .013956$

$f_{w1} = 0.003959$ $g_1 := -.78850$

$\delta_1 := .066$ $\beta := .3$ $\tau_{K1} := .168$ $\tau_{w1} := .168$

(iterated to get G_{1true} and GDP_{1true} to equal that observed, since there are no extant data for effective capital and wage tax rates for the representative worker-consumer)

Let Y = the output vector. Then

$Y := MacroEcon(GDP_0, N_0, I_0, K_0, X_0, x_0, x_1, \delta X, Imp_0, e_{i0}, e_{i1}, \delta Imp, c_{res1}, a_{vol1}, y_{vol1}, f_{w1}, a_{prod1}, \delta_1, \beta, \tau_{K1}, \tau_{w1}, g_1, GD$

Results:

$GDP_1 := Y_1$ $N_1 := Y_3$ $s_1 := Y_4$ $b_1 := Y_5$

$GDP_1 = 1.314862 \times 10^{13}$ $N_1 = 1.399 \times 10^{8}$ $s_1 = 1.752109 \times 10^{4}$ $b_1 = 2.819575 \times 10^{4}$

$w_1 := Y_6$ $c_1 := Y_7$ $C_1 := Y_8$ $K_1 := Y_9$ $X_1 := Y_{10}$

$w_1 = 6.579008 \times 10^{4}$ $c_1 = 6.067513 \times 10^{4}$ $C_1 = 8.48845 \times 10^{12}$ $K_1 = 1.736723 \times 10^{13}$ $X_1 = 1.4724 \times 10^{12}$

$Imp_1 := Y_{11}$ $NX_1 := Y_{12}$ $def_1 := Y_{13}$ $S_1 := Y_{14}$ $r_1 := Y_{15}$

$Imp_1 = 1.828 \times 10^{12}$ $NX_1 = -3.556 \times 10^{11}$ $def_1 = -3.556 \times 10^{11}$ $S_1 = 2.0956 \times 10^{12}$ $r_1 = 0.227128$

$TXR_1 := Y_{16}$ $G_1 := Y_{17}$ $G_{1check} := Y_{18}$ $z_{p1} := Y_{19}$ $CPI_1 := Y_{20}$

$TXR_1 = 2.208968 \times 10^{12}$ $G_1 = 2.564568 \times 10^{12}$ $G_{1check} = 2.564568 \times 10^{12}$ $z_{p1} = -0.533296$ $CPI_1 = 99.599958$

$Infl_1 := Y_{21}$ $gr_1 := Y_{22}$ $gr_{1true} := Y_{23}$ $r_{1true} := Y_{24}$ $wb_{1disp} := Y_{25}$

$Infl_1 = -0.400042$ $gr_1 = -1.228814$ $gr_{1true} = -0.832101$ $r_{1true} = 0.126971$ $wb_{1disp} = 7.819621 \times 10^{4}$

$b_{1true} := Y_{26}$ $c_{1true} := Y_{27}$ $C_{1true} := Y_{28}$ $K_{1true} := Y_{30}$

$b_{1true} = 2.8309 \times 10^{4}$ $c_{1true} = 6.091883 \times 10^{4}$ $C_{1true} = 8.522544 \times 10^{12}$ $K_{1true} = 1.743698 \times 10^{13}$

$NX_{1true} := Y_{31}$ $S_{1true} := Y_{32}$ $G_{1true} := Y_{33}$ $G_{1checktrue} := Y_{34}$

$NX_{1true} = -3.570283 \times 10^{11}$ $S_{1true} = 2.104017 \times 10^{12}$ $G_{1true} = 2.574868 \times 10^{12}$ $G_{1checktrue} = 2.574868 \times 10^{12}$

$g_{1opt0} := Y_{35}$ $g_{1opt0m1} := Y_{36}$ $GDP_{1true} := Y_{37}$ $v_1 := Y_{38}$

$g_{1opt0} = -0.789346$ $g_{1opt0m1} = -0.787218$ $GDP_{1true} = 1.320143 \times 10^{13}$ $v_1 = -0.454514$

Here are some ratios: $\frac{GDP_{1true}}{GDP_{1obs}} = 1.01648$

$G_{1obs} := 2564.6 \cdot 10^{9}$ $\frac{G_{1true}}{G_{1obs}} = 1.004004$ $\frac{G_{1true}}{GDP_{1true}} = 0.195045$

Now let's describe the *representative* American worker-consumer in 2009 nominal terms based on the above results:

wage or salary income: $w_1 = 65790.08$ (this obviously includes fringe benefits!)

investment income: $b_1 = 28195.75$

total income: $w_1 + b_1 = 93985.83$

taxes: $\frac{TXR_1}{N_1} = 15789.62$

disposable income: $wb_{1disp} = 78196.21$

consumption: $c_1 = 60675.13$

savings: $s_1 = 17521.09$ $\frac{s_1}{wb_{1disp}} = 0.224066$

Please note that these results are per worker, not per capita. Also, note that these results are *mean* values, not *median* values. Average depreciation rate is set at .066/year and assuming straight-line depreciation over 15 years. The amazing increase in savings rate went to increase productivity (which is much higher than the official figures).

u. worked example 5: 2010

2009 will be Year 0 (the base year) and 2010 will be Year 1 for this example. The *data calculated above are used*, and other data are known, so no extrapolations need be done.

$GDP_0 := GDP_{1obs}$ $N_0 := N_1$ $I_0 := I_1$ $K_0 := K_{1true}$ $X_0 := X_1$

$X_1 := 1723.29 \cdot 10^9$ $\delta X := \dfrac{X_1 - X_0}{GDP_0}$ $\delta X = 0.019318$ $Imp_0 := Imp_1$ $Imp_1 := 2183.88 \cdot 10^9$

$\delta Imp := \dfrac{Imp_1 - Imp_0}{GDP_0}$ $\delta Imp = 0.027402$ $x_0 := \dfrac{X_0}{GDP_0}$ $x_0 = 0.113371$ $GDP_{1obs} := 13063.1 \cdot 10^9$ (Ref. [4], p. 27) $x_1 := \dfrac{X_1}{GDP_{1obs}}$

$e_{i0} := \dfrac{Imp_0}{GDP_0}$ $e_{i0} = 0.140752$ $e_{i1} := \dfrac{Imp_1}{GDP_{1obs}}$ $e_{i1} = 0.167179$ $I_1 := I_0 \cdot 1.10$ $x_1 = 0.13192$

$c_{res1} := -\dfrac{I_1 - I_0}{GDP_0}$ $c_{res1} = -0.011762$ $Infl_1 := 1.6$ $y_{vol1} := .084527$

$N_1 := 139.1 \cdot 10^6$ $a_{vol1} := \dfrac{N_1}{N_0}$ $a_{vol1} = 0.994282$ $y_{vol1} := .061399$ (more probable)

$f_{w1} := \dfrac{57143.78 - 55615.44}{55615.44}$ (from Ref. [16], using only base wage) $a_{prod1} := .011613$

$f_{w1} = 0.02748$ $g_1 := -.12996$

$\delta_1 := .066$ $\beta := .3$ $\tau_{K1} := .161$ $\tau_{w1} := .161$ (iterated to get G_{1true} and GDP_{1true} to equal that observed, since there are no extant data for effective capital and wage tax rates for the representative worker-consumer)

Let Y = the output vector. Then

$Y := MacroEcon(GDP_0, N_0, I_0, K_0, X_0, x_0, x_1, \delta X, Imp_0, e_{i0}, e_{i1}, \delta Imp, c_{res1}, a_{vol1}, y_{vol1}, f_{w1}, a_{prod1}, \delta_1, \beta, \tau_{K1}, \tau_{w1}, g_1, GD$

Results:

$GDP_1 := Y_1$ $N_1 := Y_3$ $s_1 := Y_4$ $b_1 := Y_5$

$GDP_1 = 1.336523 \times 10^{13}$ $N_1 = 1.391 \times 10^{8}$ $s_1 = 9883.824587$ $b_1 = 2.882508 \times 10^{4}$

$w_1 := Y_6$ $c_1 := Y_7$ $C_1 := Y_8$ $K_1 := Y_9$ $X_1 := Y_{10}$

$w_1 = 6.725852 \times 10^{4}$ $c_1 = 7.073031 \times 10^{4}$ $C_1 = 9.838586 \times 10^{12}$ $K_1 = 1.766098 \times 10^{13}$ $X_1 = 1.72329 \times 10^{12}$

$Imp_1 := Y_{11}$ $NX_1 := Y_{12}$ $def_1 := Y_{13}$ $S_1 := Y_{14}$ $r_1 := Y_{15}$

$Imp_1 = 2.18388 \times 10^{12}$ $NX_1 = -4.6059 \times 10^{11}$ $def_1 = -4.6059 \times 10^{11}$ $S_1 = 9.1425 \times 10^{11}$ $r_1 = 0.22703$

$TXR_1 := Y_{16}$ $G_1 := Y_{17}$ $G_{1check} := Y_{18}$ $z_{p1} := Y_{19}$ $CPI_1 := Y_{20}$

$TXR_1 = 2.151802 \times 10^{12}$ $G_1 = 2.612392 \times 10^{12}$ $G_{1check} = 2.612392 \times 10^{12}$ $z_{p1} = -0.068929$ $CPI_1 = 101.599924$

$Infl_1 := Y_{21}$ $gr_1 := Y_{22}$ $gr_{1true} := Y_{23}$ $r_{1true} := Y_{24}$ $wb_{1disp} := Y_{25}$

$Infl_1 = 1.599924$ $gr_1 = 2.909188$ $gr_{1true} = 1.288647$ $r_{1true} = 0.108479$ $wb_{1disp} = 8.061414 \times 10^{4}$

$b_{1true} := Y_{26}$ $c_{1true} := Y_{27}$ $C_{1true} := Y_{28}$ $K_{1true} := Y_{30}$

$b_{1true} = 2.837116 \times 10^{4}$ $c_{1true} = 6.96165 \times 10^{4}$ $C_{1true} = 9.683655 \times 10^{12}$ $K_{1true} = 1.738287 \times 10^{13}$

$NX_{1true} := Y_{31}$ $S_{1true} := Y_{32}$ $G_{1true} := Y_{33}$ $G_{1checktrue} := Y_{34}$

$NX_{1true} = -4.53337 \times 10^{11}$ $S_{1true} = 8.99853 \times 10^{11}$ $G_{1true} = 2.571254 \times 10^{12}$ $G_{1checktrue} = 2.571254 \times 10^{12}$

$g_{1opt0} := Y_{35}$ $g_{1opt0m1} := Y_{36}$ $GDP_{1true} := Y_{37}$ $v_1 := Y_{38}$

$g_{1opt0} = -0.11604$ $g_{1opt0m1} = -0.107111$ $GDP_{1true} = 1.315476 \times 10^{13}$ $v_1 = -0.109855$

Here are some ratios: $\frac{GDP_{1true}}{GDP_{1obs}} = 1.007017$

$G_{1obs} := 2570.1\cdot10^9$ $\frac{G_{1true}}{G_{1obs}} = 1.000449$ $\frac{G_{1true}}{GDP_{1true}} = 0.195462$

Now let's describe the *representative* American worker-consumer in 2010 nominal terms based on the above results:

wage or salary income: $w_1 = 67258.52$ (this obviously includes fringe benefits!)

investment income: $b_1 = 28825.08$

total income: $w_1 + b_1 = 96083.59$

taxes: $\frac{TXR_1}{N_1} = 15469.46$

disposable income: $wb_{1disp} = 80614.14$

consumption: $c_1 = 70730.31$

savings: $s_1 = 9883.82$ $\frac{s_1}{wb_{1disp}} = 0.122607$

Please note that these results are per *representative* worker, not per capita. Also, note that these results are *mean* values, not *median* values. Average depreciation rate is set at .066/year--which assumes straight-line depreciation over 15 years.

v. notes on above calculations (2006-2010)

1) To test the theoretical equations, known values of N_1, I_1, X_1, Imp_1, $Infl_1$, g_1, and GDP_{1obs} have been used to calculate a_{vol1}, c_{res1}, δX, δImp, y_{vol1}, and v_1, respectively.

2) The data are given as "chained" 2005 dollars, rather than "reflected" back to 2005, as in our calculations, although the differences are slight.

3) Ups and downs in private sector savings and investment (personal plus business) are the cause of recessions and booms. For the years calculated, the personal savings rate (as a percent of disposable income) has varied from about 13% to about 22%. c_{res1} (which is the *negative of the change in the private sector investment rate* relative to GDP_0 has varied between 0.01 and -0.01.

4) The true *effective* interest rate (return on capital, average rental rates, etc.) has varied from about 11% to 14% .

5) The *effective tax rate* on capital and labor have been set *equal*. This is purely an assumption, as there is (apparently) no data on this issue. However, many economists believe that this is the correct policy, whether or not it is true in practice.

6) The government always seems to spend 4-5 percentage points more than it takes in.

7) Average disposable income has ranged from about $77000 to about $83000.

w. extrapolation to other years, in 2005 dollars

1) simple extrapolation

Mathcad's predict function can be used to do simple extrapolations of macroeconomic variables, such as GDP or I or G. As in the microeconomics section of this paper, it's convenient to put the past data into a spread sheet and then import the data in. From *Excel*:

$xlmacro := $ ...\US_econdata.xls

$xlmacro1 := xlmacro^{\langle 1 \rangle}$ $xlmacro2 := xlmacro^{\langle 2 \rangle}$ $xlmacro3 := xlmacro^{\langle 3 \rangle}$ $xlmacro4 := xlmacro^{\langle 4 \rangle}$

$xlmacro5 := xlmacro^{\langle 5 \rangle}$ $xlmacro6 := xlmacro^{\langle 6 \rangle}$ $xlmacro7 := xlmacro^{\langle 7 \rangle}$ $xlmacro8 := xlmacro^{\langle 8 \rangle}$

$xlmacro9 := xlmacro^{\langle 9 \rangle}$ $xlmacro10 := xlmacro^{\langle 10 \rangle}$ $xlmacro11 := xlmacro^{\langle 11 \rangle}$ $xlmacro12 := xlmacro^{\langle 12 \rangle}$

$xlmacro13 := xlmacro^{\langle 13 \rangle}$ $xlmacro14 := xlmacro^{\langle 14 \rangle}$ $xlmacro15 := xlmacro^{\langle 15 \rangle}$ $xlmacro16 := xlmacro^{\langle 16 \rangle}$

$xlmacro17 := xlmacro^{\langle 17 \rangle}$ $xlmacro18 := xlmacro^{\langle 18 \rangle}$ $xlmacro19 := xlmacro^{\langle 19 \rangle}$ $xlmacro20 := xlmacro^{\langle 20 \rangle}$

$xlmacro21 := xlmacro^{\langle 21 \rangle}$ $xlmacro22 := xlmacro^{\langle 22 \rangle}$ $xlmacro23 := xlmacro^{\langle 23 \rangle}$ $xlmacro24 := xlmacro^{\langle 24 \rangle}$

$xlmacro25 := xlmacro^{\langle 25 \rangle}$ $xlmacro26 := xlmacro^{\langle 26 \rangle}$ $xlmacro27 := xlmacro^{\langle 27 \rangle}$ $xlmacro28 := xlmacro^{\langle 28 \rangle}$

$xlmacro29 := xlmacro^{\langle 29\rangle}$ $xlmacro30 := xlmacro^{\langle 30\rangle}$ $xlmacro31 := xlmacro^{\langle 31\rangle}$ $xlmacro32 := xlmacro^{\langle 32\rangle}$

$xlmacro33 := xlmacro^{\langle 33\rangle}$

Now we have to form all of the submatrices. The first letter of the variable name will be *doubled* to indicate that we're dealing with a vector of values.

$GGDP_{1obs} := submatrix(xlmacro2, 2, 27, 1, 1)$

$ggr_{1obs} := submatrix(xlmacro3, 3, 27, 1, 1)$

$II_{1obs} := submatrix(xlmacro4, 2, 27, 1, 1)$

$cc_{res1} := submatrix(xlmacro5, 3, 27, 1, 1)$

$GG_1 := submatrix(xlmacro6, 2, 27, 1, 1)$

$XX_1 := submatrix(xlmacro7, 2, 27, 1, 1)$

$\delta\delta X_1 := submatrix(xlmacro8, 3, 27, 1, 1)$

$IImp_1 := submatrix(xlmacro9, 2, 27, 1, 1)$

$\delta\delta Imp := submatrix(xlmacro10, 3, 27, 1, 1)$

$eemp_1 := submatrix(xlmacro11, 2, 27, 1, 1)$ (number of employees, starting in 1985, to calculate N_1/N_0)

$N1N0ratio := submatrix(xlmacro12, 3, 27, 1, 1)$

$wwage_1 := submatrix(xlmacro13, 14, 27, 1, 1)$ (wages to calculate f_{w1})

$ff_{w1} := submatrix(xlmacro14, 15, 27, 1, 1)$

$pprod := submatrix(xlmacro15, 2, 27, 1, 1)$ (productivity to calculate a_{prod1})

$aa_{prod1} := submatrix(xlmacro16, 3, 27, 1, 1)$

$MM0 := submatrix(xlmacro17, 2, 27, 1, 1)$

$VVel := submatrix(xlmacro18, 2, 27, 1, 1)$ (velocity of circulation to calculate v_1)

$vv_{1calc1} := submatrix(xlmacro19, 3, 27, 1, 1)$ (first method for calculating v_1)

$gg_1 := submatrix(xlmacro20, 3, 27, 1, 1)$

$vv_{1calc2} := submatrix(xlmacro21, 3, 27, 1, 1)$ (second method for calculating v_1)

$zz_{p1} := submatrix(xlmacro22, 3, 27, 1, 1)$

$IInfl_1 := submatrix(xlmacro23, 2, 27, 1, 1)$

$sqrt1plcres_1 := submatrix(xlmacro24, 3, 27, 1, 1)$

$sqrt1plfw_1 := submatrix(xlmacro25, 3, 27, 1, 1)$

$sqrt1pldeltaX := submatrix(xlmacro26, 3, 27, 1, 1)$

$sqrt1plInfl_1 := submatrix(xlmacro27, 3, 27, 1, 1)$

$sqrt1pldeltaprod_1 := submatrix(xlmacro28, 3, 27, 1, 1)$

$sqrt1plg_1 := submatrix(xlmacro29, 3, 27, 1, 1)$

$sqrt1pldeltaImp_1 := submatrix(xlmacro30, 3, 27, 1, 1)$

$yy_{vol1} := \text{submatrix}(xlmacro31, 3, 27, 1, 1)$

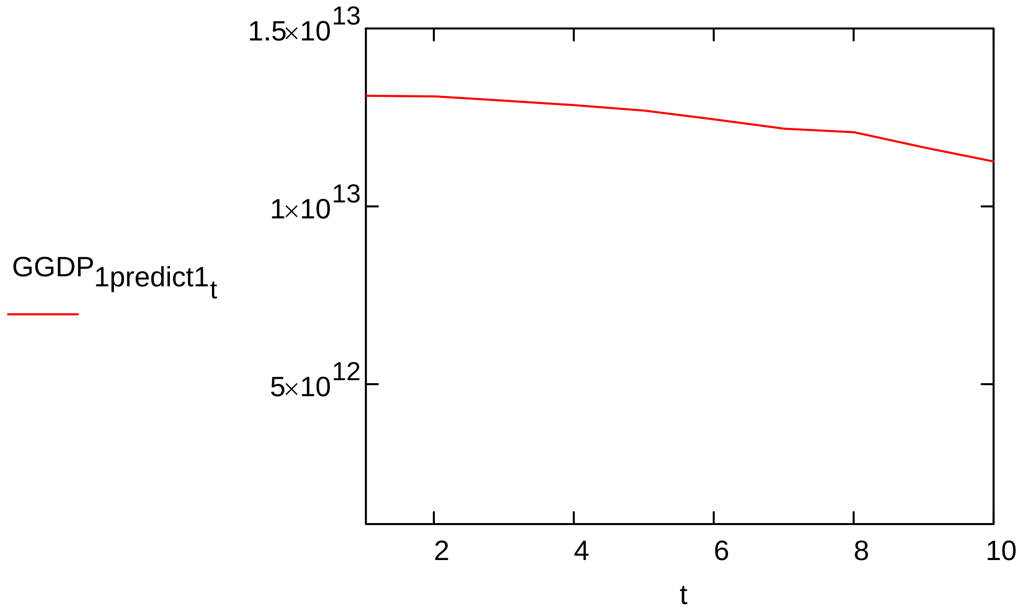
$GGDP_{1true} := \text{submatrix}(xlmacro32, 22, 27, 1, 1)$

$GGDP\_calc\_obs\_ratio := \text{submatrix}(xlmacro33, 22, 27, 1, 1)$

Not all the vectors defined above will be used in what follows, but they have all been listed for convenience.

The very simplest extrapolation is to use $GGDP_{1obs}$. We'll extrapolate 10 years from 2010, using 19 past values. The values for 2011-2015 can be compared with that observed (version 1 of this paper was written in 2016). The remaining years are predictions.

$GGDP_{1predict1} := \text{predict}\left(GGDP_{1obs}, 25, 10\right) \qquad t := 1 .. 10$



2005 dollars

Figure 3-1. Prediction 1

2010 to 2020

This is a bit of a surprise. The graph says that GDP (in 2005 dollars) will slightly fall over the years 2010-2020.

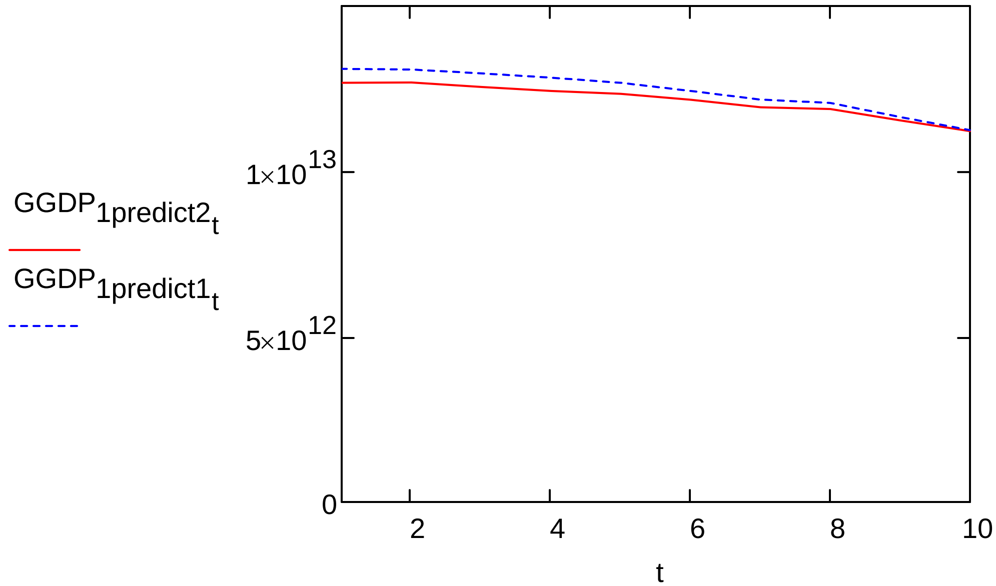
Perhaps a better approach is to use g_1 and v_1.

$t := 2 .. 25$ $\qquad GGDP_{1calcg1v1_1} := GGDP_{1obs_1}$

$$GGDP_{1calcg1v1_t} := \left[\left[\overrightarrow{\left(1 - gg_{1_t}\right)}\right] \cdot \overrightarrow{\left(1 + vv_{1calc2_t}\right)}\right] \cdot GGDP_{1calcg1v1_{t-1}}$$ (using *Mathcad*'s vectorize function)

$t := 1 .. 10$

$$GGDP_{1predict2} := \text{predict}\left(GGDP_{1calcg1v1}, 24, 10\right)$$



2010 to 2020

2005 dollars

Figure 3-2. Predictions 1, 2

Also a decline. Let's try linear regression.

$$x\_values := (1\ \ 2\ \ 3\ \ 4\ \ 5\ \ 6\ \ 7\ \ 8\ \ 9\ \ 10\ \ 11\ \ 12\ \ 13\ \ 14\ \ 15\ \ 16\ \ 17\ \ 18\ \ 19\ \ 20\ \ 21\ \ 22\ \ 23\ \ 24\ \ 25\ \ 26)^T$$

$$GDP_{1linear} := \text{line}(x\_values, GGDP_{1obs})$$

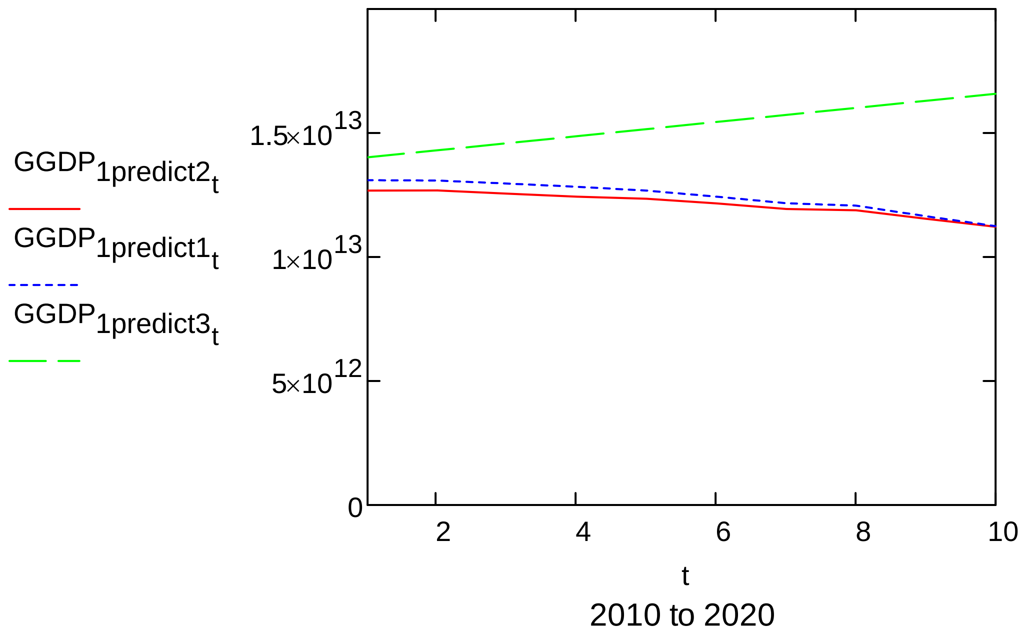
$$GDP_{1linear} = \begin{pmatrix} 6.320115 \times 10^{12} \\ 2.850809 \times 10^{11} \end{pmatrix}$$ y-intercept, slope

$$t := 1 .. 26$$

$$GGDP_{1calclinear_t} := 6.320115 \cdot 10^{12} + 2.850809 \cdot 10^{11} \cdot t \qquad GGDP_{1calclinear_{26}} = 1.373222 \times 10^{13}$$

$$t := 1 .. 10$$

$$GGDP_{1predict3_t} := 6.320115 \cdot 10^{12} + 2.850809 \cdot 10^{11} \cdot (t + 26)$$



2005 dollars

Figure 3-3. Predictions 1, 2, 3

That's certainly much more optimistic. Now let's try using the GDP growth rate.

The mean value of the GDP growth rate is

$gr\_mean := mean(ggr_{1obs})$ $\quad gr\_mean = 2.629915\ \%$

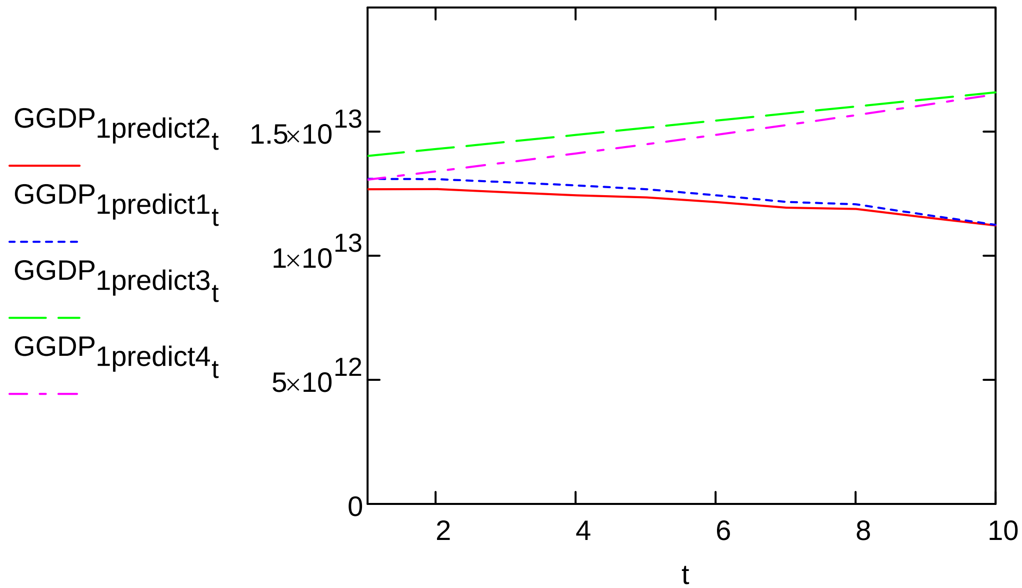
With this value, we can make another set of predictions:

$GGDP_{1calcgr_1} := GGDP_{1obs_{26}}$ $\quad t := 1..10$

$$GGDP_{1calcgr_{t+1}} := \left(1 + \frac{gr\_mean}{100}\right) \cdot GGDP_{1calcgr_t}$$

$GGDP_{1predict4_1} := GGDP_{1calcgr_1}$

$$GGDP_{1predict4_{t+1}} := \left(1 + \frac{gr\_mean}{100}\right) \cdot GGDP_{1predict4_t}$$



2010 to 2020

2005 dollars

Figure 3-4. Predictions 1, 2, 3, 4

2) complex extrapolation

Obviously, the four simple extrapolations do not work very well--because by inspection we can see that there should usually be a steady small growth in GDP. What we'll do now is extrapolate the parameters of the theory and then use those to extrapolate the GDP. Recall that

$$GDP_1 := GDP_0 \cdot (1 + c_{res1}) \cdot (a_{vol1}) \cdot (1 + f_{w1}) \cdot (1 + \delta X)$$

Using *Mathcad*'s vectorize function, we can obtain each term in parentheses (from the vectors imported from the spreadsheet) and then obtain the mean values for the series, including the inflation:

$$ccrespl1 := \overrightarrow{(1 + cc_{res1})} \qquad ccrespl1\_mean := mean(ccrespl1) \qquad ccrespl1\_mean = 0.996402$$

$$aa_{vol1} := N1N0ratio \qquad aavol1\_mean := mean(N1N0ratio) \qquad aavol1\_mean = 1.010267$$

$$ffw1pl1 := \overrightarrow{(1 + ff_{w1})} \qquad ffw1pl1\_mean := mean(ffw1pl1) \qquad ffw1pl1\_mean = 1.03643$$

$$\delta\delta X1pl1 := \left[\overrightarrow{(1 + \delta\delta X_1)}\right] \qquad \delta\delta X1pl1\_mean := mean(\delta\delta X1pl1) \qquad \delta\delta X1pl1\_mean = 1.00555$$

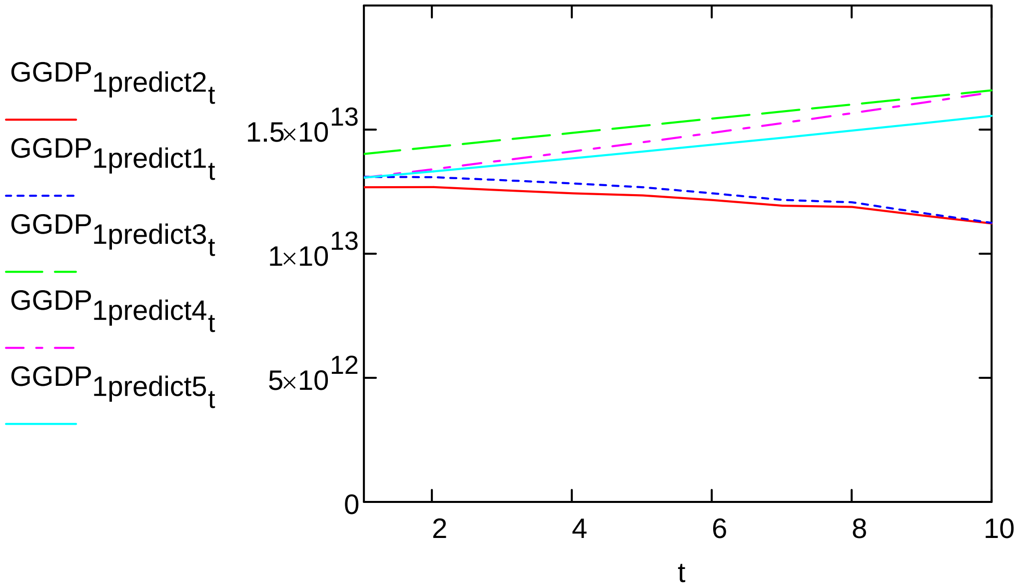
$$IInfl1pl1 := \overrightarrow{\left(1 + \frac{IInfl_1}{100}\right)} \qquad IInfl1\_mean := mean(IInfl1pl1) \qquad IInfl1\_mean = 1.028962$$

$$factors\_mean := \frac{ccrespl1\_mean \cdot aavol1\_mean \cdot ffw1pl1\_mean \cdot \delta\delta X1pl1\_mean}{IInfl1\_mean} \qquad factors\_mean = 1.019567$$

$GGDP_{1predict5_1} := 1.3063 \cdot 10^{13}$ (2010 observed GDP in 2005 dollars)

$t := 1 .. 10$

$GGDP_{1predict5_{t+1}} := factors\_mean \cdot GGDP_{1predict5_t}$



2005 dollars

Figure 3-5. Predictions 1, 2, 3, 4, 5

2010 to 2020

So the theory implies that the mean real growth rate we can achieve is about 2%.

One last method is to use the individual factors, rather than the mean values.

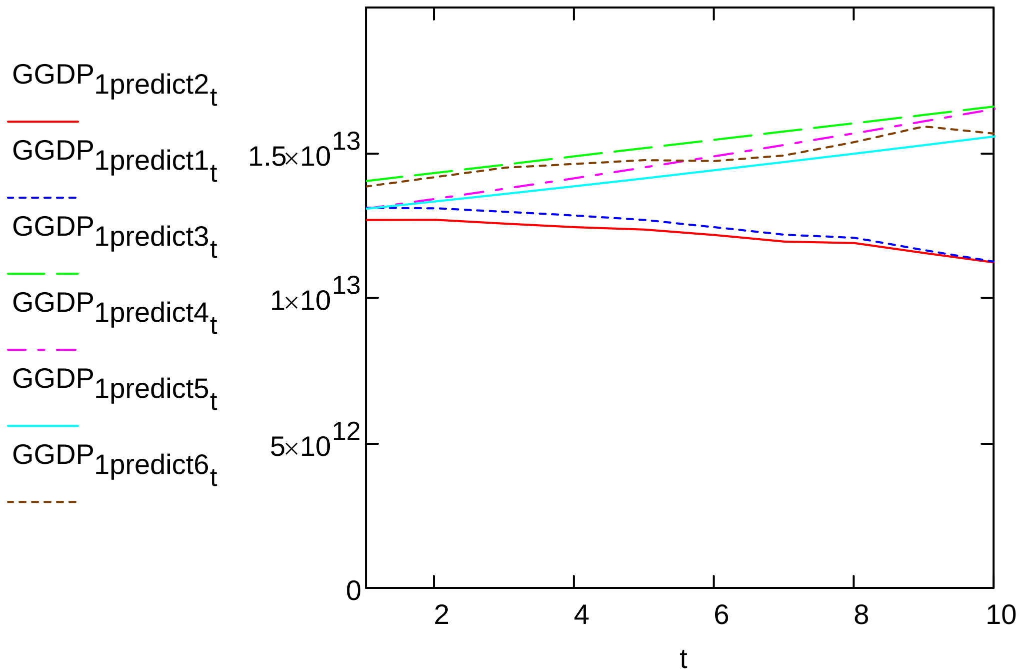
$t := 14 .. 25$

$$factors_t := \overrightarrow{\frac{ccrespl1_t \cdot aa_{vol1_t} \cdot ffw1pl1_{t-13} \cdot \delta\delta X1pl1_t}{IInfl1pl1_t}}$$

$t := 1 .. 12$

$$GGDP_{1calcfactors_t} := GGDP_{1obs_{t+12}} \cdot factors_{t+13}$$

$$GGDP_{1predict6} := predict\left(GGDP_{1calcfactors}, 11, 10\right)$$



2005 dollars

Figure 3-6. Predictions 1, 2, 3, 4, 5, 6

2010 to 2020

The sixth prediction lies between the linear regression and the mean factors prediction, and it crosses the prediction based on the mean growth rate. But the fifth prediction is probably still the most accurate.

Ref. [39] shows that the real growth rates for the US economy for 2011 to 2015 are 1.6%, 2.2%, 1.7%, 2.4%, and 2.4% respectively. The mean value is

$$mean\_gr1obs := mean(1.6, 2.2, 1.7, 2.4, 2.4) \qquad mean\_gr1obs = 2.06$$

This compares with the mean value for the factors calculated by prediction 5:

$$mean\_gr1calc := 100 \cdot (factors\_mean - 1) \qquad mean\_gr1calc = 1.96$$

Thus the theory is verified to within the observation error.

Similar calculations could be done for the prediction of government expenditures, investment rates, taxes, and all the other variables, but that can be left to a comprehensive database program to be developed in the future.

Recommendations

Based on the microeconomics theory presented here, the ratio of total cost to total revenue of the *average* firm in the sector represents the survival limit for *all* firms in the sector: the lower this ratio, the lower the survival limit for all firms in the sector. Over many periods of time, a particular firm's figure of merit (k_R/k_C) will fluctuate, but clearly it must *usually* be above the average TC/TR. When it's not, the firm's reserves will decrease. When it's way above the average TC/TR, the firm may increase its reserves.

If there is unemployment in a particular sector, the survival limit must be too high in that sector. If there is general unemployment, the general survival limit must be too high. The surplus labor is “substandard” in that its ratio of relative revenue production to relative cost is too low for it to be utilized under current conditions. Of course, for a governmental agency or for a socialist firm the survival limit is nearly zero; they are subsidized, and very little of value is produced. The correct general survival limit is that which provides *full employment* and not a whit lower, because otherwise less will be produced.

So, what is it that causes the survival limit of firms to be so high that we have unemployment? It is government taxes and laws, such as minimum wage laws and prevailing wage laws (e.g., the Davis-Bacon Act), and over-regulation of business that cause unemployment. It is also the union scale, which mandates the same labor rates across industry, regardless of differences in *productivity*. Thus unemployment is not the result of “market failure.” To have full employment, we must *reduce the survival limit* of businesses by repealing minimum wage laws and prevailing wage laws, by de-regulating business and discontinuing union scale, and by cutting taxes.

Now consider the macroeconomics theory. Everyday this author hears that the Federal Reserve is going to do this or that with interest rates. As the central bank in a country with a fiat currency, the Fed's job is to manage the amount of actual currency and coins, MB, and set the reserve ratio for banks, which determines the amount of credit money; M1 = MB + credit money, including checking accounts. The Fed also has the responsibility of setting the interest it will pay to banks for their reserve accounts at the Fed, and it has the responsibility to set the discount rate--the interest the Fed charges banks for loans. What this author disagrees with is the idea that the Fed should set the Federal Funds Rate--the interest rate banks charge each other for overnight loans; in the author's opinion, the whole community of banks, in competition, should set their own inter-bank lending rate. If they did, interest rates would be a lot higher than they are and thus much more normal. The current low interest rates prevent most bank lending, and they cause distortions in asset prices, like those of the stock market and housing. The Fed should increase the money supply when there is deflation, and decrease it when there is inflation. This author prefers zero inflation, not 2% inflation, so one has to laugh when Fed officials keep telling us that "inflation rates are too low." Eq. (3-29) can be be used on a quarterly basis to zero out inflation; using a fixed monetary rule is unwise. As far as optimal fiscal policy is concerned, the government budget should be balanced; government assets should be sold off to pay off the national debt, and there should no longer be any deficits.

Conclusion

Larsonian econophysics represents a new paradigm for microeconomics and macroeconomics. The fundamental component of the economic mechanism is *purchasing power*, which is analogous to *available energy* in physical science. Purchasing power is produced and then expended or consumed. The fundamental equation of economics is p = B/V, where p = unit price, B = purchasing power, and V = the number (or volume) of units. This equation can be applied to all levels of the economy. At the microeconomic level, the present values of representative firms (based on earnings) and workers (based on savings) are computed and projected into the future. At the national level, B = GDP, and to determine the next year's GDP the previous year's GDP and p and V are multiplied by a *series of factors* representing the change in the various economic parameters. Five sets of sample calculations for the years 2006-2010 are given and compared to the observed values; there is excellent agreement. Then the equations are applied to predict the values for GDP and the growth rate for 10 years in the future from 2010. The results for 2011 to 2015 are compared with the observations; again there is fine agreement. The theory also provides an equation which the central bank can use to zero out inflation. Larsonian econophysics therefore supersedes all other economic theories.

Acknowledgments

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[39] https://www.thebalance.com/u-s-gdp-growth-3306008

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original publishing date: 11/27/2016

$$\text{obs}) :=$$

$$GDP_{1num} \leftarrow (1 + c_{res1}) \cdot (a_{vol1}) \cdot (1 + f_{w1}) \cdot (1 + \delta X)$$

$$GDP_{1denom} \leftarrow 1$$

$$GDP_{1} \leftarrow GDP_{0} \cdot \frac{GDP_{1num}}{GDP_{1denom}}$$

$$I_{1} \leftarrow I_{0} + c_{res1} \cdot GDP_{0}$$

$$N_{1} \leftarrow N_{0} \cdot (a_{vol1})$$

$$s_{1} \leftarrow \frac{I_{1}}{N_{1}}$$

$$b_{1} \leftarrow \frac{\beta \cdot GDP_{1}}{N_{1}}$$

$$w_{1} \leftarrow \frac{(1 - \beta) \cdot GDP_{1}}{N_{1}}$$

$$tr_{1} \leftarrow 0$$

$$TXP_{1} \leftarrow N_{1} \cdot tr_{1}$$

$$c_{1} \leftarrow (1 - \tau_{w1}) \cdot w_{1} + (1 - \tau_{K1}) \cdot b_{1} + tr_{1} - s_{1}$$

$$C_{1} \leftarrow N_{1} \cdot c_{1}$$

$$K_{1} \leftarrow (1 - \delta_{1}) \cdot K_{0} + I_{1}$$

$$X_{1} \leftarrow GDP_{0} \cdot \delta X + X_{0}$$

$$Imp_1 \leftarrow GDP_0 \cdot \delta Imp + Imp_0$$

$$NX_1 \leftarrow X_1 - Imp_1$$

$$def_1 \leftarrow NX_1$$

$$S_1 \leftarrow I_1 + NX_1$$

$$r_1 \leftarrow \beta \cdot \frac{GDP_1}{K_1}$$

$$TXR_1 \leftarrow \tau_{K1} \cdot r_1 \cdot K_1 + \tau_{w1} \cdot w_1 \cdot N_1$$

$$G_1 \leftarrow GDP_1 - C_1 - I_1 - NX_1$$

$$G_{1check} \leftarrow TXR_1 - TXP_1 + \left| def_1 \right|$$

$$z_{p1} \leftarrow \frac{\left(1 + c_{res1}\right)}{\left(1 + y_{vol1}\right)} - 1$$

$$CPI_{1num} \leftarrow \left(1 + z_{p1}\right) \cdot \left(1 + f_{w1}\right) \cdot (1 + \delta X)$$

$$CPI_{1denom} \leftarrow \left(1 + y_{vol1}\right) \cdot \left(1 + a_{prod1}\right) \cdot (1 + \delta Imp) \cdot \left(1 + g_1\right)$$

$$CPI_1 \leftarrow 100 \cdot \frac{CPI_{1num}}{CPI_{1denom}}$$

$$Infl_1 \leftarrow \left(CPI_1 - 100\right)$$

$$gr_1 \leftarrow 100 \cdot \left[\frac{\left(GDP_1 - GDP_0\right)}{GDP_0}\right]$$

$$ratio \leftarrow \frac{100}{CPI_1}$$

$$GDP_{1true} \leftarrow GDP_1 \cdot ratio$$

$$v_1 \leftarrow -\frac{GDP_{1obs} + GDP_0 \cdot \left(g_1 - 1\right)}{GDP_0 \cdot \left(g_1 - 1\right)}$$

$$gr_{1true} \leftarrow 100 \cdot \left(\frac{GDP_{1true} - GDP_0}{GDP_0} \right)$$

$$r_{1true} \leftarrow \left[r_1 \cdot \left(1 - \tau_{K1}\right) - \delta_1 - \frac{Infl_1}{100} \right]$$

$$wb_{1disp} \leftarrow w_1 + b_1 - \frac{TXR_1}{N_1}$$

$$b_{1true} \leftarrow b_1 \cdot ratio$$

$$c_{1true} \leftarrow c_1 \cdot ratio$$

$$s_{1true} \leftarrow s_1 \cdot ratio$$

$$I_{1true} \leftarrow I_1 \cdot ratio$$

$$K_{1true} \leftarrow K_1 \cdot ratio$$

$$NX_{1true} \leftarrow NX_1 \cdot ratio$$

$$S_{1true} \leftarrow S_1 \cdot ratio$$

$$C_{1true} \leftarrow C_1 \cdot ratio$$

$$G_{1true} \leftarrow G_1 \cdot ratio$$

$$G_{1checktrue} \leftarrow G_{1check} \cdot ratio$$

$$g_{1opt0} \leftarrow \frac{\left(1 + f_{w1}\right) \cdot (1 + \delta X) \cdot \left(1 + z_{p1}\right)}{\left(1 + a_{prod1}\right) \cdot (1 + \delta Imp) \cdot \left(1 + y_{vol1}\right)} - 1$$

$$g_{1opt0m1} \leftarrow \frac{100}{99} \cdot \left[\frac{\left(1 + f_{w1}\right) \cdot (1 + \delta X) \cdot \left(1 + z_{p1}\right)}{\left(1 + a_{prod1}\right) \cdot (1 + \delta Imp) \cdot \left(1 + y_{vol1}\right)} \right] - 1$$

$$\begin{pmatrix} GDP_1 \\ I_1 \\ N \ldots \end{pmatrix}$$

N_1
s_1
b_1
w_1
c_1
C_1
K_1
X_1
Imp_1
NX_1
def_1
S_1
r_1
TXR_1
G_1
G_{1check}
z_{p1}
CPI_1
$Infl_1$
gr_1
gr_{1true}
r_{1true}

$$\begin{pmatrix} wb_{1disp} \\ b_{1true} \\ c_{1true} \\ C_{1true} \\ I_{1true} \\ K_{1true} \\ NX_{1true} \\ S_{1true} \\ G_{1true} \\ G_{1checktrue} \\ g_{1opt0} \\ g_{1opt0m1} \\ GDP_{1true} \\ v_1 \end{pmatrix}$$

$\cdot P_{1obs})$

$\cdot P_{1obs})$

$\cdot P_{1obs})$

$\cdot P_{1obs})$

$\cdot P_{1obs})$