Calculation of the Gravitational Limits and the Hubble Constant for the Local Group

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Abstract

This paper presents the equations for the calculation of the gravitational limits and Hubble constant for celestial bodies according to the Reciprocal System of physical theory developed by D. B. Larson. It then applies these equations to calculate the gravitational limits and the Hubble Constant for the Local Group of galaxies and compares the results to observation. It is shown that the Hubble Constant is not a fixed structural value of the universe; rather it is different for each stellar aggregate and is a function of mass.

keywords: gravitation, space-time, Hubble Constant, Reciprocal System, Local Group

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Introduction

The Reciprocal System of physical theory is described in the books by Dewey B. Larson, such as Ref. [1], [2], [3]. In this theory two oppositly-directed basic motions exist, the space-time progression and gravitation. Both are scalar, not vectorial or tensorial. The gravitational motion is inward, or negative, whereas the space-time progression is outward, or positive, in regard to macroscopic phenomena. The gravitational motion is attenuated with distance by the inverse-square law, whereas the space-time progression is not. Therefore, Larson concludes that "...the gravitational limit of a mass is the distance at which the inward gravitational motion of another mass toward the mass under consideration is equal to its outward motion due to the progression of the natural reference system relative to our stationary system of reference." (Ref. [3], p. 105). This is the first gravitational limit. The second gravitational limit is reached at the point where gravitation is eliminated entirely, and the mass entity recedes at the speed of light, the full speed of the progression. The Hubble Constant relates to the increase in speed of the mass entity between the two gravitational limits. Expanding on the work presented in Ref. [4], this paper is runnable as a *Mathcad* program, so the equations are more detailed than they otherwise would be.

Nomenclature

a_g= acceleration of arbitrary mass due to gravity of mass M, dynes/gm or cm/s^2

a_p= acceleration of arbitrary mass due to the space-time progression, dynes/gm or cm/s^2

a_n= net acceleration of arbitrary mass due to gravity and the space-time progression, dynes/gm or cm/s^2

b_1 = constant of integration for gravitational velocity

b_2 = constant of integration for velocity due to space-time progression

c = speed of light, cm/s

$conv_{cmtoly}$ = factor to convert cm to lightyears

$conv_{lytoMpc}$ = factor to convert lightears to Mpc (megaparsecs)

$conv_{Mpctokm}$ = factor to convert Mpc to km

$conv_{stoyear}$ = factor convert seconds to years

d_0 = gravitational limit at which gravitation equals the space-time progression, cm

d_{0km} = d_0, km

d_{0n} = d_0, natural space units

d_{0ly} = d_0, lightyears

d_{0Mpc} = d_0, Mpc

d_{0lyLG} = d_0 for Local Group, lightyears

d_{0MpcLG} = d_0 for Local Group, Mpc

d_1 = gravitational limit at which gravitation is eliminated entirely, cm

d_{1km} = d_1, km

d_{1ly} = d_1, lightyears

d_{1Mpc} = d1, Mpc

d_{1lyLG} = d_1 for Local Group, lightyears

d_{1MpcLG} = d_1 for Local Group, Mpc

G = universal constant of gravitation, dynes cm^2/gm^2

H = Hubble Constant, km/sec per Mpc

H_{LG1} = Hubble Constant for Local Group based on "observed mass", km/sec per Mpc

H_{LG2} = Hubble Constant for Local Group based on calculated mass, km/sec per Mpc

H_{LG3} = Hubble Constant for Local Group based on d_{0MpcLG} and d_{1MpcLG}

H_x = Hubble "Constant" as function of x close to d_0

I_{d0} = rotational ratio, dimensionless

I_{d1} = inter-regional ratio, dimensionless

j = jerk of the "universe", km/sec^3

M = mass of celestial body, gm (exept in Figures, where it's expressed in solar units)

M_n = mass of celestial body, natural mass units

M_s = mass of the sun, gm

M_{LGobs} = observed mass of Local Group of galaxies, gm

M_{LGcalc} = calculated mass of Local Group of galaxies, gm

$M_{LGobssol}$ = observed mass of Local Group of galaxies, in solar units

$M_{LGcalcsol}$ = calculated mass of Local Group of galaxies, in solar units

m_u = natural unit of mass, gm

P = universal constant of the space-time progression, dynes/gm or cm/s^2

s_u = natural unit of space, cm

t = time, s

t = time, s

$t_{LG\_0\_1\_sec}$ = lifetime of Local Group between the two gravitational limits, sec

tLG_0_1_yr = lifetime of Local Group between the two gravitational limits, yr

v_g = velocity of arbitrary mass due to gravitation, inside first gravitational limit, cm/s

v_{go} = velocity of arbitrary mass due to gravitation, outside first gravitational limit, cm/s

v_n = net velocity of arbitrary mass, inside first gravitational limit, cm/s

v_{no} = net velocity of arbitrary mass, outside first gravitational limit, cm/s

v_0 = speed of arbitrary mass due to space-time progression at first gravitational limit, cm/s

v_{0km} = speed of arbitrary mass due to space-time progression at first gravitational limit, km/s

v_p = velocity of mass due to the space-time progession, inside first gravitational limit, cm/s

v_{po} = velocity of mass due to space-time progression, outside first gravitational limit, cm/s

x = distance from celestial body to arbitrary mass, cm

y = ratio of x to first gravitational limit, dimensionless

Note: A black square in the upper right of an equation means that the equation is disabled from running in *Mathcad*. This is done because not all variables in the equation have, as yet, been given numerical values.

Unit Conversions and Physical Constants

$conv_{cmtoly} := 1.0571 \times 10^{-18}$ $conv_{lytoMpc} := 3.0674 \times 10^{-7}$

$conv_{Mpctokm} := 3.084 \cdot 10^{19}$ $conv_{stoyear} := 3.169 \cdot 10^{-8}$ $c := 2.997925 \cdot 10^{10}$ cm/s

$G := 6.67 \cdot 10^{-8}$ dynes cm^2 / gm^2

$M_s := 2 \cdot 10^{33}$ gm $M_{LGobs} := 1.03 \cdot 10^{12} \cdot M_s$ $M_{LGobs} = 2.06 \times 10^{45}$ gm (Ref. [5])

$m_u := 1.65979 \cdot 10^{-24}$ gm $s_u := 4.558816 \cdot 10^{-6}$ cm (Ref. [1], p. 160)

$I_{d1} := 156.4444$

1. Motion Inside the First Gravitational Limit

The acceleration of a body of arbitrary mass due to the gravitational force of a celestial body of mass M a distance x away (outside of it) is given by

$$a_g := \frac{-G M}{x^2} \qquad \text{dynes/gm or cm/s}^2 \qquad (1)$$

The acceleration of a body of arbitrary mass due to the force of the space-time progression is

$$a_p := P \qquad \text{dynes/gm or cm/s}^2 \qquad (2)$$

Therefore, the net acceleration of the mass at a distance x from mass M is

$$a_n := P - \frac{G \cdot M}{x^2} \qquad \text{dynes/gm or cm/s}^2 \tag{3}$$

The first gravitational limit occurs when the net acceleration is zero. To calculate the distance at which this occurs, d_0, we will have to temporarily revert to using the natural units of the Reciprocal System. With these units, there is no need to use the conversion factor G, and the value of P is 1. Thus

$$\frac{M_n}{{d_{0n}}^2} := 1 \tag{4}$$

But Ref. [3], p. 196, shows that the gravitational motion is distributed over 2.1475 x 10^9 units, only one of which is acting against the the outward progression. Larson terms this number the "rotational ratio."

$$I_{d0} := 2.1475 \cdot 10^9 \tag{5}$$

Equation 4 then becomes

$$\frac{M_n}{I_{d0} \cdot {d_{0n}}^2} := 1 \tag{6}$$

Solving for d_{0n}, we obtain

$$d_{0n} := \left(\frac{M_n}{l_{d0}}\right)^{.5} \qquad (7)$$

In terms of conventional units

$$d_0 := \left(\frac{\frac{M}{m_u}}{l_{d0}}\right)^{.5} \cdot s_u \qquad (8a)$$

$$d_0 := 76.3588 \cdot M^{.5} \quad \text{cm} \qquad (8b)$$

$$d_{0ly} := 3.6099 \cdot \left(\frac{M}{M_s}\right)^{.5} \quad \text{lightyears} \qquad (8c)$$

$$d_{0Mpc} := 2.476 \cdot 10^{-23} \cdot M^{.5} \quad \text{Mpc} \qquad (8d)$$

Knowing d_0 we can now obtain the value for P in conventional units:

$$P := \frac{G \cdot M}{d_0^{\,2}} \qquad P := \frac{G \cdot M}{76.3588^2 M} \qquad P := \frac{G}{76.3588^2} \qquad (9)$$

$$P = 1.144 \times 10^{-11} \quad \text{dynes/gm}$$

And, knowing the accelerations, we can now derive the speeds due to the gravitational and space-time progression accelerations. For gravitation:

$$a_g = dv_g/dt = (dv_g/dx) * v_g = - G * M / x^2 \qquad (10)$$

Separating variables and integrating,

$$\int v_g \, dv_g = \int \frac{-G \cdot M}{x^2} \, dx \qquad (10a)$$

we obtain

$$v_g^2 = 2 * G * M / x + b_1 \qquad (11)$$

For the acceleration due to the space-time progression:

$$a_p = dv_p/dt = (dv_p/dx) * v_p = P \qquad (12)$$

$$\int v_p \, dv_p = \int P \, dx \qquad (12a)$$

$$v_p^2 = 2 * P * x + b_2 \qquad (13)$$

At the first gravitational limit, where x = d_0, the net speed is zero, so $v_p^2 = v_g^2$.

$$2 * G * M / d_0 + b_1 = 2 * P * d_0 + b_2$$

$$b_1 - b_2 = 2 (P * d_0 - G * M / d_0) = 2 * d_0 (P - G * M / d_0^2) = 0 \qquad (14)$$

In the limit as x goes to zero, v_p should go to zero, or close to zero. Therefore b_1 is zero, or close to zero, which by eq. 14 makes b_2 zero, or close to zero, as well. Note that we are treating the masses here as essentially point masses, and therefore not including any differential gravitational effects. Also, note that if we were merely dealing with two atoms approaching each other, rather than huge aggregations of matter, the progression and gravitation would reverse directions at unit space. Right at the boundary of unit space between the two atoms, the progression is zero. Considering all of this, we will take b_1 and b_2 to be zero in what follows.

K. Nehru, Ref. [4}, terms the progression speed of the mass aggregate at d_0 the "zero-point speed", v_0.

$$v_0 := (2 \cdot P \cdot d_0)^{.5} \qquad (15a)$$

$$v_0 := 4.1797 \cdot 10^{-5} \cdot M^{.25} \quad \text{cm/s} \qquad (15b)$$

$$v_{0km} := 4.1797 \cdot 10^{-10} \cdot M^{.25} \quad \text{km/s} \qquad (15c)$$

If we let $y = x / d_0$, the distance in non-dimensional form, we can express both of the speeds in compact form:

$$v_g = -(2*G*M / x)^{.5} = -(2*G*M/d_0)^{.5} * (d_0/x)^{.5} = -v_0/y^{.5} \quad cm/s \qquad (16)$$

$$v_p = v_0 * y^{.5} \quad cm/s \qquad (17)$$

Then the net velocity, v_n, applicable from x equal zero (or near zero) to the first gravitational limit, is:

$$v_n = v_p + v_g = v_p - |v_g| = v_0 * (y^{.5} - 1/y^{.5}) \quad cm/s \qquad (18)$$

Here's a plot of v_{0km} (km/s) as a function of mass (gm) for galaxies or associations considerably smaller to considerably larger than the Milky Way.

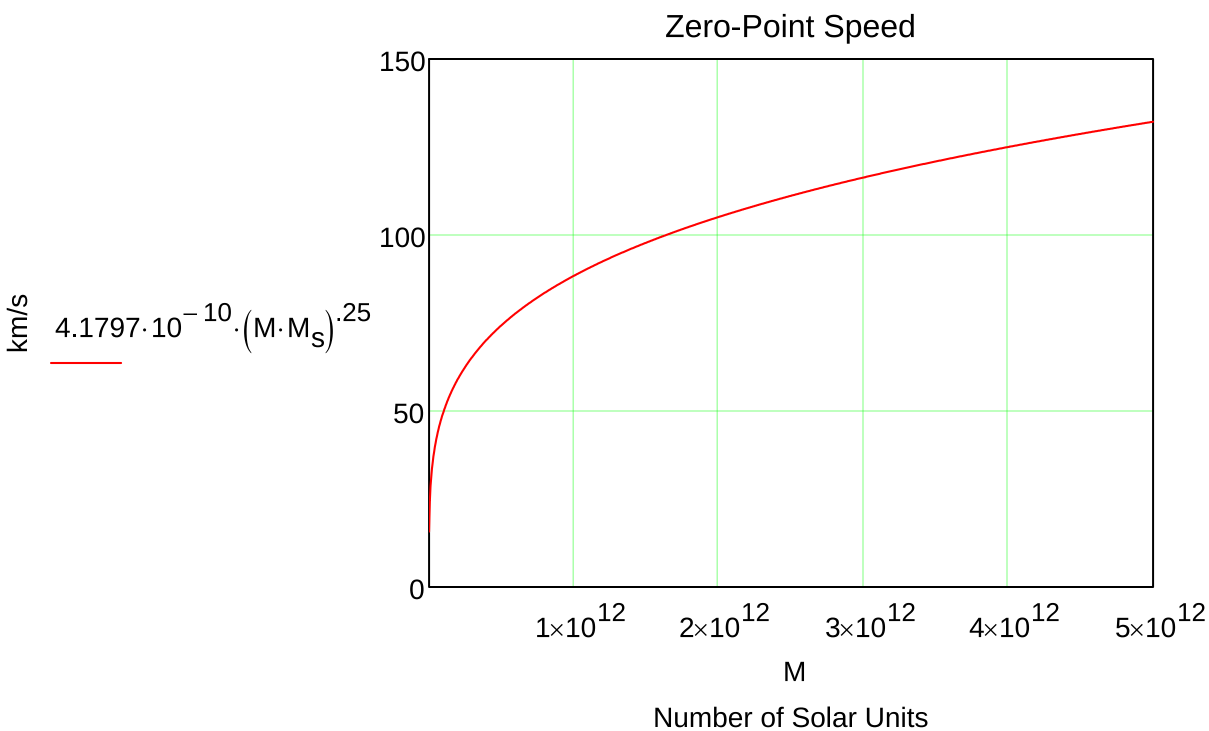

Figure 1. Zero-Point Speed as a Function of Mass

For the Local Group, the zero-point speed is (after converting to km/s)

$$v_{0LGkm} := 4.1797 \cdot 10^{-10} \cdot M_{LGobs}^{.25} \qquad v_{0LGkm} = 89.0455 \quad \text{km/s}$$

And here's a plot of the speed due to the space-time progession, the speed due to gravitation, and the net speed of an arbitrary mass as a function of distance from y (= x/d_0) equal to .05 to y = 1 for the Local Group, using the above equations. (Below approximately y = .05, local and differential gravitational effects cannot be ignored.)

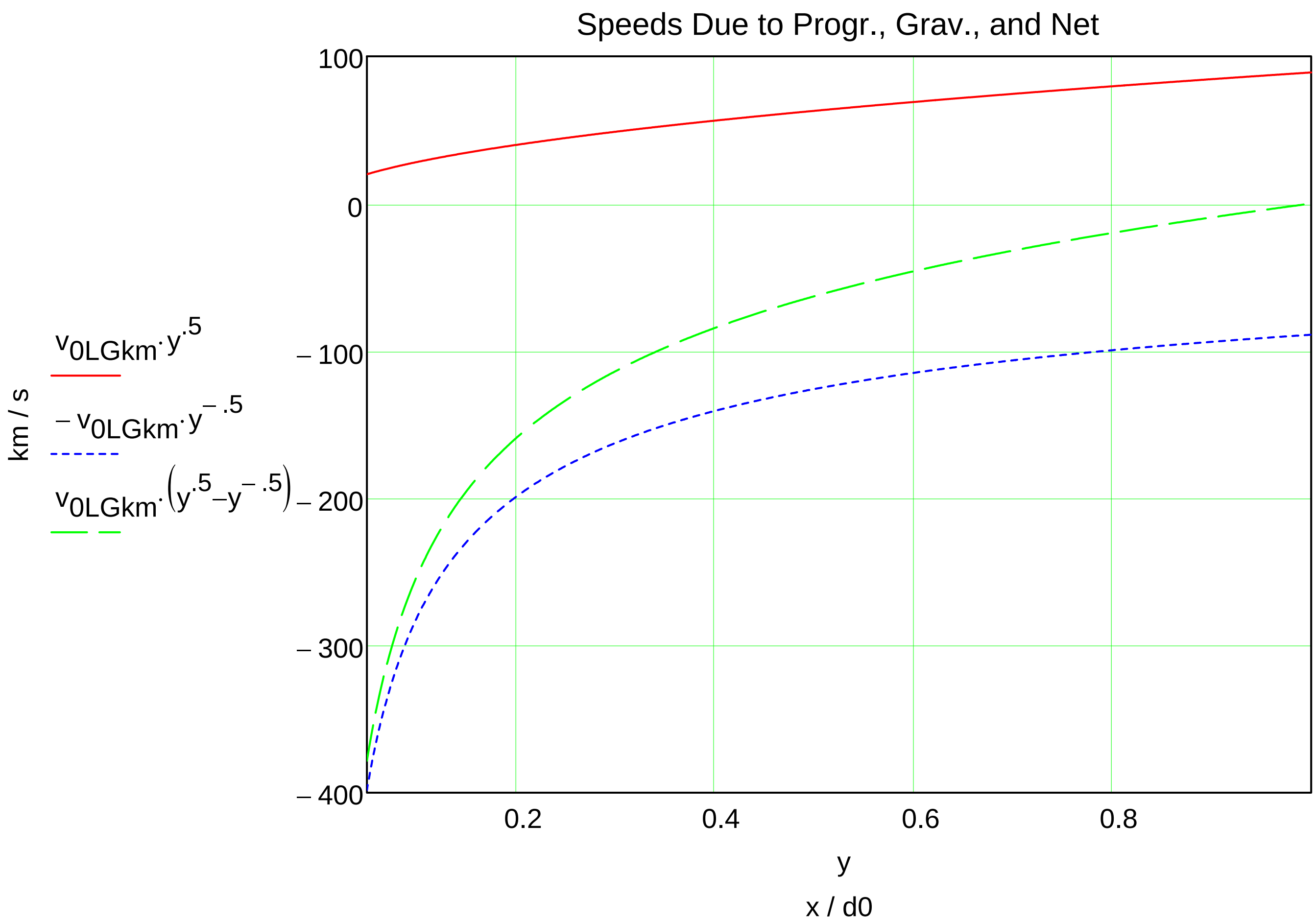

Figure 2. Speed Components of Arbitrary Mass Toward Local Group
as a Function of Distance in Terms of First Gravitational Limit

Also of great importance is the first gravitational limit as a function of mass. Here's the plot:

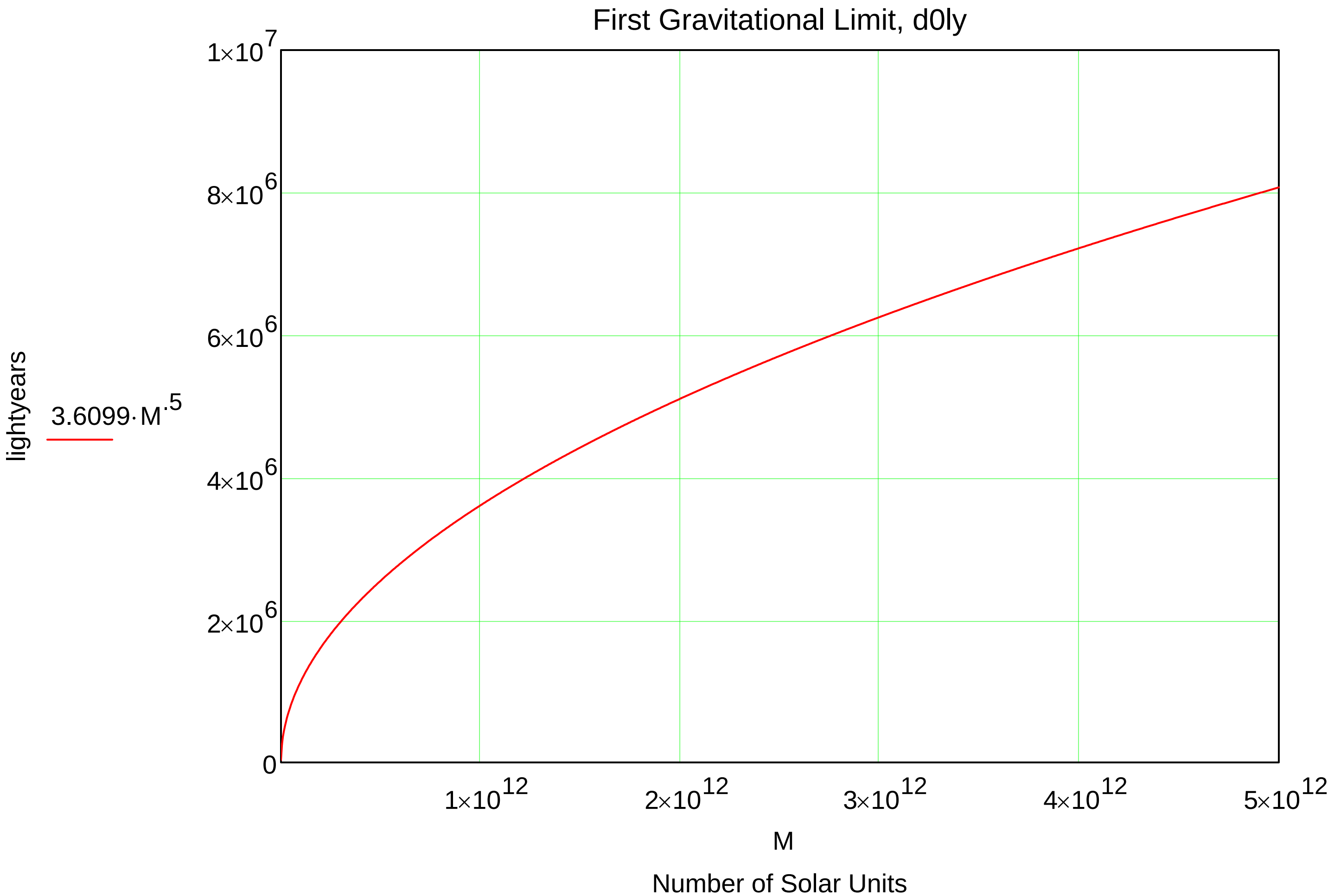

Figure 3a. First Gravitational Limit, d_{0ly}, as a Function of Mass

For the Local Group,

$$d_{0lyLG} := 3.6099 \cdot \left(\frac{M_{LGobs}}{M_S}\right)^{.5} \qquad d_{0lyLG} = 3.6636 \times 10^{6} \text{ lightyears}$$

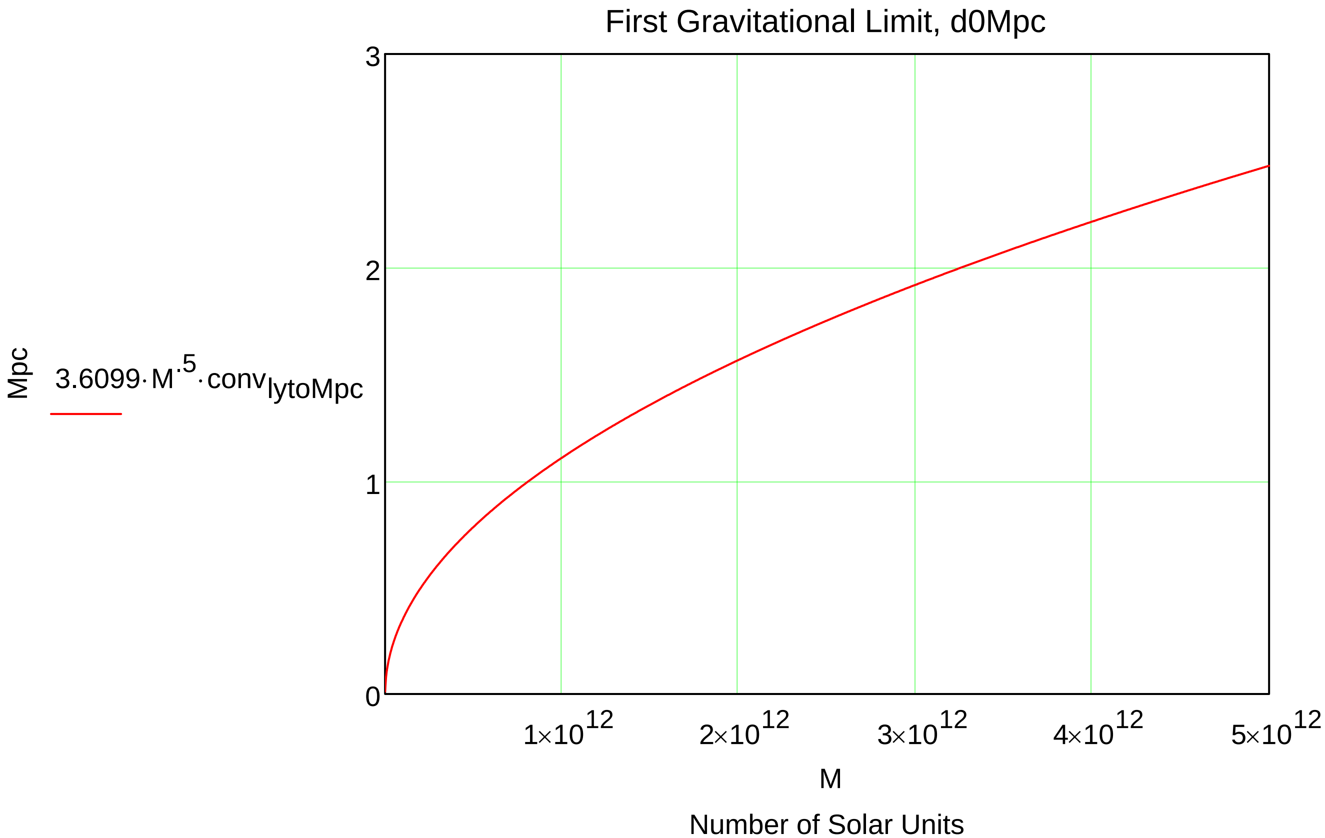

Figure 3b. First Gravitational Limit, $d_{0Mpc,}$ as a Function of Mass

For the Local Group,

$$d_{0MpcLG} := 3.6099 \cdot \left(\frac{M_{LGobs}}{M_S} \right)^{.5} \cdot conv_{lytoMpc} \qquad d_{0MpcLG} = 1.1238 \ \ \text{Mpc}$$

Ref. [5] says that "the zero-velocity surface, which separates the Local Group from the field that is expanding with the Hubble flow, has a radius R_o = 1.18 +/- 0.15 Mpc." Our result thus appears a bit high, but this is based on the observed mass--also as reported in Ref. [5]. If, instead, we take d_{0MpcLG} = 1.18, we would then calculate the mass of the Local Group to be,

$$M_{LGcalcsol} := \left(\frac{1.18}{conv_{lytoMpc} \cdot 3.6099}\right)^2 \qquad M_{LGcalcsol} = 1.1356 \times 10^{12}$$

which is close to that "observed." Ref. [5] also states that "The galaxies NGC 3109, Antlia, Sextans A and Sextans B appear to form a distinct grouping...that is located beyond the LG zero-velocity surface at a distance of 1.7 Mpc from the Local Group centroid." Both 1.18 Mpc and 1.6793 Mpc are consistent with this observation.

2. Motion Outside the First Gravitational Limit and Inside the Second Gravitational Limit

Beyond the first gravitational limit, the motion changes. Larson states that "...the three-dimensional region of space extends only to the gravitational limit...beyond this limit...the gravitational effect of the aggregate is in equivalent space rather than in actual space." (Ref. [3], p. 197). All quantities in equivalent space (whether within a single unit of space or outside the first gravitational limit) are second power expressions of those in ordinary space--"...if we measure the quantity *a* in the outside region, it is essential that the equation be expressed in the correct regional form: $a = b^2c^2$." (Ref. [1], p. 155).

First we divide the gravitational speed of the arbitrary mass, v_g, by the the zero-point speed v_0, square it, and then multiply by v_0. So

$$v_{go} := -\left(\frac{v_g}{v_0}\right)^2 \cdot v_0 \quad \text{cm/s} \qquad (19a)$$

Substituting from eq. (16),

$$v_{go} := \frac{-v_0}{y} \quad \text{cm/s} \qquad (19b)$$

Second we divide the speed of the arbitrary mass due to the space-time progression, v_p, by the zero-point speed v_0, square it, and then multiply by v_0.

$$v_{po} := \left(\frac{v_p}{v_0}\right)^2 \cdot v_0 \quad \text{cm/s} \qquad (20a)$$

Substituting from eq. (17)

$$v_{po} := v_0 \cdot y \quad \text{cm/s} \qquad (20b)$$

Eq. (18) then becomes

$$v_{no} = v_{po} + v_{go} = v_{po} - |v_{go}| = v_0 * (y - 1/y) \text{ cm/s} \qquad (21)$$

For distances large compared with the first gravitational limit, d_0, eq. (21) reduces to

$$v_{no} := v_0 \cdot y \quad \text{cm/s} \qquad (22)$$

Substituting for v_0 and y, we get

$$v_{no} := \left(\frac{2 \cdot P}{d_0}\right)^{.5} \cdot x \qquad \text{cm/s} \qquad (23)$$

Substituting for d_0, and simplifying, we get

$$v_{no} := 5.4739 \cdot 10^{-7} \cdot M^{-.25} \cdot x \qquad \text{cm/s} \qquad (24)$$

By inspection, we can see that Hubble's Constant is

$$H := 5.4739 \cdot 10^{-7} \cdot M^{-.25} \qquad s^{-1} \qquad (25)$$

Converting to the normal units for Hubble's Constant (using $conv_{Mpctokm}$), we finally obtain

$$H := 1.6882 \cdot 10^{13} \cdot M^{-.25} \qquad \text{km s}^{-1}\ \text{Mpc}^{-1} \qquad (26)$$

For the Local Group of galaxies, the result is

$$H_{LG1} := 1.6882 \cdot 10^{13} \cdot M_{LGobs}^{-.25} \qquad H_{LG1} = 79.2423 \qquad \text{km s}^{-1}\ \text{Mpc}^{-1}$$

If we use the calculated mass for the Local Group, instead, we get

$$H_{LG2} := 1.6882 \cdot 10^{13} \cdot \left(M_{LGcalcsol} \cdot M_s\right)^{-.25} \qquad H_{LG2} = 77.3318 \qquad \text{km s}^{-1}\ \text{Mpc}^{-1}$$

Either of these values of the Hubble Constant for the Local Group is close to the range of current estimates by astronomers. Ref. [6] states "Many astronomers are working hard to measure the Hubble Constant using a variety of different techniques. Until recently, the best estimates ranged from 65 km/sec/Megaparsec to 80 km/sec/Megaparsec, with the best value being about 72 km/sec/Megaparsec." But, please note that H is not a single and fixed structural constant of the universe. From the above equations, it clearly is a function of mass of the celestial body. A plot will be helpful.

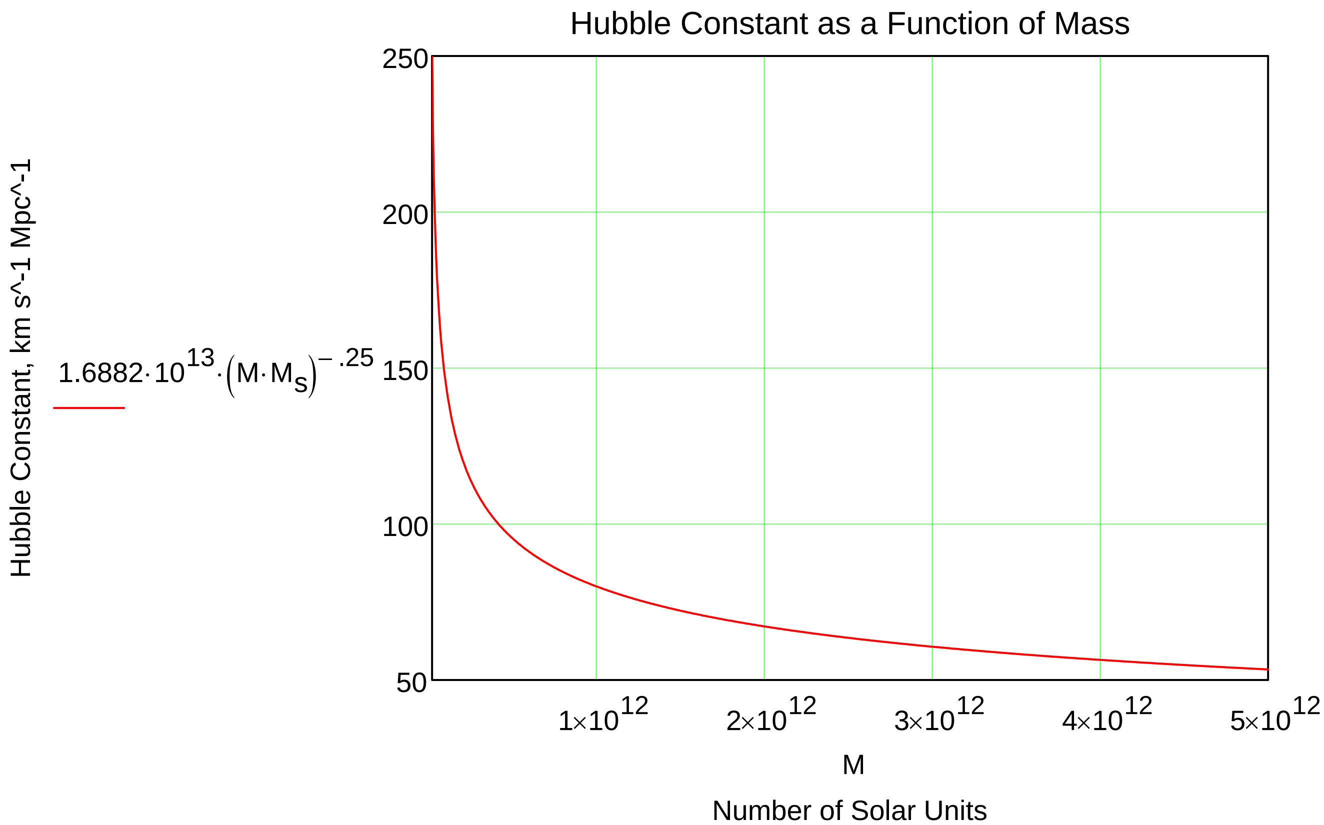

Figure 4. Hubble Constant as a Function of Number of Solar Units

The gravitational motion should not be neglected close to the first gravitational limit (< $10 \cdot d_0$).
Let's plot the speeds (converted to km/s) due to the gravitational force, the space-time progression, and the net speed for an arbitrary mass in relation to the Local Group.

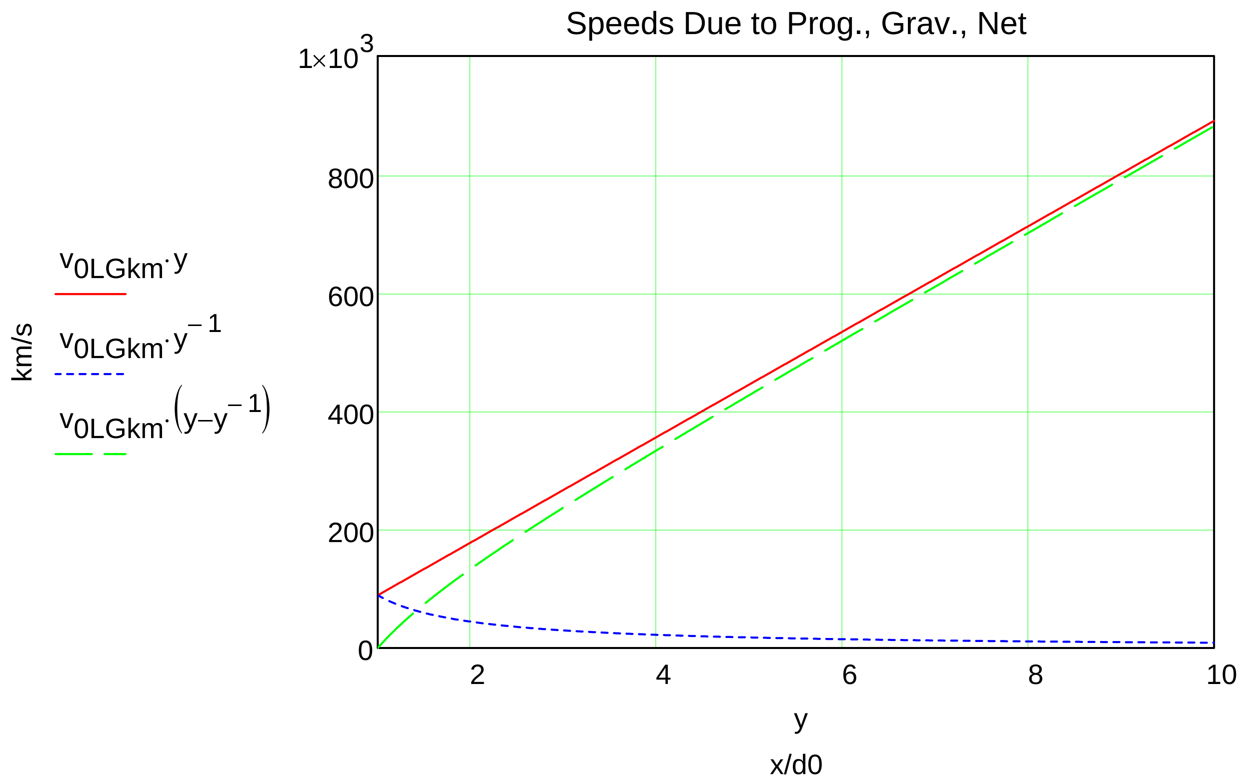

Figure 5. Speed Components of Mass Away from Local Group as a Function of Distance in Multiples of First Gravitational Limit

Obviously, close to the first gravitational limit the value of Hubble's Constant is really not constant.
This can be seen by writing eq. (21) as

$$H_x \cdot x := v_0 \cdot \left(\frac{x}{d_0} - \frac{d_0}{x} \right) \qquad (27)$$

Solving for H_x as a function of x we get

$$H_x := 4.1797 \cdot 10^{-10} \cdot M^{.25} \cdot \left(\frac{1}{2.476 \cdot 10^{-23} \cdot M^{.5}} - \frac{2.476 \cdot 10^{-23} \cdot M^{.5}}{x^2} \right) \qquad (28)$$

For the Local Group this plots as follows.

km/s

$$4.1797 \cdot 10^{-10} \cdot M_{LGobs}^{.25} \cdot \left(\frac{1}{2.476 \cdot 10^{-23} \cdot M_{LGobs}^{.5}} - \frac{2.476 \cdot 10^{-23} \cdot M_{LGobs}^{.5}}{x^2} \right)$$

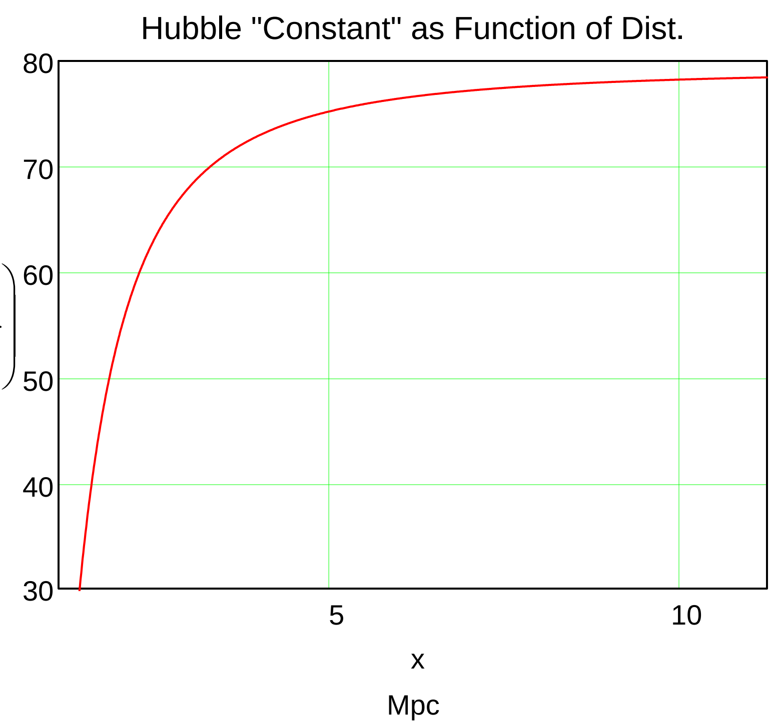

Figure 6. Hubble "Constant" as Function of Distance Near d_0

3. Second Gravitational Limit

Larson states (Ref. [3], pp.200-201), "The net outward speed increases with the distance, but it cannot continue increasing indefinitely. Eventually the attenuation of the gravitational motion by distance brings it down to the point where the remaining motion of each mass unit is sufficient to cover the distribution over the dimensional units involved in direct one-dimensional contact between motion in space and motion in time. Less than this amount...does not exit. Beyond this point, therefore, the gravitational effect is eliminated entirely, and the recession takes place at the full speed of light." Larson then goes on to derive the value of this second gravitational limit, d_1, by a method similar to that for d_0. He substitutes I_{d1}= 156.4444 (the inter-regional ratio) for I_{d0} in our eq. (6).

$$\frac{M_n}{I_{d1} \cdot d_{1n}^2} := 1 \qquad (29)$$

Note that relativistic corrections are not necessary since we're working in equivalent space, not ordinary space!

Solving for d_{1n}, we obtain

$$d_{1n} := \left(\frac{M_n}{I_{d1}}\right)^{.5} \qquad (30)$$

Therefore, in terms of conventional units,

$$d_1 := \left(\frac{\frac{M}{m_u}}{I_{d1}}\right)^{.5} \cdot s_u \qquad (31a)$$

$$d_1 := 2.8291 \cdot 10^5 \cdot M^{.5} \qquad \text{cm} \qquad (31b)$$

$$d_{1ly} := 13374 \cdot \left(\frac{M}{M_S}\right)^{.5} \qquad \text{lightyears} \qquad (31c)$$

$$d_{1Mpc} := 4.1023 \cdot 10^{-3} \cdot \left(\frac{M}{M_S}\right)^{.5} \qquad \text{Mpc} \qquad (31c)$$

For the Local Group:

$$d_{1lyLG} := 13374 \cdot \left(\frac{M_{LGobs}}{M_s}\right)^{.5} \qquad d_{1lyLG} = 1.3573 \times 10^{10} \quad \text{lightyears}$$

$$d_{1MpcLG} := 4.1023 \cdot 10^{-3} \cdot \left(\frac{M_{LGobs}}{M_s}\right)^{.5} \qquad d_{1MpcLG} = 4.1634 \times 10^{3} \quad \text{Mpc}$$

Now the plots:

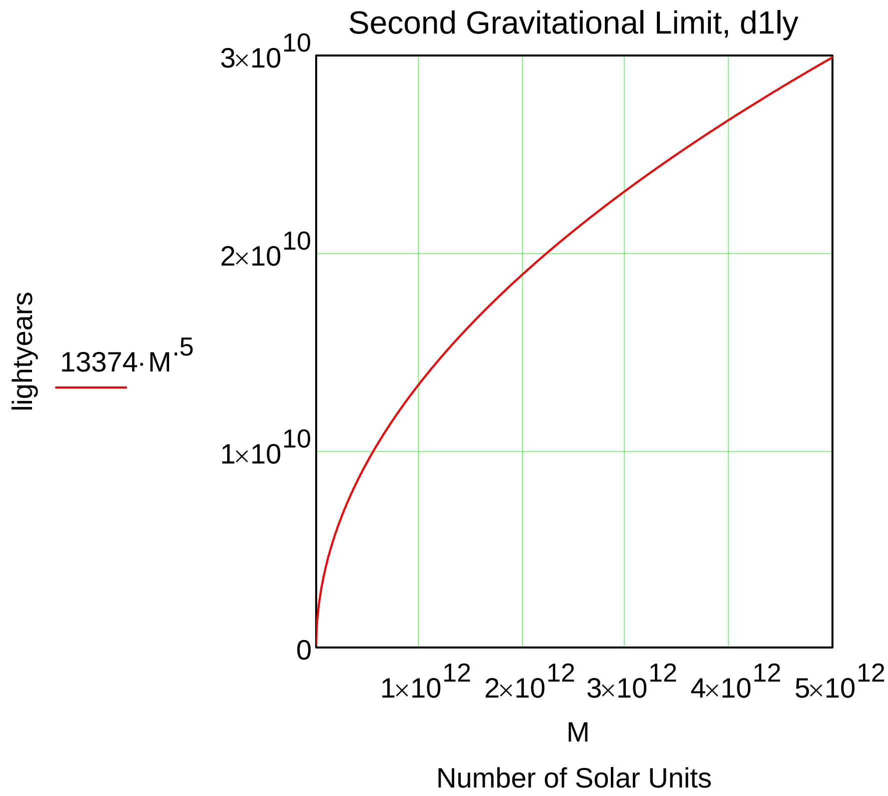

Figure 7. Second Gravitational Limit, lightyears

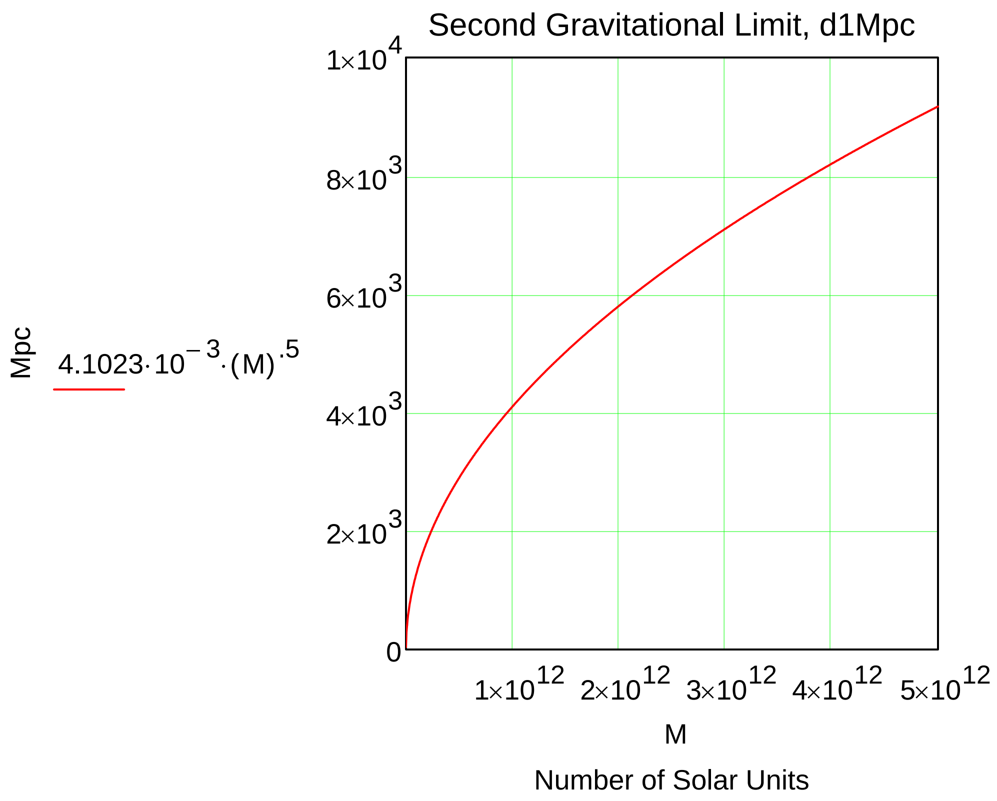

Figure 8. Second Gravitational Limit, Mpc

Knowing both the first and second gravitational limits, we have another way to calculate Hubble's Constant for the Local Group, using M_{LGobs}.

$$H_{LG3} := \frac{\left(c \cdot 10^{-5}\right)}{\left(d_{1MpcLG} - d_{0MpcLG}\right)} \qquad H_{LG3} = 72.0264 \qquad (32)$$

which is very close to the best current estimate of the astronomers.

Discussion

As the above equations show, Hubble's Constant is a function of mass. It is not a fixed structural constant of the universe. It's also a function of distance close to the first gravitational limit of our Local Group of galaxies. Both the first and second gravitational limits are a function of mass, as well. The space-time progression is the source of the expansion.

Conventional astrophysical theory posits that the universe is expanding because of a big bang explosion which occurred at 1/H seconds ago. According to Ref. [6], if the universe has a very low density of matter then the age of the universe is extrapolated to (If H is taken to be 72 km/s/Mpc)

$$\left(\frac{72}{conv_{Mpctokm}}\right)^{-1} \cdot conv_{stoyear} = 1.3574 \times 10^{10} \quad \text{years}$$

or 13.6 billion years. But, if the universe is flat and composed mostly of matter then the age of the universe is, according to Ref. [6], 2/(3*H), or

$$\left(\frac{3 \cdot 72}{2 \cdot conv_{Mpctokm}}\right)^{-1} \cdot conv_{stoyear} = 9.0493 \times 10^{9} \quad \text{years}$$

or about 9 billion years. This latter calculation puts astronomers into a conundrum because that's less than the age of the oldest stars!

Ref. [6] concludes that *if* the second calculation were correct then either 1) our measurement of the Hubble Constant is incorrect, 2) the Big Bang theory is incorrect, or 3) that we need a form of matter like a cosmological constant that implies an older age for a given observed expansion rate.

Ref. [6] then says that NASA's WMAP satellite has come to the rescue and measured the expansion rate such that the first calculation is correct.

None of this affects the Reciprocal System of theory. The Hubble Constant is not, in fact, a determinate of the age of the universe. The observed expansion is due to the space-time progression overpowering gravitation at the first gravitational limit and causing the mass to accelerate to the speed of light at the second gravitational limit. There was no cosmic egg that exploded 13 billion years ago.

Acknowledgments

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References

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[5] S. van den Bergh, "The Local Group of Galaxies," *The Astronomy and Astrophysics Review*, Vol. 9, No. 3-4, 1999, p. 273.

[6] NASA, http://map.gsfc.nasa.gov/m_uni/uni_101.age.html, 12/15/2005.

Updated 12/29/2015 to revise calculation of second gravitational limit
Updated 08/05/2015 to add Appendix C

Appendix A: Acceleration of "the Universe"

From Eq. (21) and Eq. (15c) we have

$$v := 4.1797\cdot 10^{-10}\cdot M^{.25}\cdot\left(\frac{x}{d_0} - \frac{d_0}{x}\right) \qquad \text{km/sec} \qquad \text{(A-1)}$$

Then

$$a := \frac{d}{dt}v \qquad \text{or} \qquad a := \frac{d}{dx}v\cdot v \qquad \text{(A-2)}$$

$$\frac{d}{dx}\left[4.1797\cdot 10^{-10}\cdot M^{.25}\cdot\left(\frac{x}{d_0} - \frac{d_0}{x}\right)\right] \rightarrow 4.1797\text{e-}10\cdot M^{0.25}\cdot\left(\frac{d_0}{x^2} + \frac{1}{d_0}\right)$$

so

$$a := 4.1797\text{e-}10\cdot M^{0.25}\cdot\left(\frac{d_0}{x^2} + \frac{1}{d_0}\right)\cdot\left[4.1797\cdot 10^{-10}\cdot M^{.25}\cdot\left(\frac{x}{d_0} - \frac{d_0}{x}\right)\right]\cdot\left(\frac{1}{conv_{Mpctokm}}\right) \qquad \text{km/sec}^2 \qquad \text{(A-3)}$$

where, for convenience, we use Mpc for the distances.

Set

$M_1 := M_{LGobs}$ $M_1 = 2.06 \times 10^{45}$ gm

(two different determinations for mass of Local Group)

$M_2 := M_{LGcalcsol} \cdot M_s$ $M_2 = 2.2712 \times 10^{45}$ gm

$d_{0\_1} := d_{0MpcLG}$ $d_{0\_1} = 1.1238$ Mpc

(resultant gravitational limit for the two mass choices)

$d_{0\_2} := 1.18$ Mpc

We'll make two functions to represent the two different rates:

$$RS\_1(M_1, d_{0\_1}, x) := 4.1797e\text{-}10 \cdot M_1^{0.25} \cdot \left(\frac{d_{0\_1}}{x^2} + \frac{1}{d_{0\_1}}\right) \cdot \left[4.1797 \cdot 10^{-10} \cdot M_1^{.25} \cdot \left(\frac{x}{d_{0\_1}} - \frac{d_{0\_1}}{x}\right)\right] \cdot \left(\frac{1}{conv_{Mpctokm}}\right)$$

$$RS\_2(M_2, d_{0\_2}, x) := 4.1797e\text{-}10 \cdot M_2^{0.25} \cdot \left(\frac{d_{0\_1}}{x^2} + \frac{1}{d_{0\_2}}\right) \cdot \left[4.1797 \cdot 10^{-10} \cdot M_2^{.25} \cdot \left(\frac{x}{d_{0\_2}} - \frac{d_{0\_2}}{x}\right)\right] \cdot \left(\frac{1}{conv_{Mpctokm}}\right)$$

Here's the plot, showing how the acceleration is increasing for both cases, together with the accelerations from different values of Hubble's Law, 65, 72, and 77 (km/sec)/Mpc.

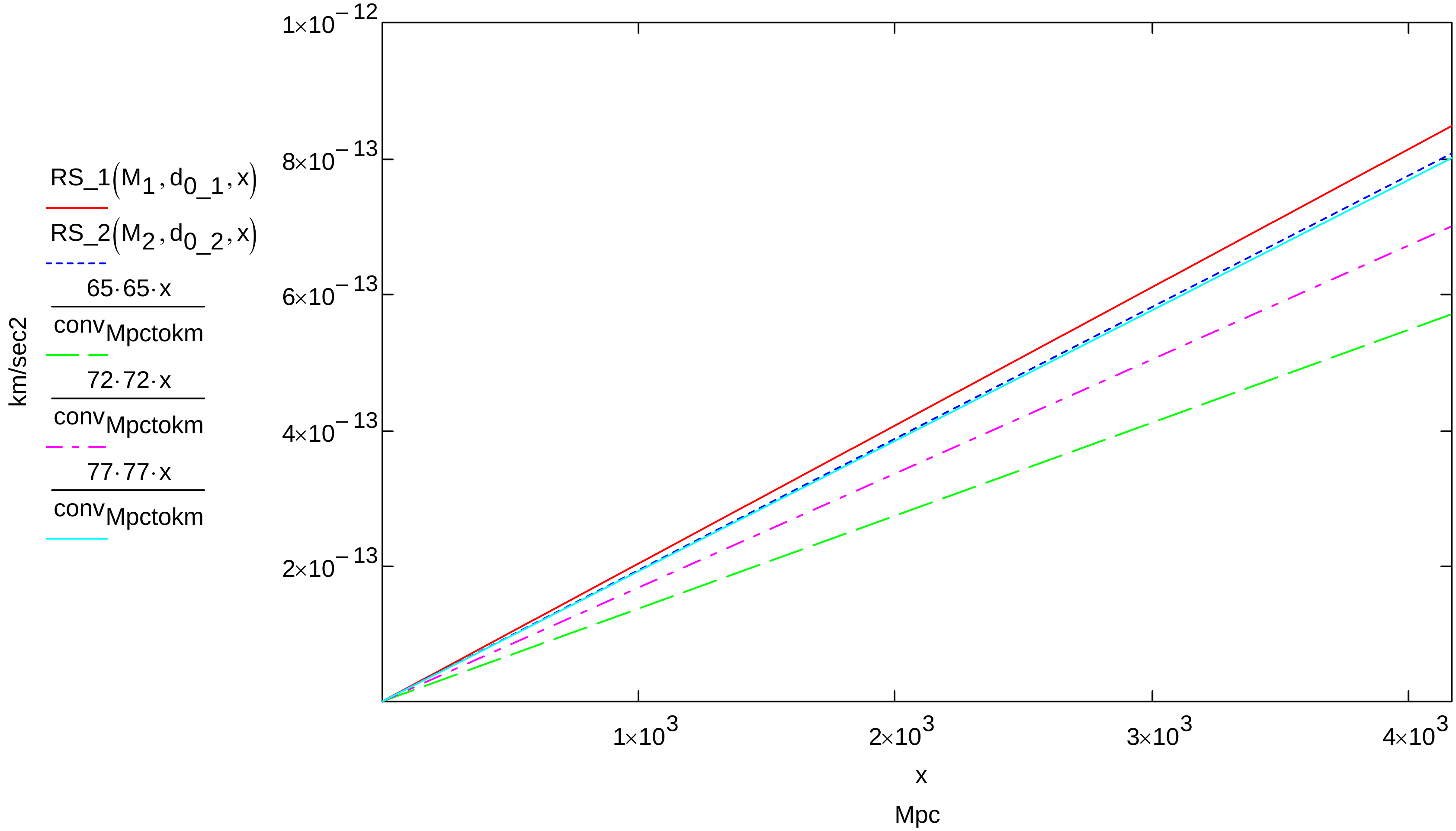

Figure 9. Acceleration of "the Universe"

Appendix B: Jerk of "the Universe"

The jerk is defined as the rate of change with time of acceleration. Therefore:

$$a := \frac{d}{dt}v \qquad \text{or} \qquad a := \frac{d}{dx}v \cdot v$$

$$j := \frac{d}{dt}a \qquad \text{or} \qquad j := \frac{d}{dx}a \cdot v \qquad \text{(B-1)}$$

$$a := 4.1797\text{e-}10 \cdot M^{0.25} \cdot \left(\frac{d_0}{x^2} + \frac{1}{d_0}\right) \cdot \left[4.1797 \cdot 10^{-10} \cdot M^{.25} \cdot \left(\frac{x}{d_0} - \frac{d_0}{x}\right)\right] \cdot \left(\frac{1}{conv_{Mpctokm}}\right) \qquad \text{km/sec}^2$$

$$\frac{d}{dx}\left[4.1797\text{e-}10 \cdot M^{0.25} \cdot \left(\frac{d_0}{x^2} + \frac{1}{d_0}\right) \cdot \left[4.1797 \cdot 10^{-10} \cdot M^{.25} \cdot \left(\frac{x}{d_0} - \frac{d_0}{x}\right)\right] \cdot \left(\frac{1}{conv_{Mpctokm}}\right)\right] \rightarrow 5.66468615110246433\text{2e-}39 \cdot M^{0.5} \cdot ($$

$$RS\_3\left(M_1, d_{0\_1}, x\right) := \left[5.66468615110246433\text{2e-}39 \cdot M_1^{0.5} \cdot \left(\frac{d_{0\_1}}{x^2} + \frac{1}{d_{0\_1}}\right)^2 + \frac{1.13293723022049286\text{64e-}38 \cdot M_1^{0.5} \cdot d_{0\_1} \cdot (}{x^3}\right.$$

$$RS\_4\left(M_2, d_{0\_2}, x\right) := \Bigg[5.66468615110246433 2e\text{-}39 \cdot M_2^{0.5} \cdot \left(\frac{d_{0\_2}}{x^2} + \frac{1}{d_{0\_2}} \right)^2 + \frac{1.13293723022049286 64e\text{-}38 \cdot M_2^{0.5} \cdot d_{0\_2} \cdot \Big(\frac{d}{\;}}{x^3}$$

The graphs follow.

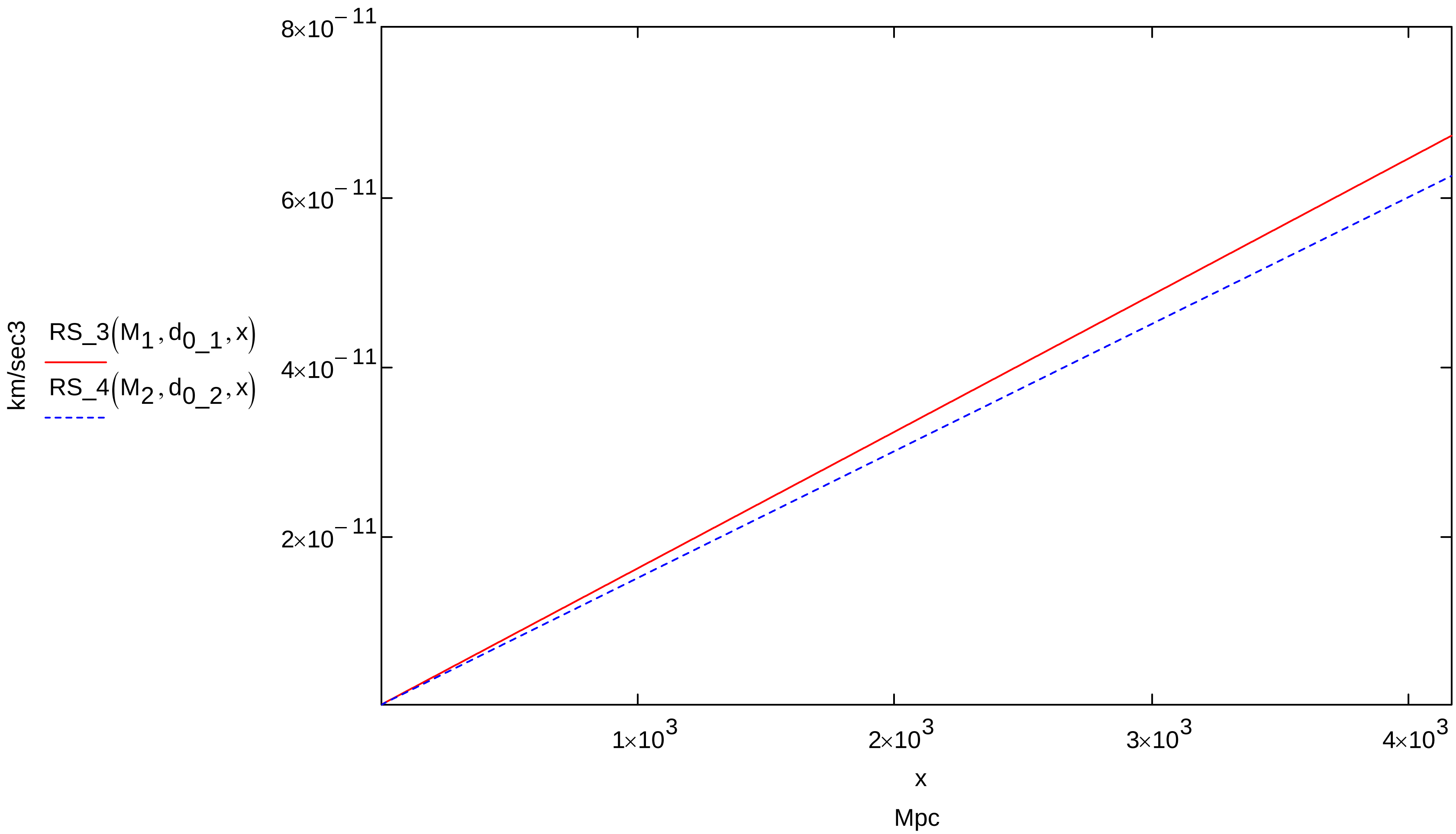

Figure 10. Jerk of "the Universe"

Appendix C: Time Span Between First and Second Gravitational Limits

From elementary kinematics:

$$t := \int_{x_1}^{x_2} \frac{1}{v}\,dx \qquad \text{(C-1a)}$$

Let

$d_{0kmLG} := d_{0MpcLG} \cdot conv_{Mpctokm}$ $\qquad d_{0kmLG} = 3.4658 \times 10^{19}$ km

$d_{1kmLG} := d_{1MpcLG} \cdot conv_{Mpctokm}$ $\qquad d_{1kmLG} = 1.284 \times 10^{23}$ km

For the Local Group, using the observed mass, and neglecting the time near the first gravitational limit:

$$\int_{d_{0kmLG}}^{d_{1kmLG}} \frac{1}{4.1797 \cdot 10^{-10} \cdot \left(M_{LGobs}{}^{.25}\right) \cdot \left(\frac{x}{d_{0kmLG}}\right)}\,dx \rightarrow 3.19830520104061637\text{02e18} \quad \text{sec} \qquad \text{(C-1b)}$$

$t_{LG\_0\_1\_sec} := 3.19830520104061637\text{02e18}$ sec

$t_{LG\_0\_1\_yr} := t_{LG\_0\_1\_sec} \cdot conv_{stoyear}$ $t_{LG\_0\_1\_yr} = 1.0135 \times 10^{11}$ years

This indicates that the Local Group will be discombobulated far before reaching a recession speed equal to that of light. If we assume that 1) there is a limit of four Type I supernovae explosions for the same matter and 2) that the time between each Type 1 supernova is 10 billion years, that gives 40 billion years. If we add another five billion years until the Type II supernova explosion, which terminates the matter's existence in our sector, that gives a total of 45 billion years. The quasars ejected from the Local Group, when a whole series of Type II supernovae go off, might last another five billion years before themselves exploding and creating the matter rays which leave our sector of the universe. The total then is approximately 50 billion years--considerably less than 119 billion years. If we assume the equivalent lifetime of cosmic matter in the cosmic sector to be 50 billion years also, this gives the cyclical period of the universe to be 100 billion years.

As a check on the above calculations, we can use the values calculated for the Hubble Constant (where distance is in Mpc).

$$t_{LG\_0\_1\_sec} := \int_{d_{0MpcLG}}^{d_{1MpcLG}} \frac{conv_{Mpctokm}}{H_{LG1} \cdot x} \, dx \qquad \text{(C-1c)}$$

$t_{LG\_0\_1\_sec} = 3.1981 \times 10^{18}$ sec (close enough to above result)

The main point of this discussion is that there are *limits in space and time* to the matter in our sector of the universe, the material sector. In terms of lightyears, the second gravitational limit is at

$$d_{1lyLG} = 1.3573 \times 10^{10} \quad \text{ly}$$

The mass of the Local Group may or may not be representative of other clusters of galaxies or giant spheroidal galaxies. The *largest* known cluster of galaxies has a mass of approximately 10^{15} M_s. For such a *rich* cluster:

$$d_{1ly\_rich\_cluster} := 13374 \cdot \left(10^{15}\right)^{.5} \qquad d_{1ly\_rich\_cluster} = 4.2292 \times 10^{11} \quad \text{ly}$$

But this calculation would *not* really be right because the rich cluster would actually be comprised of *many* galaxy groups, which would be slowly *moving apart*.

Possibly the *largest galaxy group* would comprise the equivalent of 1.4×10^{13} suns. Then

$$d_{1ly\_rich\_galaxy\_group} := 13374 \cdot \left(1.4 \cdot 10^{13}\right)^{.5} \qquad d_{1ly\_rich\_galaxy\_group} = 5.0041 \times 10^{10}$$

This then would be the actual "radius of the universe"--about 50 billion light years.

$$\frac{d_0}{x^2} + \frac{1}{d_0}\Bigg)^2 + \frac{1.132937230220492866 4e\text{-}38 \cdot M^{0.5} \cdot d_0 \cdot \left(\frac{d_0}{x} - \frac{x}{d_0}\right)}{x^3}$$

$$\frac{\left.\frac{d_{0\_1}}{x} - \frac{x}{d_{0\_1}}\right)\Bigg]}{} \cdot \left[4.1797 \cdot 10^{-10} \cdot M_1^{.25} \cdot \left(\frac{x}{d_{0\_1}} - \frac{d_{0\_1}}{x}\right)\right]$$

$$\left.\frac{\left.{}_{0\_2}\right. \ldots - \frac{x}{d_{0\_2}}\right)}{\ldots}\right] \cdot \left[4.1797 \cdot 10^{-10} \cdot M_2^{\cdot 25} \cdot \left(\frac{x}{d_{0\_2}} - \frac{d_{0\_2}}{x}\right)\right]$$