Motion of the Sun in the Milky Way Galaxy

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Abstract

This paper presents the simple equations for the calculation of the motion of the Sun in the Milky Way galaxy. It is shown that there are two factors: the rotation around the galactic core of the *spiral arm* in which the Sun is situated and the motion of the Sun *along* the spiral arm.

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Introduction

Larson, in Ref. [1], pp. 17-18, says this about the spiral structure of the Milky Way and similar galaxies:

"The mixing of this large amount of dust and gas with the stars of the galaxy alters the dynamics of the rotation, and causes a change in the galactic structure. If the dust cloud is captured while the galaxy is still quite small, the result is likely to be a reversion to the irregular status until further growth of the galaxy takes place. Because of the relative scarcity of the immature clusters, however, most captures of these objects occur after the [small] elliptical galaxy has grown to a substantial size. In this case the result is that the structure of the galaxy opens up and a spiral form develops.

"There has been a great deal of speculation as to the nature of the forces responsible for the spiral structure, and no adequate mathematical treatment of the subject has appeared. But from a qualitative standpoint there is actually no problem, as the forces which are definitely known to exist—the rotational forces and the gravitational attraction—are sufficient in themselves to account for the observed structure. As already noted, the galactic aggregate has the general characteristics of a heterogeneous viscous liquid. A spiral structure in a rotating liquid is not unusual; on the contrary, a striated or laminar structure is almost always found in a rapidly moving heterogeneous fluid, whether the motion is rotational or translational. Objections have been raised to this explanation, generally known as the "'coffee cup'" hypothesis, on the ground that the spiral in a coffee cup is not an exact replica of the galactic spiral, but it must be remembered that the coffee cup lacks one force that plays an important part in the galactic situation: the gravitational attraction toward the center of the mass. If the experiment is performed in such a manner that a force simulating gravitation is introduced, as, for instance, by replacing the coffee cup by a container that has an outlet at the bottom center, the resulting structure of the surface of the water is very similar to the galactic spiral In this kind of a rotational structure the spiral is the last stage, not an intermediate form."

Thus, obviously, Kepler's laws do *not* apply here, and those astrophysicists who attempt to do so come to the conclusion that the galactic mass is insufficient to account for the observed stellar motions and therefore there must be "dark matter." Nonsense. The Milky Way spiral is a very slowly rotating vortex, with the stars gradually moving into the core along the spiral arms. Each star (or binary or multiple system) is outside the gravitational limit of other stars, but attracted inward by the mass of stars in the galactic core. The stars are *not* "orbiting" the core like planets around the Sun. The structure is fluid, and the stars are following one another, very slowly, on their particular spiral.

The astrophysics literature has many complicated proposals for the motion, like that of Prof. Shu in Ref. [4], pp. 275-284. The model presented in this paper is much simpler and is intended for practical use.

Nomenclature

a = parameter of spiral, kpc

c = multiplicative constant for the spiral angle (dimensionless)

$conv_{kpctokm}$ = conversion factor from kpc to km

$conv_{sectoyears}$ = conversion factor from seconds to years

p = time for one revolution of the Sun around the galactic center, years

r = Sun's radial distance to galactic center, kpc

s = arclength of spiral between any two angles, kpc

t = time for Sun to reach galactic center, years

v_{along_spiral} = velocity of Sun along the third arm of the galactic spiral, kpc/sec

v_{Sun} = tangential velocity of the Sun, kpc/sec

θ = angle between perpendicular to the center of the spiral and a point on an arm, rad

ω_{arm} = rotational speed of the third arm of the galactic spiral, rad/sec

ω_{arm_years} = rotational speed of the third arm of the galactic spiral, rad/years

Unit Conversions

$conv_{sectoyears} := \frac{1}{3600} \cdot \frac{1}{24} \cdot \frac{1}{365.25}$

$conv_{sectoyears} = 3.1688 \times 10^{-8}$

$conv_{kpctokm} := 1000 \cdot \left(3.08567758128 \cdot 10^{13}\right)$

$conv_{kpctokm} = 3.0857 \times 10^{16}$

1. Basic Structure of the Milky Way Galaxy

We assume a *logarithmic* spiral for the Milky Way Galaxy:

$$r := a \cdot e^{-c \cdot \theta} \qquad (1)$$

where r is the radial distance, θ is the tangential angle, and a and c are constants to be determined. For the Sun, we have (from Ref. [3, p. 478])

$r := 8.5$ kpc from galactic center

Then, at $\theta = 0$ as our initial condition, we have

$$a := r \qquad (2)$$

Now we will construct a polar plot of the Milky Way (top view).

After some iteration, we obtain: $c := -.044$ $\theta\theta := -6\pi, -5.95\cdot\pi .. 6\cdot\pi$ (3)

The galactic radius is 15 kpc; the Sun is at 8.5 kpc, close to the third arm

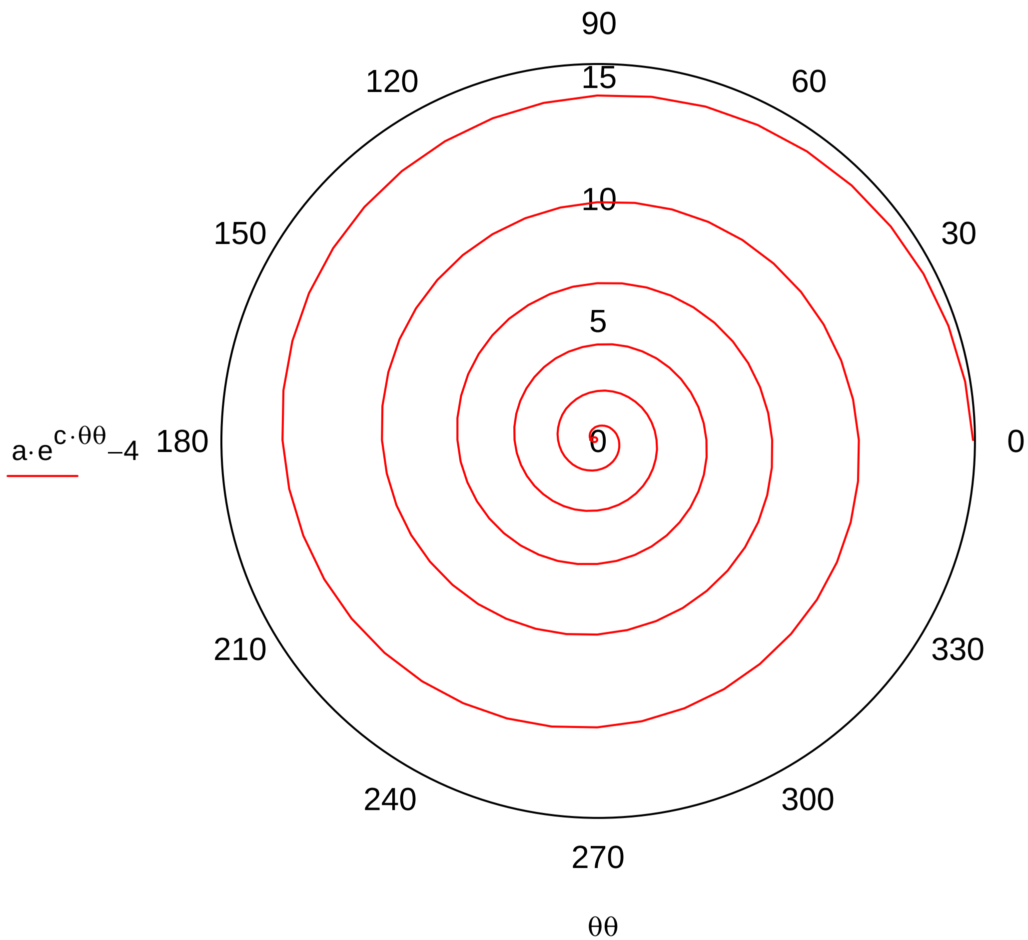

Figure 1. Approximate top view of Milky Way Galaxy.

2. Motion of the Sun

The motion of the Sun has two components (to be verified): 1) it rotates with the third arm of the spiral around the galactic core, and 2) it moves *along* this third arm of the spiral, toward the galactic core.

From the basic mathematics of a logarithmic spiral (see Ref. [2], p. 304), we can express component 2) as follows:

$$v_{along\_spiral} = \sqrt{\left(\frac{d}{dt}r\right)^2 + r^2 \cdot \left(\frac{d}{dt}\theta\right)^2} \qquad (4a)$$

$$\frac{d}{dt}\theta = \sqrt{\frac{v_{along\_spiral}^2 - \left(\frac{d}{dt}r\right)^2}{r^2}} \qquad (4b)$$

But $\frac{d}{dt}r = \frac{v_{along\_spiral}}{\sqrt{1 + \frac{1}{c^2}}}$ (5)

So

$$\frac{d}{dt}\theta = \sqrt{\frac{v_{along\_spiral}^2}{r^2 \cdot \left(c^2 + 1\right)}} \qquad (6)$$

But this is only the *change in* θ due to motion of the Sun *along the arm*. We must therefore add the *rotation of the galaxy itself*--or at least the *rotation of the arm* relative to the z-axis perpendicular to the center of the galaxy, because observations show that the *rotation of the different arms is inverse to the radius*, thus keeping the tangential velocity of stars approximately *constant* accros the disk of the galaxy. (The rotation curve of the Milky Way is given in Ref. [4], p. 269.)

$$\left(\frac{d}{dt}\theta\right)_{tot} = \sqrt{\frac{{v_{along\_spiral}}^2}{r^2\cdot\left(c^2+1\right)}} + \omega_{arm} \qquad (7)$$

Then

$$v_{Sun} = r\cdot\left[\sqrt{\frac{{v_{along\_spiral}}^2}{r^2\cdot\left(c^2+1\right)}} + \omega_{arm}\right] \qquad (8)$$

$v_{Sun} := \dfrac{220}{conv_{kpctokm}}$ $\quad v_{Sun} = 7.1297\times10^{-15}$ kpc/sec (*observed*, Ref. [3], p. 478)

$v_{along\_spiral} := \dfrac{7.5}{conv_{kpctokm}}$ $\quad v_{along\_spiral} = 2.4306\times10^{-16}$ kpc/sec (*assumed* for this calculation and verified by result)

Solving for the required value of ω_{arm}:

$$\omega_{arm} := \frac{v_{Sun} - r\cdot\sqrt{\frac{v_{along\_spiral}^2}{r^2\cdot\left(c^2+1\right)}}}{r} \qquad \omega_{arm} = 8.102\times10^{-16} \quad \text{rad/sec} \qquad (9)$$

$$\omega_{arm\_years} := \frac{\omega_{arm}}{conv_{sectoyears}} \qquad \omega_{arm\_years} = 2.5569\times10^{-8} \quad \text{rad/years}$$

So one revolution will take: $p := \frac{1}{\omega_{arm\_years}}\cdot 2\cdot\pi \qquad p = 2.4574\times10^{8}$ years (10)

The arc length of this arm for three revolutions or 6π is

$$s := \int_0^{6\cdot\pi} \sqrt{\left(a\cdot e^{c\cdot\theta}\right)^2 + \left(a\cdot c\cdot e^{c\cdot\theta}\right)^2}\, d\theta \qquad s = 108.9982 \quad \text{kpc} \qquad (11)$$

Therefore the time for the Sun to get to the galactic core is

$$t := \frac{s}{v_{along\_spiral}} \cdot conv_{sectoyears} \qquad t = 1.421 \times 10^{10} \quad \text{years} \qquad (12)$$

which is probably within the "ballpark", verifying the assumptions

Over many, many billions of years the stars of the Milky Way *spiral into the core*, and so eventually a large spiral galaxy becomes a *gigantic spheroidal galaxy*. Therefore, the Sun is *not* actually in a "circular" orbit around the galactic core described by Kepler's laws. The Sun's motion more resembles a very slow fluid vortex spiralling into a "sink."

References

[1] D. Larson, *The Universe of Motion* (Portland, OR: North Pacific Publishers, 1984).

[2] E. Mendelson, *3000 Solved Problems in Calculus* (New York, NY: McGraw-Hill, 1988).

[3] K. Lang, *Essential Astrophysics* (Berlin, Germany: Springer, 2013).

[4] F. Shu, *The Physical Universe* (Mill Valley, CA: University Science Books, 1982).

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