Theory of Dielectrics, Diamagnets, Paramagnets, and Ferromagnets, including the Calculation of Electric and Magnetic Susceptibilities

by

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Abstract

This paper presents the derivation of the equations for the properties of dielectrics, diamagnets, paramagnets, and ferromagnets according to the Reciprocal System of physical theory developed by D. B. Larson. The factors include: atomic electric and magnetic rotational displacements, electric rotational vibration frequency, magnetic rotational vibration frequency, Planck's constant, Rydberg's constant, temperature, and molecular or crystal structure.

keywords: scalar motion, dielectrics, diamagnets, paramagnets, ferromagnets, gravitational force, electrical force, magnetic force, electric susceptibility, magnetic susceptibility, RLC circuits, capacitors, permittivity, permeability, index of refraction, magnetic charge, magnetic saturation, Curie temperature, magnetic hysteresis, magnetic energy product

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Introduction and Literature Survey

Thousands of scientists and engineers have worked on the experimental and theoretical aspects of electricity and magnetism, but D. B. Larson was the first and only one to reduce all electric, magnetic, and gravitational concepts and equations to space-time terms alone. This paper translates Larson's work, in Ref. [1] (particularly pp. 133-260), Ref. [2] (pp. 50-89, first ed.), Ref. [3] (pp. 153-190), Ref. [4], and Ref. [5], into the language of mathematical physics. It is runnable as a Mathcad program, so it is a highly computational paper.

Conventional physical theory can be found in any university physics textbook, such as Ref. [6], or at a somewhat higher level, in the nine volume Encyclopaedic Dictionary edited by Thewlis, Ref. [7].

The properties of matter are discussed, at an elementary level, in Ref. [8] and Ref. [9], and at a higher level in Ref. [10] and Ref. [11].

Tables of material data can be found in a variety of handbooks, such as Ref. [12] - [17].

Specific dielectric properties and equations are discussed in detail in Von Hippel's three works, Ref. [18], Ref. [19], and Ref. [20].

Electric circuits, specifically DC RC, DC RL, DC RLC, AC RC, AC RL, and AC RLC, are conventionally analyzed in Ref. [21].

The standard work in Quantum Mechanics for the calculation of electric and magnetic susceptibilities is by Van Vleck, Ref. [22].

A fairly recent work in the Quantum Mechanics of magnetism in solids is by Martin, Ref. [23].

The atomic/molecular beam experiments, used to determine atomic and "nuclear" electric and magnetic moments, are described by Fraser, Ref. [24], and Ramsey, Ref. [25].

Two major works in magnetochemistry are those by Bhatnagar and Mathur, Ref. [26], and Selwood, Ref. [27].

General works in magnetism include Ref. [28], Ref. [29], and Ref. [30].

Applied magnetism is the focus of Ref. [31] and Ref. [32].
Field theory is described and applied in Ref. [33] - [42]. As will be explained, the Reciprocal System is compatible with field theory calculations, but not with the conventional physical interpretations.

Computer animation can be helpful in understanding electrical and magnetic phenomena. Prof. Goodstein's series of DVD's, *The Mechanical Universe and Beyond*, Ref. [43], is excellent in this regard, though it's naturally biased toward conventional theory. On the Web, P. Falstad, Ref. [44], has displays of numerous applets showing many different kinds of fields in action. There is not, as of yet, much in the way of animations of various aspects of the Reciprocal System, but the 17 figures in the author's first book, *The Unmysterious Universe*, Ref. [45], should be helpful to newcomers of the theory. Also, some of the author's previous papers, Ref. [46]-[54], may be of assistance in understanding this paper (but where there is a difference between the previous papers and this one, this one supercedes).

Last, but not least, mention must be made of a pioneer experimenter and theoretician, Sir James A. Ewing (1855-1935). His major work in ferromagnetism is now online, Ref. [55]. It was Ewing who conceived of atoms as acting like bar magnets, somewhat like the picture we get from the Reciprocal System! His understanding of ferromagnetic hysteresis was quite accurate, as we shall see.

Additional references are given, as needed, following Ref. [55], at the end of this paper.
Nomenclature

\( a \) = general symbol for acceleration

\( a_u \) = natural Reciprocal System unit of acceleration, stated in cm/sec\(^2\)

\( a_{u\text{-cgs}} \) = natural cgs unit of acceleration = 1 cm/sec\(^2\)

\( B \) = general symbol for magnetic flux density (context determines whether units are cgs, SI, or Reciprocal Sys.)

\( B_{\text{ext}} \) = external magnetic flux density impressed on solid, tesla = webers/m\(^2\)

\( B_{\text{ext}_c} \) = external magnetic flux density at coercivity point in hysteresis cycle, tesla

\( B_{\text{ext}_{\text{max}}} \) = maximum external magnetic flux density in hysteresis cycle, tesla

\( BH \) = magnetic energy product, J/m\(^3\) (subscript max = maximum, subscript may have mat.; calc = calculated, obs = observed)

\( B_{\text{int}} \) = internal magnetic flux density (that within the material), tesla

\( B_{\text{nat}_{\text{SI}}} \) = natural Reciprocal System unit of magnetic flux density (time-space region), stated in tesla

\( B_r \) = remanence internal magnetic flux density (context determines whether at 0 K or temperature \( T \)), tesla

\( B_s \) = saturation internal magnetic flux density at 0 K, tesla (at room temperature for hysteresis curves); subscript has material

\( B_{s\_T} \) = saturation internal magnetic flux density at temperature \( T \), tesla

\( C \) = capacitance, farads

\( C \) = Curie constant in Curie-Weiss magnetic susceptibility equation, K if using non-dimensional \( \chi \)
$C_M = \text{Curie constant on molar basis}$

$C_u = \text{natural Reciprocal System unit of capacitance, stated in farads}$

$c = \text{velocity of light (subscript has units)}$

$c_u = \text{natural Reciprocal System unit of velocity} = 1 \text{ unit of space} / 1 \text{ unit of time}$

$c_{\text{cgs}} = \text{speed of light expressed as cm/sec}$

$c_{\text{SI}} = \text{speed of light expressed as m/sec}$

$c_0, c_1, c_2 = \text{constants in electrical circuit differential equations}$

$\text{conv}_{\text{Atomm}} = \text{factor to convert angstroms to mm}$

$\text{conv}_{\text{caltoev}} = \text{factor to convert calories to electron-volts}$

$\text{conv}_{\text{cmtofarad}} = \text{factor to convert cm to farads (now obsolete)}$

$\text{conv}_{\text{CV}} = \text{factor to convert CV to joules (now obsolete)}$

$\text{conv}_{\text{faradtocm}} = \text{factor to convert farads to cm (now obsolete)}$

$\text{conv}_{\text{IC}} = \text{conversion constant for RLC circuit}$

$\text{conv}_{\text{jouletoev}} = \text{factor to convert joules to electron-volts}$

$\text{conv}_{\text{RCV}} = \text{factor to convert RC/V to seconds (now obsolete)}$

$\text{conv}_{\text{seccmtojoule}} = \text{factor to convert sec/cm to joules}$
conv_{volttoseccm^2} = factor to convert volts to sec/cm^2

D = diameter of atom, m (subscript has element symbol)

D_u = natural Reciprocal System unit of atomic diameter, stated in m

d = number of natural Reciprocal System units of density

d_u = natural Reciprocal System unit of density = 1 unit of mass / 1 unit of volume, stated in g/cm^3

d_{cgs} = density in cgs units

d_{u_cgs} = natural cgs unit of density = 1 g/cm^3

d_Bdz = gradient of external magnetic flux density, B_{ext}, with respect to z-direction, tesla/m

E = energy from voltage source, joules

E_C = energy stored in capacitor, joules

E_L = energy stored in inductor, joules

E_{I_atom_elec} = ionization energy of atom-electron pair, eV

E_{I_pos_mole} = ionization energy for atoms (not including the electrons) in a solid, kJ/mole

E_R = energy dissipated in resistor, joules

E_{\lambda_0} = energy of incident photons on medium, eV

E_{\lambda} = energy of photons after travelling through x cm of medium, eV
edge_a = edge_a of the crystal volume unit cell (could be in angstroms, cm, or m, by context)

edge_b = edge_b of the crystal volume unit cell (could be in angstroms, cm, or m, by context)

edge_c = edge_c of the crystal volume unit cell (could be in angstroms, cm, or m, by context)

edge_uc = general symbol for any of the three edges of the crystal volume unit cell (could be in angstroms, cm, or m, by context)

F = general symbol for force (subscript has direction)

F_E = electrostatic force (subscript has units)

F_M = magnetostatic force (subscript has units)

F_G = gravitational force (subscript has units)

F_v = refraction vibration factor (dimensionless)

f = frequency, cycles/sec

G = gravitational constant (actually dimensionless whether stated in cgs or SI)

h = Planck's constant, eV-sec

I = steady-state current in electrical circuit, amps

I_M = transmission ratio for internal magnetic flux density (dimensionless)

i = instantaneous current in electrical circuit, amps

i_RLC = function to compute current for RLC circuit, amps
\( i_c \) = complementary solution for instantaneous current, amps

\( i_{\text{eff}} \) = effective current for AC circuit, amps

\( i_p \) = particular solution for instantaneous current, amps

\( i_u \) = natural Reciprocal System unit of current, stated in amps

\( j \) = imaginary unit (square root of -1)

\( k \) = extinction coefficient (dimensionless)

\( k_0, k_1, ... \) = constants for use in the electric circuit equation

\( k_B \) = Boltzmann's Constant, joule/K; \( k_{B_{\text{ev}}} \) = Boltzmann's Constant, eV/K

\( k_c \) = factor relating coercivity of ferromagnet to the coercivity of its predominant ferromagnetic material (dimensionless)

\( k_{\text{dpm}} \) = geometric coefficient for magnetic dipole length (dimensionless)

\( k_G \) = crystal unit cell volume geometric factor (dimensionless)

\( k_r \) = refraction constant (dimensionless) (subscript has element or compound symbol)

\( k_{r0} \) = number of 1/9 initial units for index of refraction equation (dimensionless) (subscript has element symbol)

\( L \) = inductance in inductor, henries

\( L_u \) = natural Reciprocal System unit of inductance, stated in henries

\( M_1 \) or \( M_2 \) = magnetic charge of body (subscript has units)
\( M_P \) = magnetic polarization (dipole moments per unit volume), webers/m²

\( M_{P_{nat\_t}} \) = natural Reciprocal System unit of magnetic polarization (time region), stated in tesla = webers/m²

\( M_u \) = natural Reciprocal System unit of magnetic charge (subscript has units)

\( m \) = general symbol for mass (if atomic, subscript has element symbol)

\( m_1 \) or \( m_2 \) = mass of body (subscript has units)

\( m_u \) = natural Reciprocal System unit of mass, stated in sec³/cm³

\( m_{u\_g} \) = natural Reciprocal System unit of mass, stated in g

\( n \) = index of refraction (dimensionless) or real component of complex index of refraction

\( n^* \) = complex index of refraction (dimensionless)

\( n_1 \) = net number of atoms of element 1 of compound in unit cell

\( n_2 \) = net number of atoms of element 2 of compound in unit cell

\( n_a \) = number of atoms of element in formula molecule (subscript has element symbol)

\( n_f \) = fraction of maximum possible magnetic dipole moments induced in crystal volume unit cell, 0 to 1

\( n_M \) = number of magnetic charges on each rotational system of an atom, 0 to \( t_p \) (subscript may have material)

\( n_{M\_r} \) = number of magnetic charges on each rotational system of an atom at remanence (subscript may have material)

\( n_r \) = factor relating remanence of ferromagnet to the remanence of its predominant ferromagnetic base material (dimensionless)
n_{uc} = \text{net total number of atoms in unit cell}

p = \text{instantaneous power from voltage source, watts}

p_C = \text{instantaneous power supplied to capacitor, watts}

p_L = \text{instantaneous power supplied to inductor, watts}

p_R = \text{instantaneous power supplied to resistor, watts}

Q_1 \text{ or } Q_2 = \text{electric charge of body (subscript has units)}

Q_u = \text{natural Reciprocal System unit of electric charge (subscript has units)}

Q_{u\text{.cgs}} = \text{natural cgs unit of charge}= 1 \text{ esu}

R = \text{resistance of resistor, ohms}

R = \text{Rydberg frequency (hydrogen), cycles/sec}

R_u = \text{natural Reciprocal System unit of resistance, stated in ohms}

R_v = \text{Rydberg frequency (hydrogen), half-cycles/sec}

s = \text{space dimension, m in SI or cm in cgs (subscript has units)}

s_0 = \text{interatomic distance (subscript has units)}

s_d = \text{deflection of vapor atoms from centerline, m}

s_{tot \text{.sep}} = \text{total separation between traces in Stern-Gerlach experiment, m}
s_{u} = \text{natural Reciprocal System unit of space in time region, stated in cm}

s_{u} = \text{natural Reciprocal System unit of space in time-space region, stated in cm}

s_{u}_{cgs} = \text{natural cgs unit of space = 1 cm}

T = \text{time period of AC voltage cycle, sec}

T = \text{temperature of substance, K (subscript has element symbol)}

T_0 = \text{zero-point temperature, K (Ref. [1], p. 84)}

T_1 = \text{temperature at end of first specific heat line segment, K (Ref. [1], p. 86)}

T_c = \text{Curie temperature, K (actual transition from ferromagnet to paramagnet takes place over a large zone of temperature)}

te = \text{electric rotational displacement of an atom or sub-atom (dimensionless)}

te_{mod} = \text{modified electric rotational displacement of an atom to account for neutrinos within and thus for isotopic differences in properties (dimensionless)}

te_{eff} = \text{effective atomic rotational displacement (both magnetic and electric) for use in refraction calculations (dimensionless)}

t_p = \text{principal (or primary) magnetic rotational displacement of an atom or sub-atom (dimensionless)}

t_r = \text{equivalent net added time to motion of photons or electrons through atoms (natural units of time)}

t_s = \text{subordinate (or secondary) magnetic rotational displacement of an atom or sub-atom (dimensionless)}

\text{Note: } t_{p \text{ eff}}, t_{s \text{ eff}}, t_{e \text{ eff}} \text{ are listed in three column headings of magnetic susceptibility calculations in Table V (from ) --theoretically they should be identical to } t_p, t_s, t_e
\[ T_{sc} = \text{superconducting temperature, K} \]
\[ T_{T_u} = \text{natural Reciprocal System unit of temperature in the time region (solid, liquid states), stated in K} \]
\[ T_{V_u} = \text{natural Reciprocal System unit of temperature at boundary between time and time-space regions (vapor state), K} \]
\[ t = \text{time dimension, sec} \]
\[ t_u = \text{natural Reciprocal System unit of time, stated in sec} \]
\[ U_{\text{cohes}_\text{molec}_\text{mole}} = \text{cohesive energy of atomic or molecular solid, kJ/mole} \]
\[ U_{\text{ionic}_\text{atom}_\text{pair}} = \text{Coulombic potential energy of pair of ions, J} \]
\[ V = \text{DC voltage of electric circuit source, volts} \]
\[ V_{\text{max}} = \text{amplitude of AC voltage, volts} \]
\[ V_u = \text{natural Reciprocal System unit of voltage, states in volts} \]
\[ V_{UC} = \text{volume of crystal unit cell (subscript has units)} \]
\[ v = \text{velocity of body or light (subscript has units or element symbol)} \]
\[ v_C = \text{instantaneous voltage across capacitor, volts} \]
\[ v_{\text{elec}} = \text{atomic electric rotational velocity, natural units} \]
\[ v_L = \text{instantaneous voltage across inductor, volts} \]
\[ v_{\text{mag}} = \text{atomic magnetic rotational velocity, natural units} \]
\( v_R \) = instantaneous voltage across resistor, volts

\( v_u \) = velocity of body or light in natural units

\( W_H \) = magnetic hysteresis energy loss, \( J/m^3 \) (subscript may have material name; calc=calculated, obs=observed)

\( w \) = atomic weight, amu (atomic mass units)

\( x \) = distance traveled by light within medium (cm for cgs, m for SI) (subscript has units)

\( x = T/T_c \) for graph show variation of saturation internal magnetic flux density versus temperature

\( y \) = length of path of vapor atoms through poles of electromagnet, m

\( Z_{atno} \) = atomic number (subscript has element symbol)

\( Z_{uc} \) = number of formula molecules (atoms for an element) in a crystal unit cell

\( \alpha \) = absorption coefficient (\( cm^{-1} \) for cgs, \( m^{-1} \) for SI) (subscript has units)

\( \alpha, \beta \) = statistical parameters for argument of erf for magnetic hysteresis curves (subscript i for initial magnetization)

\( \Delta \) = Weiss constant in denominator of Curie-Weiss Law, K

\( \delta \) = dielectric loss angle, rad

\( \varepsilon \) = permittivity of medium separating electric charges (electrostatics) or electrons (currents), units should be in \( s^2/t \) but are not in SI or cgs

\( \varepsilon' \) = real component of complex permittivity

\( \varepsilon'' \) = imaginary component of complex permittivity
$\varepsilon_r^*$ = complex relative permittivity (dimensionless)

$\varepsilon_r'$ = complex relative permittivity coefficient (dimensionless)

$\varepsilon_0$ = permittivity of free space, unity in both the Reciprocal System ($s^2/t$) and cgs, but *incorrect* dimensions in both SI and cgs ([Ref. [1], p. 172, p. 184) (subscript has units)]

$\varepsilon_r$ = relative permittivity or dielectric constant (dimensionless)

$\varepsilon_{r1}$ = real component of complex relative permittivity (dimensionless)

$\varepsilon_{r2}$ = imaginary component of complex relative permittivity (dimensionless)

$\lambda$ = wavelength of light within medium (cm in cgs, m in SI) (subscript has units)

$\lambda_0$ = wavelength of light incident to medium (cm in cgs, m in SI) (subscript has units)

$\lambda_{0\_n}$ = wavelength of light incident to medium, natural units

$\mu$ = permeability of medium separating magnetic charges (magnetostatics), abhenry/cm for cgs and henry/m in SI (subscript has units)

$\mu_0$ = permeability of free space, unity in both the Reciprocal System and cgs, abhenry/cm for cgs and henry/m in SI (subscript has units; $\mu_{0\_SI}$ must be used for magnetic equations expressed in SI)

$\mu_{adpm}$ = atomic magnetic dipole moment, weber-m (subscript has element symbol)

$\mu_{adpm\_el\_som}$ = Sommerfeld atomic magnetic dipole moment, joule/tesla, for element "el", joule/tesla
μ_{adpm_u} = natural Reciprocal System unit for atomic magnetic dipole moment, weber-m

μ_B = Bohr magneton for Quantum Mechanics

μ_{dpm} = magnetic dipole moment, weber-meter

μ_r = relative permeability (dimensionless); μ_{r_0} = natural relative permeability of free space = 1

μ_{r_avg} = average relative permeability of ferromagnet in second quadrant of hysteresis curve (dimensionless)

μ_r^* = complex relative permeability (dimensionless)

μ_r' = complex relative permeability real coefficient (dimensionless)

μ_r'' = complex relative permeability imaginary coefficient (dimensionless)

μ_W = Weiss magneton

ν = frequency (natural units)

ν_{M_0} = physical "zero" for the rotational vibration frequency of magnetic charge, cycles/sec

ν_{M_n} = rotational vibration frequency of an n magnetic charge unit, cycles/sec

ν_{phot} = frequency of photons used in photomagnetization experiments, cycles/sec

ρ_e = electrical resistivity of medium, ohm-cm for cgs and ohm-m for SI (subscript has units)

τ_C = time constant for capacitor, sec (subscript "legacy" for conventional value)
\( \tau_L \) = time constant for inductor, sec (subscript "legacy" for conventional value)

\( \phi \) = magnetic loss angle, rad

\( \phi_{ac} \) = phase angle for AC voltage, rad

\( \chi \) = general symbol for magnetic susceptibility (dimensionless, subscript has compound name)

\( \chi_{cgs\_mass} \) = magnetic susceptibility, stated in cm\(^3\)/g

\( \chi_{cgs\_mol} \) = magnetic susceptibility, stated in cm\(^3\)/mole

\( \chi_{obs} \) = observed value of magnetic susceptibility (dimensionless, subscript indicates source or compound name)

\( \chi_{SI\_mass} \) = magnetic susceptibility, stated in m\(^3\)/kg

\( \chi_{SI\_mol} \) = magnetic susceptibility, stated in m\(^3\)/mole

\( \chi_{u\_SI} \) = natural Reciprocal System unit of magnetic susceptibility, computed using SI value of \( c \)

\( \omega \) = angular frequency, rad/sec

Note: A black square in the upper right of an equation means that the equation is disabled from running in Mathcad. This is done because not all variables in the equation have, as yet, been given numerical values at that point in the program. In a few cases, equations (like those with apostrophes) have to be given as text because Mathcad cannot represent the symbols properly; in this case, there is no black square, but the equation is not computational.
Reciprocal System Physical Constants (from Ref. [1]-[2], or derived therefrom)

\[ s_u := 4.558816 \times 10^{-6} \text{ cm} \quad t_u := 1.520655 \times 10^{-16} \text{ sec} \quad a_u := \frac{s_u}{t_u^2} \quad a_u = 1.971472 \times 10^{26} \text{ cm/sec}^2 \]

\[ l_R := 156.4444 \quad \text{(inter-regional ratio)} \]

\[ t_{u_cgs} := 1 \text{ sec} \quad t_{u_{SI}} := 1 \text{ sec} \]

\[ s_{t_u} := \frac{s_u}{l_R} \quad s_{t_u} = 2.914 \times 10^{-8} \text{ cm} \]

\[ m_u := \frac{t_u^3}{s_u^3} \quad m_u = 3.7114 \times 10^{-32} \text{ sec}^3/\text{cm}^3 \quad m_{u_g} := 1.65979 \times 10^{-24} \text{ g} \]

\[ G_{cgs} := 3 \cdot \frac{m_u}{m_{u_g}} \cdot \frac{1}{1.00639} \quad G_{cgs} = 6.6656 \times 10^{-8} \quad \text{(dimensionless!)} \quad \text{(Ref. 1, pp. 162-163)} \]

\[ G_{SI} := G_{cgs} \cdot 10^{-3} \quad G_{SI} = 6.6656 \times 10^{-11} \quad \text{(dimensionless!)} \]

\[ s_{u_{cgs}} := 1 \text{ cm} \quad a_{u_{cgs}} := 1 \text{ cm/sec}^2 \quad m_{u_{cgs}} := 1 \text{ g} \quad Q_{u_{cgs}} := 1 \text{ esu} \]

\[ s_{u_{SI}} := 1 \text{ m} \quad a_{u_{SI}} := 1 \text{ m/sec}^2 \quad m_{u_{SI}} := 1 \text{ kg} \quad Q_{u_{SI}} := 1 \text{ C} \]

\[ Q_{u_{esu}} := 4.80287 \times 10^{-10} \text{ esu} \quad \text{(electric charge)} \]

\[ Q_{u_{coul}} := 1.602062 \times 10^{-19} \text{ coulombs} \quad \text{(electric charge)} \]
\[ R_v := 6.576115 \cdot 10^{15} \text{ half-cycles/sec (Ryderberg frequency for H)} \quad R := \frac{R_v}{2} \text{ cycles/sec} \]

\[ c_{cgs} := 2.997925 \cdot 10^{10} \text{ cm/sec} \quad c_{SI} := 2.997925 \cdot 10^8 \text{ m/sec} \quad c_u := 1 \]

\[ M_{cgs} := 1 \text{ emu} \quad M_{SI} := 1 \text{ weber} \quad M_{u \text{-emu}} := \frac{Q_{u \text{-esu}}}{c_{cgs}} \quad M_{u \text{-emu}} = 1.6021 \times 10^{-20} \text{ emu} \]

\[ d_{u \text{-cgs}} := 1 \text{ g/cm}^3 \quad M_{u \text{-weber}} := M_{u \text{-emu}} \cdot 299.7925 \quad M_{u \text{-weber}} = 4.8029 \times 10^{-18} \text{ weber} \]

(magnetic charge)

\[ j := \sqrt{-1} \quad \text{conv}_{\text{farad to cm}} := 8.98758 \cdot 10^{11} \quad \text{conv}_{\text{volt to sec cm}^2} := 7.85944 \cdot 10^{-15} \quad \text{conv}_{\text{sec cm to joule}} := 4.472162 \]

\[ \text{conv}_{\text{cm to farad}} := 1.112647 \cdot 10^{-12} \quad \text{conv}_{\text{CV}} := \text{conv}_{\text{farad to cm}} \cdot \text{conv}_{\text{volt to sec cm}^2} \cdot \text{conv}_{\text{sec cm to joule}} \cdot R \]

\[ \text{conv}_{\text{CV}} = 4.9412 \quad (\text{so if } V = 5, .5 \times C \times V \times \text{conv}_{\text{CV}} = .5 \times C \times V^2 \text{ approx.}) \]

\[ R_u := 8.83834 \cdot 10^{11} \text{ ohms} \quad V_u := 9.31146 \cdot 10^8 \text{ volts} \quad C_u := \frac{t_u}{R_u} \quad C_u = 1.7205 \times 10^{-28} \text{ farads} \]

\[ L_u := R_u \cdot t_u \quad L_u = 0.0001 \text{ henries} \quad i_u := 1.05353 \cdot 10^{-3} \text{ amps} \]
conv$_{RCV}$ := $\frac{R_u \cdot C_u}{V_u \cdot t_u}$  

conv$_{RCV}$ = $1.0739 \times 10^{-9}$  

(for use in RC time constant calculations—now obsolete)

conv$_{iC}$ := $i_u \cdot C_u \cdot V_u$  

conv$_{iC}$ = $1.6878 \times 10^{-22}$  

(conversion constant for RLC circuit—now obsolete)

conv$_{Atomm}$ := $10^{-7}$  

(converting angstroms to mm for dielectric strength calculations)

Av := $6.02486 \cdot 10^{23}$ molecules/g-mole  
h := $4.14 \cdot 10^{-15}$ eV·sec  

(Planck's constant)

M$_{nat\_SI}$ := $V_u \cdot t_u$  

M$_{nat\_SI}$ = $1.416 \times 10^{-7}$ webers  

µ$_r$ := 1  

µ$_0$$_{\_SI}$ := $4\pi \cdot 10^{-7}$ henry/m

B$_{nat\_SI}$ := $\frac{M_{nat\_SI}}{(s_u \cdot 10^{-2})^2}$  

B$_{nat\_SI}$ = $6.8131 \times 10^7$ webers/m$^2$  

(time-space region)

T$_{t\_u}$ := 510.8 K  

k$_B$ := $1.38 \cdot 10^{-23}$ J/K  

(Boltzmann Constant)

conv$_{caltoev}$ := $2.613 \cdot 10^{19}$

k$_{B\_ev}$ := $8.617 \cdot 10^{-5}$ eV/K

conv$_{joulestoev}$ := $6.242 \cdot 10^{18}$

Note (06/19/2012): From experiment on DC RC circuits, the dimensions of capacitance are now known to be $s^3/t$, not $s$; this is a revision of the Reciprocal System, and therefore sections of this paper which treated capacitance as $s$ have had to be updated.
1. Force Definitions and Calculations

a. Inertial Force

Force, not mass, is defined by Newton's Second Law of Motion:

\[ F := m \cdot a \]  

(1a)

In the Reciprocal System, there are no so-called "fundamental forces"; rather, *motion* or *space-time* in the most general sense is what is "fundamental." Thus force is merely a property of motion, not the other way around. All physical quantities in the Reciprocal System therefore reduce to expressions involving space, s, and time, t, *only*. In space-time terms, Eq. (1a) is

\[ \frac{t}{s^2} := \frac{t^3}{s^3} \cdot \frac{s}{t^2} \]  

(1b)

All forces, with the exception of magnetomotive force (for reasons to be discussed later), have the dimensions t/s^2. Force may be scalar or vectorial and may be positive or negative.

b. Scalar vs. Vectorial Motions; Reference Systems

The "fundamental motions" in the Reciprocal System are *scalar* and are either *linear* or *rotational*, either *uniform* or *vibrational*, and either one-, two-, or three-dimensional. Mass is comprised of one- and two-dimensional scalar rotational motions, totalling three dimensions, so the correct dimensions of mass are t^3/s^3. As can be inferred from these dimensions, mass resists a change of motion in any dimension—this is the property of "inertia." But the same rotational motion which gives rise to mass also gives rise to gravitation, and so inertial and gravitational mass are precisely equal, which shows that the Reciprocal System derives the "Principle of Equivalence" easily and naturally.
Vectorial motion cannot occur until a gravitationally-bound system comes into existence; this establishes an inertial reference frame. Therefore, scalar motion precedes, and is thus more fundamental, than vectorial motion. By its nature, the scalar motion involved in gravitation or electrostatics or magnetostatics is *mutual*; when we couple such motion to the standard Cartesian reference system (with three coordinates of space and one of time and with a specific origin), the motion may be mis-attributed to one or the other of the (apparently) interacting entities. This standard reference system can display only one scalar dimension of motion (with three space coordinates), even though the motion may actually be two- or three-dimensional. Such multi-dimensional motion does have indirect effects which can be detected, as we will see later. The methods of field theory may be used to calculate the results of gravitational, electric, and magnetic forces, but there is nothing "physical" in the field—just coordinate space (which is, itself, generated by clock space). Each of the (apparently) interacting entities is pursuing its own course relative to its surrounding space-time locations. There is no "action at a distance" and there are no "gravitons" or "virtual photons" being exchanged. Mass, electric charge, and magnetic charge have the effect of "concentrating" space-time locations and thus "redirecting" scalar motion *automatically* and *instantly*. There are no explicit time terms in the three basic scalar motion equations given in the next section.

Vectorial motion (kinetic energy) may be added to the scalar motion, and can be properly displayed in the conventional reference system. Vectorial motion is *not* a mutual motion.
b. Gravitational Force

According to Eq. (1a), force is proportional to the first power of mass. Therefore, Newton's gravitational equation, which has the gravitational force equal to the second power of mass, has to be "re-dimensionalized" to be made correct. The gravitational constant, G, to which dimensions have traditionally been attached, needs to be changed to a pure number or ratio, for the same reason that permittivity and permeability are, respectively, in the electrostatic and magnetostatic equations of the Reciprocal System. Also, there needs to be an acceleration term, but this is numerically equal to unity in the system of units used to represent the force and distance. Finally, because of the distribution of scalar motion over the surface of an imaginary sphere enveloping the mass (or electric charge or magnetic charge) there must be an inverse square distance term, but this must be non-dimensional, like the acceleration term. Therefore, using cgs units,

\[
F_{G\_dynes} := -\frac{m_{1\_g} \cdot \frac{m_{2\_g}}{m_{u\_cgs}} \cdot a_{u\_cgs}}{\frac{1}{G_{cgs}} \left( \frac{s_{cm}}{s_{u\_cgs}} \right)^2}
\]  

(2a)

where the inverse of the gravitational constant has been put in the denominator of the expression to be consistent with the other scalar force equations. (A negative sign is prepended to the RHS to indicate that the force is attractive.) In dimensional terms, as expected:

\[
\frac{t}{s^2} := \frac{t^3}{s^3 \cdot t^2}
\]  

(2b)

But, since in cgs units, \(m_{u\_cgs}\) and \(a_{u\_gcs}\) and \(s_{u\_cgs}\) are all unity, by definition, this equation numerically reduces to
This is Newton's gravitational equation, of course, but now we understand it much more profoundly!

In SI units, the equation is

\[
F_{G_N} = \frac{-m_1 \cdot m_2}{G_{SI} \cdot s_m^2}
\]  \hspace{1cm} (2d)

Ref. [46] shows that at a high relative speed of the masses, the gravitational equation (in SI units) is

\[
F_{G_N} \sqrt{1 - \frac{v_{SI}^2}{c_{SI}^2}} = \frac{-m_1 \cdot m_2}{G_{SI} \cdot s_m^2}
\]  \hspace{1cm} (2e)

Therefore, as velocity increases, the effective force declines, with the masses remaining constant!

**Note:** Newton's gravitational equation is not as "universal" as traditionally claimed—beyond a certain distance, the space-time progression begins to dominate. For details, see one of the author's papers, Ref. [60].
c. Electrostatic Force

Whereas mass is uniform three-dimensional rotational motion, electric charge is vibrational one-dimensional rotational motion. Its dimensions are therefore t/s. To be made correct, Coulomb's equation for electrostatics needs to be "re-dimensionalized" as follows.

\[
F_{E\_dynes} := \frac{Q_1\_esu \cdot Q_2\_esu}{Q_u\_cgs \cdot s_u\_cgs} \cdot \frac{1}{\varepsilon_r \left(\frac{s_{cm}}{s_u\_cgs}\right)^2}
\]  

(3a)

where the symbols have their usual meanings, and \(Q_1\) and \(Q_2\) may be positive or negative. In dimensional terms, as expected,

\[
\frac{t}{s^2} := \frac{t}{s} \cdot \frac{1}{s}
\]  

(3b)

But, since in cgs units, \(Q_u\_cgs\) and \(s_u\_cgs\) are unity, by definition, this equation *numerically* reduces to

\[
F_{E\_dynes} := \frac{Q_1\_esu \cdot Q_2\_esu}{\varepsilon_r \cdot s_{cm}^2}
\]  

(3c)
This is Coulomb's electrostatic equation. In air, the relative permittivity $\varepsilon_r = 1.006$ and is often taken to be 1. In SI units, a $4\pi \varepsilon_{0\_SI}$ factor is necessary due to the way coulombs ($= 3 \times 10^9$ esu) and $\varepsilon_{0\_SI}$ are defined:

$$F_{E\_N} := \frac{Q_1\_C \cdot Q_2\_C}{4 \cdot \pi \cdot \varepsilon_{0\_SI} \cdot \varepsilon_r \cdot s^2}$$  \hspace{1cm} (3d)

Of course, at a high relative speed of the electric charges, Eq. (3d) becomes

$$F_{E\_N} \sqrt{1 - \frac{v_{SI}^2}{c_{SI}^2}} := \frac{Q_1\_C \cdot Q_2\_C}{4 \cdot \pi \cdot \varepsilon_{0\_SI} \cdot \varepsilon_r \cdot s^2}$$  \hspace{1cm} (3e)

with the charges remaining constant. In contrast to the SI system of units, both the cgs system and the Reciprocal System have the permittivity of free space equal to one:

$$\varepsilon_{0\_cgs} := 1 \hspace{1cm} \varepsilon_{0\_u} := 1$$  \hspace{1cm} (4a)

$$\varepsilon_{0\_SI} := 8.85 \times 10^{-12} \text{ 'farad/m'}$$  \hspace{1cm} (4b)

But in the cgs system of units, permittivity (not just relative permittivity) is dimensionless, and in the SI system the wrong units, farad/m, are used (which from the perspective of the Reciprocal System means dimensionless, as well). The correct dimensions of permittivity are $s^2/t$. Nonetheless, it's clear that the cgs system of units is closer to the natural system of units of the Reciprocal System. The relative permittivity, also termed the dielectric constant, is defined as
\[ \epsilon_r := \frac{\epsilon}{\epsilon_0} \]  

(5)

This is a basic property of matter and its calculation will be given later in this paper. \( \epsilon_r \) is dimensionless, of course.

The natural unit of charge is that of the charged electron. There are no fractional charges in the Reciprocal System and therefore no quarks!

d. Magnetostatic Force

Whereas mass is uniform three-dimensional rotational motion and electric charge is vibrational one-dimensional rotational motion, magnetic charge is vibrational two-dimensional rotational motion. Its dimensions are therefore \( t^2/s^2 \). To be made correct, Coulomb's equation for magnetostatics needs to be "re-dimensionalized" as follows.

\[
F_{M\_dynes} := \frac{M_1\_emu \cdot M_2\_emu \cdot 1}{M_u\_cgs \cdot t_u\_cgs \cdot \mu_r \left( \frac{s_{cm}}{s_u\_cgs} \right)^2}
\]

(6a)

where the symbols have their usual meanings, and \( M_1 \) and \( M_2 \) may be positive (north pole) or negative (south pole). In dimension terms, as expected,

\[
\frac{t}{s^2} := \frac{t^2}{s^2} \cdot \frac{1}{t}
\]

(6b)
A magnetic charge really is two-dimensional! But, since in cgs units, $\mu_{u\text{cgs}}$ and $s_{u\text{cgs}}$ are unity, by definition, this equation numerically reduces to

$$F_{M\text{ dynes}} := \frac{M_{1\text{ emu}} \cdot M_{2\text{ emu}}}{\mu_r \cdot s_{cm}^2}$$

This is Coulomb’s magnetostatic equation. In air, the relative permeability $\mu_r = 1.00000037$ and is almost always taken to be 1. In SI units, a $4\pi\mu_0_{SI}$ factor is necessary due to the way webers ($= 10^8 / 4\pi$ emu) and $\mu_0_{SI}$ are defined:

$$F_{M\text{ N}} := \frac{M_{1\text{ weber}} \cdot M_{2\text{ weber}}}{4\cdot\pi\cdot\mu_0_{SI}\cdot\mu_r \cdot s_{m}^2}$$

Of course, at a high relative speed of the magnetic charges, Eq. (6d) becomes

$$F_{M\text{ N}} \cdot \sqrt{1 - \frac{v_{SI}^2}{c_{SI}^2}} := \frac{M_{1\text{ weber}} \cdot M_{2\text{ weber}}}{4\cdot\pi\cdot\mu_0_{SI}\cdot\mu_r \cdot s_{m}^2}$$

with the charges remaining constant.

See McCaig (Chapter 2 of Ref. [31]) for a thorough discussion of magnetic units and definitions; some authors actually put the permeability in the numerator! In contrast to the SI system of units, both the cgs system and the Reciprocal System have the permeability of free space equal to one:

$$\mu_{0\text{ cgs}} := 1 \quad \mu_{0\text{ u}} := 1$$

(although, in cgs, the units should be abhenry/cm)
\[ \mu_0_{\text{SI}} := 4\pi \cdot 10^{-7} \text{ henry/m} \] \hspace{1cm} (7b)

It's clear, then, that once again, the cgs system of units is closer to the natural system of units of the Reciprocal System. The relative permeability is defined as

\[ \mu_r := \frac{\mu}{\mu_0} \] \hspace{1cm} (8)

This is a basic property of matter and its calculation will be given later in this paper. \( \mu_r \) is obviously dimensionless.

The natural unit of magnetic charge, \( M_{u,\text{weber}} = 4.80287 \times 10^{-18} \) weber, should be that of a magnetically-charged subatom with a single rotational system, like a proton (an independent charge or "monocharge"). But, in the case of the proton, the measurements seem to be all based on the hydrogen in water, so apparently we don't have a direct particle confirmation. (Hydrogen has two rotating systems.) The macroscopic verification will be given later.
Sample Calculation Comparing Gravitational, Electrical, and Magnetic Forces

Assuming natural unit masses, unit electric charges (opposite signs), and unit magnetic charges (opposite signs), separated by one natural unit of distance in vacuum, the following values apply, in cgs:

\[
m_{u\_g} = 1.6598 \times 10^{-24} \quad G_{\text{cgs}} = 6.6656 \times 10^{-8} \quad m_{1\_g} := m_{u\_g} \quad m_{2\_g} := m_{u\_g} \quad s_{\text{cm}} := s_{u}
\]

\[
F_{G\_\text{dynes}} := \frac{m_{1\_g} \cdot m_{2\_g}}{G_{\text{cgs}}} \cdot s_{\text{cm}}
\]

\[
F_{G\_\text{dynes}} = -8.8357 \times 10^{-45}
\]

\[
Q_{u\_\text{esu}} := 4.80287 \cdot 10^{-10} \quad \varepsilon_{r} := 1 \quad Q_{1\_\text{esu}} := Q_{u\_\text{esu}} \quad Q_{2\_\text{esu}} := -Q_{u\_\text{esu}}
\]

\[
F_{E\_\text{dynes}} := \frac{Q_{1\_\text{esu}} \cdot Q_{2\_\text{esu}}}{\varepsilon_{r} \cdot s_{\text{cm}}} \cdot s_{\text{cm}}
\]

\[
F_{E\_\text{dynes}} = -1.1099 \times 10^{-8}
\]

\[
M_{u\_\text{emu}} = 1.6021 \times 10^{-20} \quad \mu_{r} := 1 \quad M_{1\_\text{emu}} := M_{u\_\text{emu}} \quad M_{2\_\text{emu}} := -M_{u\_\text{emu}}
\]

\[
F_{M\_\text{dynes}} := \frac{M_{1\_\text{emu}} \cdot M_{2\_\text{emu}}}{\mu_{r} \cdot s_{\text{cm}}} \cdot s_{\text{cm}}
\]

\[
F_{M\_\text{dynes}} = -1.235 \times 10^{-29}
\]

Therefore:

\[
\frac{F_{G\_\text{dynes}}}{F_{E\_\text{dynes}}} = 7.9605 \times 10^{-37}
\]

\[
\frac{F_{G\_\text{dynes}}}{F_{M\_\text{dynes}}} = 7.1546 \times 10^{-16}
\]

\[
\frac{F_{M\_\text{dynes}}}{F_{E\_\text{dynes}}} = 1.1126 \times 10^{-21}
\]

(as expected, \(1/c_{\text{cgs}}^2\))
2. The Speed of Light, Permittivity, Permeability, and Index of Refraction

Maxwell's famous formula relating the speed of light, permittivity, and permeability, all in free space, is

\[
c := \frac{1}{\sqrt{\varepsilon_0 \cdot \mu_0}}
\]  

(9a)

In Reciprocal System space-time terms:

\[
\frac{s}{t} := \frac{1}{\sqrt{s^2 \cdot t^3}}
\]  

(9b)

This equation is dimensionally correct. The speed of light is the natural unit of velocity in the Reciprocal System—and this comports with the deduction that the natural unit of permittivity (in free space) is one and the natural unit of permeability (in free space) is one:

\[
1 := \frac{1}{\sqrt{1 \cdot 1}}
\]  

(9c)
Ref. [56], pp. 118-119, tabulates the "rationalized natural" system of units in which $c = 1$, $\varepsilon_0 = 1$, and $\mu_0 = 1$, and presents Maxwell's electromagnetic equations with these values. This is as close as other authors have come to the same conclusions as the Reciprocal System.

Incidentally, and unlike conventional physicists, we don't conclude from Eq. (9a) that light is "electromagnetic radiation" with fluctuating electric and magnetic fields. Energy changes in the radiation occur only at the time of emission or absorption of the photons, not during transit. Therefore, Eq. (9a) is really just a spatio-temporal relation.

According to Ref. [7], Vol. 5, p. 342, "The terms 'permittivity' and 'refractive index' represent merely alternative ways of attaching numerical values to one and the same property of a dielectric medium. The customary use of the former in electrical problems and of the latter in optical ones arises only from convenience, and the historical development of the subject. Permittivity is a somewhat simpler concept in problems involving wave-lengths large compared with the dimensions of the medium concerned, so that the spatial change of the electric gradient can be ignored, while refractive index becomes simpler for problems having the converse relation, of a wave-length very small compared with the extent of the medium."

The speed of light "slows down" in matter with a relative permittivity of $\varepsilon_r$ and a relative permeability of $\mu_r$; the speed is

$$v := \frac{c}{\sqrt{\varepsilon_r \cdot \mu_r}}$$

(assuming no losses). By definition, the index of refraction is

$$n := \frac{c}{v}$$

(11a)

so, for a loss-free medium,

$$n := \sqrt{\varepsilon_r \cdot \mu_r}$$

(11b)
In many situations involving dielectrics, the relative permeability can be neglected (set equal to $\mu_0$, which is 1 in cgs or Reciprocal System units), so Eq. (11b) becomes

$$n := \sqrt{\varepsilon_r} \quad \varepsilon_r := n^2$$ \hspace{1cm} (11c)

But all real media are dispersive, and possess a corresponding dielectric loss angle, $\delta$, and magnetic loss angle, $\phi$. Letting $j$ be the imaginary unit (rather than $i$), the complex permittivity becomes

$$\varepsilon_r^* := \varepsilon_r'(1 - j \tan(\delta)) \quad \text{or} \quad \varepsilon_r^* := \varepsilon_r' - j \varepsilon_r'' \quad \text{with} \tan(\delta) = \varepsilon_r''/\varepsilon_r' \quad \text{(Von Hippel's notation)}$$ \hspace{1cm} (12)

Likewise, the complex permeability becomes

$$\mu_r^* := \mu_r'(1 - j \tan(\phi)) \quad \text{or} \quad \mu_r^* := \mu_r' - j \mu_r'' \quad \text{with} \tan(\phi) = \mu_r''/\mu_r' \quad \text{(Von Hippel's notation)}$$ \hspace{1cm} (13)

Therefore, the complex refractive index is

$$(n^*)^2 := \varepsilon_r' \mu_r' (1 - j \tan(\delta)) (1 - j \tan(\phi)) \quad \text{or} \quad n^* = n (1 - j k) \quad \text{(Von Hippel's notation)}$$ \hspace{1cm} (14)

where $k$ is the extinction coefficient. The article in Ref. [7] given above goes on to point out that for anisotropic media, there would be three principal values of $\varepsilon_r^*$ and $\mu_r^*$, not necessarily with coincident electric and magnetic axes, or with equal values of dispersion. "The most general linear case therefore requires 12 constants and two direction cosines for its description. Magnetic properties, however, are in general non-linear, and there may also be interaction effects between the magnetic and optical constants, so that an even higher number of constants may be involved."

Ref. [10] provides tables of values for $\tan(\delta)$ for various dielectric materials. Unfortunately, there do not seem to be readily available tables of values for $\tan(\phi)$ for various magnetic materials.

Where we can neglect the permeability but not the dielectric losses, Eq. (14) becomes
Other references, like Ref. [13], p. 6-118, and Ref. [16], p. 12-145, define the index of refraction and permittivity somewhat differently, so that care must be used when consulting various data sources. Suppose a light beam of energy $E_{\lambda 0}$, in eV, and wavelength $\lambda_0$, in cm, impinges on a dielectric. The beam loses energy as it traverses a length $x$ (cm) of material. Let $\alpha$ be the absorption coefficient and $k$ be the extinction coefficient. Then

\[ E_\lambda := E_{\lambda 0} \cdot e^{-\alpha_{\text{cgs}} \cdot \chi_{\text{cgs}}} \quad \text{eV} \]  

\[ \lambda_{0\_\text{cgs}} := \frac{1.2398 \cdot 10^{-4}}{E_{\lambda 0}} \quad \text{cm} \]  

\[ k := \frac{\alpha_{\text{cgs}} \cdot \lambda_{0\_\text{cgs}}}{4 \cdot \pi} \]  

\[ \varepsilon_{r1} := n^2 - k^2 \]  

\[ \varepsilon_{r2} := 2 \cdot n \cdot k \]  

\[ n^* := n - j k \quad \text{(which is different from Von Hippel; the k here = n x Von Hippel's k)} \]  

\[ \varepsilon_r^* := \varepsilon_{r1} - j \varepsilon_{r2} \quad \varepsilon_r^* := (n^*)^2 \quad \varepsilon_r^* := n^2 - k^2 - 2 \cdot n \cdot k \]  

Ref. [57] provides an equation in SI for the extinction coefficient, $k$, in terms of the electrical resistivity, $\rho_e$, of the material:
Note the $n$ in the denominator. In space-time terms:

$$k := \frac{\mu_{\text{SI}} \cdot \lambda_{0_{\text{SI}}} \cdot c_{\text{SI}}}{4 \cdot \pi \cdot \rho_{e_{\text{SI}}} \cdot n}$$  \hspace{1cm} (23a)$$

so it's dimensionally correct. In most practical situations, $\mu_{\text{SI}} = \mu_{0_{\text{SI}}}$, so

$$k := \frac{\mu_{0_{\text{SI}}} \cdot c_{\text{SI}}}{4 \cdot \pi \cdot \rho_{e_{\text{SI}}} \cdot n}$$  \hspace{1cm} (23b)$$

The author's previous paper on electrical resistivity, Ref. [53], can be used to determine $\rho_{e_{\text{SI}}}$. Note that the values given there for metals will have to be multiplied by the factor $10^{-8}$, and the values given there for semiconductors will have to be multiplied by $10^{-2}$, to convert to ohm-m, for use in Eq. (23c).

Eq. (22) now becomes
\[
\varepsilon_r^* := n^2 - \left( \frac{\mu_0 \text{SI} \cdot \text{C_SI}}{4 \cdot \pi \cdot \rho_{\text{e_SI}} \cdot n} \right)^2 - 2 j n \left( \frac{\mu_0 \text{SI} \cdot \text{C_SI}}{4 \cdot \pi \cdot \rho_{\text{e_SI}} \cdot n} \right)
\] (24)

To use Eq. (24), we need to determine \( n \), the component of the index of refraction not involved with losses. We will use Larson's method described in Ref. [2], 1st ed., pp. 125-130, to calculate \( n \).

Larson says, p. 125: "Since matter is a time displacement, the space-time ratio involved in the passage of radiation through matter or matter through radiation is not unity, but a modified value resulting from the addition of the time displacement to the time component of the original unit velocity. Addition of more time to the ratio \( s/t \) decreases the numerical value and the apparent velocity of radiation in matter is therefore less than unity."

Larson continues, p. 126: "...on this basis the index of refraction relative to a vacuum is equal to the total time associated with unit space in the motion of the radiation. For present purposes we will be interested in the time displacement, rather than the total time and since the time per unit space in undisplaced space-time is unity, the displacement is \( n - 1 \). The displacement due to the presence of any specific atom or quantity of matter is independent of temperature but there is a temperature variation in the refractive index due to the accompanying change in density. We may eliminate this effect by dividing each displacement by the corresponding density, obtaining a temperature independent quantity \((n - 1)/d\)."

Continuing, p. 126: "The refractive displacement is the sum of two components, one due to the motion of matter through radiation (the apparent transitory motion of the radiation) and the other due to the vibratory motion of the radiation through matter. In the first of these we have a simple motion at unit velocity in the time region. We have previously determined that the three-dimensional distribution of motion in the time region reduces the component parallel to one-dimensional time-space region motion to \(1/8\) of the total \([1/2^3]\), and the vibratory nature of the motion of matter in the time region introduces an additional factor of \(1/2\). We therefore find the displacement on a time-space region basis to be \(1/16\) of the effective time region displacement units.

Continuing, p. 126: "If the magnetic rotational displacement is unity the refractive displacement is also unity, but where the rotational displacement is \( t_{\text{eff}} \) [our notation] the radiation travels through only one of the \( t_{\text{eff}} \) displacement units and the effective displacement is \(1/t_{\text{eff}}\). If we represent the average value of \(1/t_{\text{eff}}\) as \( k_r \), the refractive displacement due to the translatory motion is"
This equation must be modified to account for the geometry of the crystal unit cells. The volume of a unit cell is equal to the cube of the nearest neighbor distance multiplied by the geometric factor, $k_G$:

\[
V_{uc_{cgs}} := k_G s_0_{cgs}^3
\]  

(26)

Here's a table of values of $k_G$ for ideal crystal types (calculated as the cube root of the values given on p. 98 of Ref. [1]).

<table>
<thead>
<tr>
<th>Crystal Type</th>
<th>$k_G$</th>
</tr>
</thead>
<tbody>
<tr>
<td>FCC</td>
<td>.7071</td>
</tr>
<tr>
<td>HCP</td>
<td>.7071</td>
</tr>
<tr>
<td>BCC</td>
<td>.7700</td>
</tr>
<tr>
<td>Simple (NaCl) Cube</td>
<td>1.00</td>
</tr>
<tr>
<td>Diamond (ZnS) Cube</td>
<td>1.5396</td>
</tr>
</tbody>
</table>

These values are for atomic crystals only--not molecular crystals, like ice. For molecular crystals, one has to attempt to figure out the distance between molecular force centers, not individual atoms.

Eq. (25) is expressed in natural units; to convert to cgs, the density must be multiplied by the cgs values of unit density:
\[
\frac{d_{\text{cgs}}}{d_{\text{u,cgs}}} = \frac{d_u \cdot m_{u,g}}{s_{t,u}^3}, \quad \frac{m_{u,g}}{s_{t,u}^3} = 0.0671
\]  

This value may be inverted and placed on the RHS of Eq. (25) and then multiplied by \(k_G\):

\[
\frac{(n - 1)}{d_{\text{cgs}}} = 0.9317 \cdot k_G \cdot k_r
\]  

(28)

If we have, as is the usual case,

\[k_G := 0.7071\]

then

\[
\frac{(n - 1)}{d_{\text{cgs}}} = 0.6588 \cdot k_r
\]  

(translation)  

(29)

which is, in essence, Eq. (148) of Ref. [2], first ed., but with a more explicit derivation of the coefficient!

Larson continues, Ref. [2], first ed., pp. 126-127: "In the second refractive component, that due to the vibratory motion of the radiation, we are not dealing with unit velocity but with a lower velocity (frequency), and the refractive component is reduced by the ratio \(v/1\). This component is also modified by the geometrical relationship between the path of the radiation and the structure of the material medium through which it passes. We will call this modifying factor the vibration factor, \(F_v\). The vibrational refraction is then"
\[
\frac{(n - 1)}{d_{\text{cgs}}} = .9317 \cdot k_G \cdot \nu \cdot F \cdot k_r \\
\text{(vibration)}
\]

Adding Eq. (28) and Eq. (30) we obtain

\[
\frac{(n - 1)}{d_{\text{cgs}}} = .9317 \cdot k_G \cdot k_r \cdot (1 + \nu \cdot F \nu) \\
\text{(translation and vibration)}
\]

But \( \nu := \frac{1}{\lambda_{0_n}} \) (where we are using natural units) so

\[
\frac{(n - 1)}{d_{\text{cgs}}} = .9317 \cdot k_G \cdot k_r \cdot \left(1 + \frac{1}{\lambda_{0_n}} \cdot F \nu\right) \\
\text{(31b)}
\]

or

\[
n := \frac{d_{\text{cgs}}}{d_{u_{\text{cgs}}}} \left[ .9317 \cdot k_G \cdot k_r \cdot \left(1 + \frac{1}{\lambda_{0_n}} \cdot F \nu\right) \right] + 1 \\
\text{(31c)}
\]

The refraction constant, \( k_r \), remains to be determined. This is the reciprocal of the effective time displacement minus the initial level in 1/9 increments.

\[
k_r := \frac{1}{t_{\text{eff}}} \left(1 - k_r 0 \cdot \frac{1}{9}\right) \\
\text{(32a)}
\]

Ref. [53], the author's paper on electrical resistivity in the Reciprocal System, shows that
\[ t_{\text{eff}} := \left[ \frac{\text{te}_{-\text{mod}}}{1} \right]^2 \left( \frac{t_p^2 \cdot t_s}{3} \right) \]  

Eq. (32a) now becomes

\[ k_r := \left( \frac{t_p^2 \cdot t_s}{3} \right) \left( 1 - k_{r0} \cdot \frac{1}{9} \right) \]  

(Larson simply used \( t_{\text{eff}} = t_p \) or \( t_s \) for H, C, and O in the published work, but it's necessary to use the full expression for accuracy, because the time involved must be the same or close to that used for the electrical resistivity calculations. Appendix B of Ref. [2], first ed., p. 218, states, "The omitted portions [of Section XXXIV] include a discussion of the more complex refraction patterns and numerical calculations of both refraction and dispersion for approximately 500 substances." Unfortunately, this material has, to date, not been found, so we are on our own.)

Larson says on p. 30, Ref. [2], first ed., "This initial unit is distributed over three dimensions and the one-third unit in the dimension parallel to the space-time progression is again distributed over the three dimensions of the time region. The resultant is 1/9 unit in each magnetic dimension, a total of 2/9 units." Therefore,

\[ k_{r0} := \begin{array}{l}
2 \quad \text{(for electropositive elements, usually)} \\
2 \quad \text{or} \quad k_{r0} := 1 \quad \text{or} \quad k_{r0} := 0 \quad \text{(for electronegative elements, usually)}
\end{array} \]  

(same as for the specific heat patterns) but there may be deviations, as shown in Larson's tabulations of \((n-1)/d\) for
various organic compounds, pp. 128-129. (Atoms of the same element in a compound may have different values of $k_{r0}$.)

To get the average index of refraction for a compound or an alloy we must weight the number of atoms for each element, $n_{a_i}$, by its atomic number, $Z_{atno_i}$. The atomic number is the net number of equivalent electric time displacement units of the atom, and is thus is more appropriate to use here, rather than atomic weight (which is what conventional physics and chemistry use).

$$k_{r\text{-compound}} := \frac{\sum_i (n_{a_i} Z_{atno_i} k_{r_i})}{\left[\sum_i (n_{a_i} Z_{atno_i})\right]}$$  \hspace{1cm} (35)

The most commonly used wavelength for refraction measurements is that of the sodium D line, $5893 \times 10^{-8}$ cm. So, usually,

$$\lambda_{0\text{-n}} := \frac{c_{cgs}}{5893 \cdot 10^{-8}} \quad R_{\nu} \quad \lambda_{0\text{-n}} = 0.0774 \quad \text{(natural units)}$$  \hspace{1cm} (36)

Therefore, for this wavelength,

$$E_{\lambda 0} := \frac{1.2398 \cdot 10^{-4}}{5893 \cdot 10^{-8}} \quad E_{\lambda 0} = 2.1039 \quad \text{eV}$$  \hspace{1cm} (36)
Also, for many substances, $F_\nu := .75$ \hspace{1em} (Ref. [2], p. 127)

The theoretical justification for this factor is as follows. There are two two-dimensional rotational systems of the atom. Each has two dimensions or directions, four total, for the radiation to go. But one of these must be collinear, because there are only three dimensions available. Therefore, the ratio of available paths to total paths is $3/4 = .75$. By this argument, other allowed values of $F_\nu$ would be .5 and .25. This deduction needs more confirmation, of course.
Sample Calculation for the Simple Index of Refraction of Water or Ice, H$_2$O

H: 2-1-(1)  
\[ t\text{e}_{\text{mod}}_H := -1 \quad t_p_H := 2 \quad t_s_H := 1 \quad t_{\text{eff}}_H := \left[ \frac{t\text{e}_{\text{mod}}_H}{1} \right]^2 \]  
\[ t_{\text{eff}}_H = 0.3969 \]

\[ k_{r0} := 2 \quad \text{(for both H and O)} \]

\[ k_{r_H} := \frac{1}{t_{\text{eff}}_H} \cdot \left[ 1 - k_{r0} \frac{1}{9} \right] \quad k_{r_H} = 1.9599 \quad n_a_H := 2 \]

\[ Z_{\text{atno}_H} := 1 \quad \text{(but could be .5, if D is given at. no. 1)} \]

O: 2-2-(2)  
\[ t\text{e}_{\text{mod}}_O := -2 \quad t_p_O := 2 \quad t_s_O := 2 \quad t_{\text{eff}}_O := \left[ \frac{t\text{e}_{\text{mod}}_O}{1} \right]^2 \]  
\[ t_{\text{eff}}_O = 1 \]

\[ k_{r_O} := \frac{1}{t_{\text{eff}}_O} \cdot \left[ 1 - k_{r0} \frac{1}{9} \right] \quad k_{r_O} = 0.7778 \quad n_a_O := 1 \]

\[ Z_{\text{atno}_O} := 8 \]

\[ k_{r_{H2O}} := \frac{n_a_H \cdot Z_{\text{atno}_H} \cdot k_{r_H} + n_a_O \cdot Z_{\text{atno}_O} \cdot k_{r_O}}{n_a_H \cdot Z_{\text{atno}_H} + n_a_O \cdot Z_{\text{atno}_O}} \quad k_{r_{H2O}} = 1.0142 \]

\[ d_{H2O} := 1 \quad \text{g/cm}^3 \quad \text{(approx.)} \]

\[ \lambda_0_n := \frac{1}{.0774} \quad F_V := .75 \]

The value of $k_G$ is somewhat difficult to determine, because we're dealing with molecules, not atoms. There are four molecules of H$_2$O per unit hexagonal cell. What is needed is the average separation of the molecules from one
another, not the distance between individual atoms of hydrogen or oxygen. From Ref. [58], p. H-190, for ice Ih, the observed a-, b-, and c-edge lengths of the cell (at 0° C) are 4.5135 Å, 4.5135 Å, and 7.3521 Å. The computed unit cell volume at this temperature is 129.709 Å³. If we assume the ice or water molecules are at an average separation of the c-edge, 7.3521 Å (as opposed to the closest O atoms which are 4.5135 Å apart), which seems reasonable, then

\[ k_{G\text{-H}_2}\text{O} := \frac{129.709}{(7.3521)^3} \]

\[ k_{G\text{-H}_2}\text{O} = 0.3264 \]

\[ n := \frac{d_{\text{H}_2\text{O}}}{d_{\text{u\text{-}cgs}}} \cdot 0.9317 \cdot k_{G\text{-H}_2}\text{O} \cdot k_{r\text{-H}_2}\text{O} \left(1 + \frac{1}{\lambda_0 \cdot n} \cdot F_{\nu}\right) + 1 \]

\[ n = 1.3263 \]

Ref. [17], p. 1.95, gives the measured value as 1.333 for temperatures ranging from 0° C to 25° C. So, perhaps, the actual effective molecular separation is a little less than the c-edge.

Sample Calculations for the Complex Index of Refraction and Permittivity of Semiconductor Elements

Use of the above equations in an Excel spreadsheet results in the following table, with the observed values of n and k from Ref. [1]
append. The values of $k_G$ come from the Reciprocal system Data Base, so there are some deviations from ideality.

Element Rot. Displ. tp ts te te_mod_rho te_mod t_eff kr0 kr Density $k_G$ Fv n n_obs rho_e k k_obs
B 2-2-(5) 2 2 -5 -7 -7 12.2500 0 0.0816 2.288 2.013 0.75 1.3706 8.7443E+04 0.0003
C 2-2-(4) 2 2 -4 -4 -4 4.0000 0 0.2500 3.515 1.558 0.75 2.3496 2.4150 8.1040E+94 0.0000 0.0000
Si 3-2-(4) 3 2 -4 -4 -3 -1.7778 0 0.5625 5.321 1.471 0.75 5.3402 5.7480 4.7318E-01 11.8642 1.6340
S 3-2-(2) 3 2 -2 -2 -2 1.7778 0 0.5625 5.321 1.471 0.75 5.3402 5.7480 4.7318E-01 11.8642 1.6340
Ge 3-3-(4) 3 3 -4 -1 -4 1.7778 0 0.5625 5.321 1.471 0.75 5.3402 5.7480 4.7318E-01 11.8642 1.6340
Te 4-3-(2) 4 3 -2 -7 -6 2.7257 0 0.3669 6.267 1.461 0.75 4.3114 4.6700 2.9580E-03 2350.7191 4.6700

Table I. Complex Index of Refraction and Permittivity of Semiconductor Elements

The agreement is fair for $n$, but not for $k$. The column labeled $te_{mod\_rho}$ comes from Ref. [53]—these are the values of $te_{mod}$ used in the calculation for resistivity (which includes the isotopic effect); the column labeled $te_{mod}$ includes the values used here; they are nearly the same except for Ge. All values of $kr_0$ for these electronegative semiconductor elements are zero. Ref. [16] does not have observed values for B or S. The extinction coefficients are not in agreement, but then the resistivities for these elements are in dispute—see Ref. [53] for a particular discussion about Si. Also, Ref. [16] uses inconsistent units for the extinction coefficient, so there’s a lot to disagree with there. Von Hippel (Ref. [19], p. 302) says that for sulfur "$\tan(\delta) < 5$"; no other elements are discussed there. Using Eq. (21) and Eq. (22) we have for these elements:

$$n^*_B = 1.3706 - j 0.003$$  
$$n^*_C = 2.3496 - j 0$$  
$$n^*_Si = 1.570 - j 27.7014$$  
$$n^*_S = 5.0828 - j 0$$  
$$n^*_Ge = 5.3402 - j 11.8642$$  
$$n^*_Te = 4.3114 - j 2350.7191$$  

$$\varepsilon_{r\_B}^* = 1.8785 - j 8.2236E-04$$  
$$\varepsilon_{r\_C}^* = 5.5206 - j 0$$  
$$\varepsilon_{r\_Si}^* = -764.9027 - j 86.9824$$  
$$\varepsilon_{r\_S}^* = 25.0828 - j 0$$  
$$\varepsilon_{r\_Ge}^* = -112.2415 - j 126.7144$$  
$$\varepsilon_{r\_Te}^* = 5.5259E6 - j 2027E4$$  

Note 1: Here is an alternative derivation for $n$, using natural units:
The Reciprocal System Data Base calculates the index of refraction and permittivity for elements, compounds, and alloy/mixtures. The anisotropic aspect of this property is contained in \( k_G \).

**Note 2:** Von Hippel ran the Laboratory for Insulation Research at MIT for many years and generated much data on dielectrics. (And he lived to be 105!) His equation for dielectric conductivity (Ref. [18], p. 5) is

\[
\sigma_e := \omega \varepsilon''
\]

Converting to our symbolism and assuming that dielectric conductivity = electrical conductivity:

\[
\rho_{e\_SI}^{-1} := 2 \pi c_{SI} \varepsilon'' / \lambda_{0\_SI}
\]

Solving for \( \varepsilon'' \):

\[
\varepsilon'' := \lambda_{0\_SI} / (\rho_{e\_SI} 2 \pi c_{SI})
\]

For the sodium D line and sulfur: \( \lambda_{0\_SI} := 5893 \cdot 10^{-10} \) m \( \rho_{e\_SI} := 3.0706 \cdot 10^{12} \) ohm-m

\[
\varepsilon'' := 1.0189 \cdot 10^{-28}
\]

which is zero for all intents and purposes. Von Hippel's equations should be used if you're using his data or his laboratory's.

For \( \varepsilon' \), Von Hippel has

\[
\varepsilon' := \varepsilon_{0\_SI} n^2
\]
And so

\[ \varepsilon^* := \varepsilon' - j \varepsilon'' \quad \varepsilon^* := \varepsilon_{0,\text{SI}} n^2 - j \lambda_{0,\text{SI}} / (\rho_{e,\text{SI}} 2 \pi c_{\text{SI}}) \]  
(Von Hippel's notation)

\[ \tan(\delta) := \frac{\lambda_{0,\text{SI}}}{\varepsilon_{0,\text{SI}} n^2} \quad \tan(\delta) := \frac{\rho_{e,\text{SI}} 2 \pi c_{\text{SI}}}{\varepsilon_{0,\text{SI}} n^2} \]  
(Von Hippel's notation)

For sulfur,

\[ n_S := 5.0828 \quad \tan(\delta) := \frac{\lambda_{0,\text{SI}}}{\varepsilon_{0,\text{SI}} n_S^2} \quad \tan(\delta) = 4.4562 \times 10^{-19} \]

Von Hippel (Ref. [18], p. 27) defines the extinction coefficient (for negligible magnetic loss) in terms of \( \tan(\delta) \):

\[ \tan(\delta) = \frac{2 \cdot k}{1 - k^2} \quad \text{which gives} \quad k := \frac{1}{2 \cdot \tan(\delta)} \left[ -2 + 2 \cdot \left( 1 + \tan(\delta)^2 \right)^{1/2} \right] \]

For sulfur,

\[ k = 0 \quad \text{(as before)} \]

\[ n^*_S := n_S (1 - j k) \quad n^*_S := 5.0828 (1 - j 0) \]  
(Von Hippel's notation)

### 3. Dielectrics and Electric Circuits

Dielectrics are used in capacitors. In the Reciprocal System, an ordinary electric current is comprised of massless, uncharged electrons, not charged electrons, and therefore, our analysis of electric circuits with capacitors is different from that of conventional physics and electrical engineering. Before continuing with the derivation of the electric and magnetic susceptibility equations, we'll now take a little detour to analyze the transient and steady state operation of six electric circuits,
three DC and three AC, as shown in Figure 1, (a)-(f). In what follows, we will often make use of Mathcad’s symbolic equal sign, 

\[ \equiv \]

This will help with the symbolic differentiation and integration of the expressions. All variables in this section are in the usual SI units: volts, amps, ohms, so no subscripts for units will be used here. However, where there is a difference between the Reciprocal System equations and the conventional ones, we will use the subscript "legacy" to denote the conventional equations.
Figure 1. DC and AC Circuits. As is customary in the Reciprocal System, actual electron flows are shown, rather than assumed positive charge flow.
a. DC RC Series Circuit (Fig. 1(a))

The electric current through the resistor and capacitor, according to the Reciprocal System, is

\[ i = C \cdot \left( \frac{d}{dt}v_C \right) \]  \hspace{1cm} \text{(Note: this equation is different from that in the previous version of this paper.)} \hspace{1cm} (37a)

Dimensionally:
\[
\frac{s}{t} = \left(\frac{s^3 \cdot 1 \cdot t}{t \cdot t \cdot s^2}\right) \rightarrow 1 \quad \text{(therefore, correct dimensions)}
\]

(37b)

Using Kirchhoff's voltage law around the circuit:

\[ V = C \cdot \left(\frac{d}{dt}v_C\right) \cdot R + v_C \quad \text{(38a)} \]

or

\[ \frac{d}{dt}v_C = \left( V - v_C \right) \cdot \frac{1}{R \cdot C} \quad \text{(38b)} \]

This is a simple first order differential equation, so we can separate variables and integrate:

\[
\int 1 \, dt = \int \frac{R \cdot C}{\left(V - v_C\right)} \, dv_C
\]

(38c)

and, using Mathcad's symbolic integration routine, we get:

\[
\int 1 \, dt \rightarrow t \quad \int \frac{R \cdot C}{\left(V - v_C\right)} \, dv_C \rightarrow -C \cdot R \cdot \ln(v_C - V)
\]

(38d)

without the constant of integration. But we'll solve for \( v_C \) first:
\[-\ln(V - v_C) \cdot R \cdot C = t\]  

\[v_C := V - \exp\left(\frac{-t}{R \cdot C}\right)\]  

(38f)

But, at \(t = 0\) the capacitor is uncharged, \(v_C = 0\), so we must prefix the exponential term with \(V\), which is effectively the constant of integration:

\[v_C := V - V \cdot \exp\left(\frac{-t}{R \cdot C}\right)\]  

(39a)

which simplifies to

\[v_C := V \left(1 - \exp\left(\frac{-t}{R \cdot C}\right)\right)\]  

(39b)

According to conventional physics, as given in Ref. [21], p. 245:

\[v_{C\_legacy} := V \left(1 - \exp\left(\frac{-t}{R \cdot C}\right)\right)\]

\[i_{\text{legacy}} := \frac{V}{R} \cdot \exp\left(\frac{-t}{R \cdot C}\right)\]

The capacitor time constant is the same in both the Reciprocal System and conventional theory:
\[
\tau_C := R \cdot C \\
\frac{t^2}{s^3} \frac{s^3}{t} \rightarrow t \quad \text{(therefore, correct dimensions)} \quad (40)
\]

\[
\tau_{C\text{\_legacy}} := R \cdot C
\]

Sample DC RC Circuit

Now let's plot the results for a specific example from Ref. [21], p. 255, problem 16.5. Here,

\[
V := 100 \text{ volts} \quad R := 5000 \text{ ohms} \quad C := 20 \cdot 10^{-6} \text{ farads}
\]

The capacitor has no initial "charge." At \( t = 0 \), the switch is closed.
Reciprocal System curves are in red; conventional theory curves are in blue.

Figure 2. $v_C$ and $i$ Graphs for DC RC Series Circuit
\( \tau_C := R \cdot C \quad \tau_C = 0.1 \ \text{sec} \)

\( \tau_{C\_legacy} := R \cdot C \quad \tau_{C\_legacy} = 0.1 \ \text{sec} \)

The instantaneous power supplied to the capacitor is defined as the product of the instantaneous voltage and current:

\[
p_C := V \cdot \left(1 - \exp\left(-t \cdot \frac{1}{R \cdot C}\right)\right) \cdot \left(\frac{V}{R} \cdot \exp\left(-t \cdot \frac{1}{R \cdot C}\right)\right)
\]

(41)

The conventional expression is the same:

\[
p_{C\_legacy} := V \cdot \left(1 - \exp\left(-t \cdot \frac{1}{R \cdot C}\right)\right) \cdot \left(\frac{V}{R} \cdot \exp\left(-t \cdot \frac{1}{R \cdot C}\right)\right)
\]

Here are the plots for the example problem:
$V \cdot \left(1 - \exp\left(-\frac{t}{RC}\right)\right) \cdot \left(\frac{V}{R} \cdot \exp\left(-\frac{t}{RC}\right)\right)$

Reciprocal System curve is in red; conventional theory curve is in blue.

Figure 3. $p_C$
Graphs for DC RC Series Circuit.
The total energy supplied to the capacitor is the integral of the power from \( t = 0 \) to \( t = \infty \). Carrying out the integration (by hand), we obtain

\[
E_C := \frac{0.5 \cdot C \cdot V^2}{t \cdot \left(\frac{t}{s}\right)^2} \rightarrow \frac{t}{s}
\]

(which are the correct dimensions of energy) \((42)\)

Conventional theory (Ref. [21], p. 246) gives the same numeric result:

\[
E_{C\_legacy} := \frac{0.5 \cdot C \cdot V^2}{\text{farads x volts}^2} = \left(\sec^2 x \text{ coul}^2 / (\text{kg x m}^2)\right) \times (\text{kg x m}^2 / (\sec^2 x \text{ coul}))^2 = \text{joules}
\]

For the sample problem

\[
E_C = 0.1 \quad \text{(see below for units)}
\]

\[
E_{C\_legacy} = 0.1 \quad \text{joules}
\]

The energy can also be directly calculated from the integral, using Mathcad.

\[
\int_{0}^{\infty} V \cdot \left(1 - \exp\left(-t \cdot \frac{1}{R \cdot C}\right)\right) \cdot \left(\frac{V}{R} \cdot \exp\left(-t \cdot \frac{1}{R \cdot C}\right)\right) \, dt \rightarrow \frac{1}{10} \quad \text{joules}
\]

(For symbolic calculations, Mathcad uses double precision.)

This is the same as the conventional value.
The voltage across the resistor is

\[ v_R := V \cdot \exp\left(-t \cdot \frac{1}{R \cdot C}\right) \]  

(43)

The instantaneous power supplied to the resistor is

\[ p_R := v_R \cdot \left(\frac{V}{R} \cdot \exp\left(-t \cdot \frac{1}{R \cdot C}\right)\right) \]  

(44)

The energy delivered to the resistor from \( t = 0 \) to \( t = \infty \) is

\[ E_R := \int_{0}^{\infty} v_R \cdot \left(\frac{V}{R} \cdot \exp\left(-t \cdot \frac{1}{R \cdot C}\right)\right) \, dt \]  

(45)

Here are the voltage and power plots for the resistor, both for the Reciprocal System and conventional theory:
Figure 4. $v_R$ and $p_R$ Graphs for DC RC Series Circuit
For the Reciprocal System, the total energy delivered to the resistor (or load) is

\[
\int_0^\infty V \cdot \exp\left(-t \cdot \frac{1}{R \cdot C}\right) \cdot \left(\frac{V}{R} \cdot \exp\left(-t \cdot \frac{1}{R \cdot C}\right)\right) \, dt = 0.1 \text{ joules}
\]

For conventional theory, the total energy delivered to the resistor (or load) is

\[
\int_0^\infty V \cdot \exp\left(-t \cdot \frac{1}{R \cdot C}\right) \cdot \left(\frac{V}{R} \cdot \exp\left(-t \cdot \frac{1}{R \cdot C}\right)\right) \, dt = 0.1 \text{ joules}
\]

It's evident that an equal amount of energy goes to the capacitor and resistor according to both theories. The total energy supplied by the DC voltage source is

\[
\int_0^\infty V \cdot \left(\frac{V}{R} \cdot \exp\left(-t \cdot \frac{1}{R \cdot C}\right)\right) \, dt = 0.2 \text{ joules for the Reciprocal System}
\]

\[
\int_0^\infty V \cdot \left(\frac{V}{R} \cdot \exp\left(-t \cdot \frac{1}{R \cdot C}\right)\right) \, dt = 0.2 \text{ joules for the conventional theory}
\]
b. DC RL Series Circuit (Fig. 1(b))

Our derivation here is the same as that in Ref. [21], pp.242-244. The electric current through the resistor and inductor is

\[ i = \frac{1}{L} \int v_L \, dt \quad (46a) \]

Dimensionally:

\[ \frac{s}{t} = \frac{1}{t^3} \cdot \frac{t}{s^2} \cdot t \rightarrow 1 \quad (\text{therefore, correct dimensions}) \quad (46b) \]

Using Kirchhoff's voltage law around the circuit:

\[ V = R \cdot i + L \cdot \left( \frac{d}{dt} i \right) \quad (47a) \]

or, letting \( D \) be the differential operator, \( d/dt \)

\[ \left( D + \frac{R}{L} \right) i = \frac{V}{L} \quad (47b) \]

This is a first order, linear differential equation and its solution is:

\[ i = c_0 \cdot \exp \left( -\frac{R}{L} \cdot t \right) + \frac{V}{R} \quad (47c) \]
which can be verified by substitution, and where \( c_0 \) is a constant. At \( t = 0 \), \( i = 0 \), so

\[
c_0 := \frac{-V}{R}
\]

(47d)

Eq. (44c) becomes

\[
i = \frac{-V}{R} \exp\left(\frac{-R}{L} \cdot t\right) + \frac{V}{R} \quad \text{or} \quad i = \frac{V}{R} \left(1 - \exp\left(\frac{-R}{L} \cdot t\right)\right)
\]

(47e)

The voltage across the inductor is then

\[
v_L = L \cdot \left(\frac{d}{dt} i\right) \quad v_L = V \cdot \exp\left(\frac{-R}{L} \cdot t\right)
\]

(48)

The inductor time constant is, in both conventional theory and the Reciprocal System, by inspection,

\[
\tau_L := \frac{L}{R} \quad \frac{t^3}{s^3} \rightarrow t \quad \text{(therefore, correct dimensions)}
\]

(49)
Sample DC RL Circuit

Now let's plot the results for a specific example from Ref. [21], p. 253, problem 16.1. Here,

\[ V := 100 \text{ volts} \quad R := 50 \text{ ohms} \quad L := 10 \text{ henries} \]

The switch is closed at \( t = 0 \).
Reciprocal System and conventional theory are in agreement here.

Figure 5. $v_L$ and $i$ Graphs for DC RL Series Circuit
\[ \tau_L := \frac{L}{R} \quad \tau_L = 0.2 \text{ sec} \]

The instantaneous power supplied to the inductor is defined as the product of the instantaneous voltage and current:

\[ p_L := V \cdot \exp \left( -\frac{R}{L} \cdot t \right) \left[ \frac{V}{R} \left( 1 - \exp \left( -\frac{R}{L} \cdot t \right) \right) \right] \]

(50)

Here is the plot for the example problem:

Reciprocal System and conventional theory are in agreement here.

Figure 6. \( p_L \) Graph for DC RL Series Circuit
The total energy supplied to the inductor is the integral of the power from $t = 0$ to $t = \infty$. Carrying out the integration (by hand), we obtain

$$E_L := \frac{1}{2} L I^2$$

where $I$ is the steady current.

For the sample problem

$$I \equiv \frac{V}{R} \quad I = 2 \text{ amps}$$

$$E_L := \frac{1}{2} L I^2 \quad E_L = 20 \text{ joules}$$

We can also use the integral:

$$\int_0^{\infty} \left[ V \cdot \exp\left( -\frac{R}{L} \cdot t \right) \cdot \frac{V}{R} \cdot \left( 1 - \exp\left( -\frac{R}{L} \cdot t \right) \right) \right] \, dt = 20 \text{ joules}$$

Here, the Reciprocal System agrees numerically with the conventional theory. But the interpretation of "where" this energy is stored is very different. Quoting Larson from two of his books:

Ref. [2], first ed., pp. 76-77: "An equivalent mass $L$, moving with a velocity $I$ must have a kinetic energy of $0.5 LI^2$ and we find experimentally when a current $I$ flowing in an inductance $L$ is destroyed an amount of energy $0.5 LI^2$ does make its appearance. The explanation on the basis of existing theory is that this energy is 'stored in the electromagnetic field', but the dimensional clarification shows that it is actually the kinetic energy of the moving electrons."

Ref. [1], p. 220: "It follows that $m$ (mass) and $L$ (inductance) are equivalent....Just as inertia resists any change in speed or velocity, inductance resists any change in the electric current."
The voltage across the resistor is

\[ v_R := V \cdot (1 - \exp\left(\frac{-R}{L} \cdot t\right)) \]  

(53)

The instantaneous power supplied to the resistor is

\[ p_R := v_R \cdot \left[\frac{V}{R} \cdot (1 - \exp\left(\frac{-R}{L} \cdot t\right))\right] \]  

(54)

Here's the plot:

![Graph showing the power supplied to the resistor over time. The power reaches a constant value of 200 watts in approximately 1 second. A larger value of L would cause a longer delay.](Figure 7. p_R Graph for DC RL Series Circuit)

The power to the resistor gets to its constant value, 200 watts, in approx. 1 second. A larger value of L would cause a longer delay.
c. DC RLC Circuit (Fig. 1(c))

Using Kirchoff’s voltage law around the circuit:

\[ V = R \cdot i + L \cdot \left( \frac{d}{dt} \right) i + \frac{1}{C} \int i \, dt \]  

(55)

Differentiating with respect to \( t \):

\[ 0 = R \cdot \left( \frac{d}{dt} i \right) + L \cdot \frac{d^2}{dt^2} i + \frac{i}{C} \]  

(56a)

Rearranging and using the differential symbol \( D \):

\[ \left( D^2 + \frac{R}{L} \cdot D + \frac{1}{L \cdot C} \right) i = 0 \]  

(56b)

The particular solution for \( i \) is zero. As for the complementary solution, the coefficients in the characteristic equation within the parentheses above are constants, and so the roots of the equation are

\[ D_1 := \sqrt{\frac{R^2}{4 \cdot L^2} - \frac{1}{L \cdot C}} - \frac{R}{2 \cdot L} \quad D_2 := \frac{R}{2 \cdot L} - \sqrt{\frac{R^2}{4 \cdot L^2} - \frac{1}{L \cdot C}} \]  

(56c)
Let
\[
\alpha := \frac{-R}{2 \cdot L} \quad \quad \quad \beta := \sqrt{\frac{R^2}{4 \cdot L^2} - \frac{1}{L \cdot C}} \quad \quad \quad D_1 := \alpha + \beta \quad \quad \quad D_2 := -\alpha - \beta
\]

Case 1: \[\frac{R^2}{4 \cdot L^2} > \frac{1}{L \cdot C}\] (overdamped)

\[
i := e^{\alpha \cdot t} \left( c_1 \cdot e^{\beta \cdot t} + c_2 \cdot e^{-\beta \cdot t} \right)
\]  \hspace{1cm} (57a)

where \(c_1\) and \(c_2\) are constants to be determined from initial conditions. The voltage across the capacitor is

\[
v_C = \frac{1}{CC} \cdot \int e^{\alpha \cdot t} \left( c_1 \cdot e^{\beta \cdot t} + c_2 \cdot e^{-\beta \cdot t} \right) \, dt \rightarrow v_C = \frac{c_1 \cdot e^{t \cdot \alpha + t \cdot \beta \beta} + c_2 \cdot e^{t \cdot \alpha - t \cdot \beta \beta}}{\alpha \alpha + \beta \beta} + \frac{c_2 \cdot e^{t \cdot \alpha - t \cdot \beta \beta}}{\alpha \alpha - \beta \beta}
\]  \hspace{1cm} (57b)

(where the symbols have been doubled to get a purely symbolic result).

\[
v_R := e^{\alpha \cdot t} \left( c_1 \cdot e^{\beta \cdot t} + c_2 \cdot e^{-\beta \cdot t} \right) \cdot R
\]  \hspace{1cm} (57c)

\[
v_L := L \left[ e^{t \cdot \alpha \alpha} \left( c_1 \cdot \beta \beta \cdot e^{t \cdot \beta \beta} - c_2 \cdot \beta \beta \cdot e^{-t \cdot \beta \beta} \right) + \alpha \alpha \cdot e^{t \cdot \alpha \alpha} \left( c_1 \cdot e^{t \cdot \beta \beta} + c_2 \cdot e^{-t \cdot \beta \beta} \right) \right]
\]  \hspace{1cm} (57d)
Case 2: \( \frac{R^2}{4L^2} = \frac{1}{L\cdot C} \) (damped)

\[
i := e^{\alpha \cdot t} \cdot (c_1 + c_2 \cdot t)\tag{58a}
\]

\[
v_C = \frac{1}{CC} \int e^{\alpha \cdot t} \cdot (c_1 + c_2 \cdot t) \, dt \rightarrow v_C = -\frac{e^{t \cdot \alpha \cdot \alpha} \cdot \left( \frac{c_2 \cdot c_1 \cdot \alpha \cdot \alpha - c_2 \cdot t}{\alpha \cdot \alpha^2} \right)}{CC} \tag{58b}
\]

(where the symbols have been doubled to get a purely symbolic result).

\[
v_R := e^{\alpha \cdot t} \cdot (c_1 + c_2 \cdot t) \cdot R \tag{58c}
\]

\[
v_L := L \left[ c_2 \cdot e^{t \cdot \alpha \cdot \alpha} + \alpha \cdot e^{t \cdot \alpha \cdot \alpha} \cdot (c_1 + c_2 \cdot t) \right] \tag{58d}
\]
Case 3:

\[
\frac{R^2}{4L^2} < \frac{1}{L \cdot C} \quad \text{(underdamped)}
\]

\[
\beta := \sqrt{\frac{1}{L \cdot C} - \frac{R^2}{4L^2}}
\]

\[i := e^{\alpha t} \left( c_1 \cdot \cos(\beta t) + c_2 \cdot \sin(\beta t) \right)\]  

(59a)

\[v_C = \frac{1}{CC} \int e^{\alpha t} \left( c_1 \cdot \cos(\beta t) + c_2 \cdot \sin(\beta t) \right) dt \rightarrow v_C = \frac{e^{t \cdot \alpha \beta} \left( c_1 \cdot \alpha \cdot \cos(\beta \cdot t) - c_2 \cdot \beta \cdot \sin(\beta t) + c_2 \cdot \alpha \cdot \sin(\beta t) \right)}{CC \left( \alpha^2 + \beta^2 \right)}\]

(59b)

\[v_R := e^{\alpha t} \left( c_1 \cdot \cos(\beta t) + c_2 \cdot \sin(\beta t) \right) \cdot R\]

(59c)

\[v_L := L \left[ e^{t \cdot \alpha \beta} \left( c_2 \cdot \beta \cdot \cos(\beta \cdot t) - c_1 \cdot \beta \cdot \sin(\beta \cdot t) \right) + \alpha \cdot e^{t \cdot \alpha \beta} \left( c_1 \cdot \cos(\beta \cdot t) + c_2 \cdot \sin(\beta \cdot t) \right) \right]\]

Obviously, we should obtain

\[V = v_R + v_L + v_C\]
Sample DC RLC Circuit

Now let's plot the results for a specific example from Ref. [21], p. 258, problem 16.11. Here,

\[ V := 100 \text{ volts} \quad R := 50 \text{ ohms} \quad L := 0.1 \text{ henries} \quad C := 50 \cdot 10^{-6} \text{ farads} \]

The switch is closed at \( t = 0 \).

\[ \frac{R^2}{4L^2} = 62500 \quad \frac{1}{L \cdot C} = 2 \times 10^5 \] Therefore, this RLC circuit is underdamped.

\[ \alpha := \frac{-R}{2L} \quad \alpha = -250 \quad \beta := \sqrt{\frac{1}{L \cdot C} - \frac{R^2}{4L^2}} \quad \beta = 370.8099 \]

\[ i := e^{\alpha t} \left( c_1 \cdot \cos(\beta t) + c_2 \cdot \sin(\beta t) \right) \]

At \( t = 0 \), the current is zero:

\[ t := 0 \quad 0 = e^{\alpha \cdot 0} \left( c_1 \cdot \cos(\beta \cdot 0) + c_2 \cdot \sin(\beta \cdot 0) \right) \quad \text{Therefore,} \quad c_1 := 0 \quad \text{So,} \quad i := e^{\alpha \cdot t} c_2 \cdot \sin(\beta \cdot t) \]

Differentiating this expression,

\[ \frac{d}{dt} \left( e^{\alpha t} c_2 \cdot \sin(\beta \cdot t) \right) \rightarrow c_2 \cdot \beta \cdot e^{\alpha t} \cdot \cos(t \cdot \beta) + c_2 \cdot \alpha \cdot e^{\alpha t} \cdot \sin(t \cdot \beta) \]

At \( t = 0 \), only the inductor is operative:
\[ c_2 \cdot \beta \cdot e^{0 \cdot \alpha \cdot \cos(0 \cdot \beta)} + c_2 \cdot \alpha \cdot e^{0 \cdot \alpha \cdot \sin(0 \cdot \beta)} = \frac{V}{L} \]

Solving for \( c_2 \):
\[ c_2 := \frac{V}{L \cdot \beta} \quad \Rightarrow \quad c_2 = 2.6968 \]

Carrying out the integration for \( v_C \):
\[ v_C := \frac{c_2 \cdot \left( \beta - \beta \cdot e^{t \cdot \alpha \cdot \cos(t \cdot \beta)} + \alpha \cdot e^{t \cdot \alpha \cdot \sin(t \cdot \beta)} \right)}{C \cdot (\alpha^2 + \beta^2)} \]

(The reference doesn't provide an equation for the voltage.)

Figure 8. \( v_C \) Graph for DC RLC Series Circuit
Here is the plot for current for both the Reciprocal System and conventional theory given in the above reference:

\[ e^{\alpha \cdot t} c_2 \cdot \sin(\beta \cdot t) \]

Figure 9. Graph for DC RLC Series Circuit
Now here's the plot for $v_L$:

$$L \cdot \left( c_2 \cdot e^{\alpha \cdot t} \cdot \cos(t \cdot \beta) + c_2 \cdot e^{\alpha \cdot t} \cdot \sin(t \cdot \beta) \right)$$

Figure 10. $v_L$ for RLC Series Circuit
And here's the plot for the voltage across the resistor:

\[ V_R = R \left( e^{\alpha t} c_2 \sin(\beta t) \right) \]

Figure 11. \( v_R \) for RLC Series Circuit
The sum of the voltages across the resistor, inductor, and capacitor should equal $V$:

\[
\frac{c_2 \left( \beta - \beta \cdot e^{t \cdot \alpha} \cdot \cos(t \cdot \beta) + \alpha \cdot e^{t \cdot \alpha} \cdot \sin(t \cdot \beta) \right)}{C \left( \alpha^2 + \beta^2 \right)} + L \left( c_2 \cdot \beta \cdot e^{t \cdot \alpha} \cdot \cos(t \cdot \beta) + c_2 \cdot \alpha \cdot e^{t \cdot \alpha} \cdot \sin(t \cdot \beta) \right) 
\]

\[
+ R \left( e^{t \cdot \alpha} \cdot c_2 \cdot \sin(\beta \cdot t) \right) \]

Figure 12. $v_C + v_L + v_R$ for DC RLC Series Circuit
Now we'll plot the instantaneous power supplied to the resistor, inductor, and capacitor.

\[ p_R := v_R \cdot i \]  \hspace{1cm} (60) \\
\[ p_L := v_L \cdot i \]  \hspace{1cm} (61) \\
\[ p_C := v_C \cdot i \]  \hspace{1cm} (62)

(The graph has the equations expanded out. Because of the length of the equations, the graph spills over to another page.)
Obviously, the voltage and power to the inductor and resistor go to zero; the capacitor then stays at the voltage of the source, and there is no further current in the circuit. The energy stored in the capacitor is then
\[ E_C := 0.5 \cdot C \cdot V^2 \]

\[ E_C = 0.25 \text{ joules} \]  

(63)
d. AC RC Circuit (Fig. 1(d))

The voltage from the source is

$$v := V_{\text{max}} \cdot \sin(\omega \cdot t + \phi_{ac})$$

(64a)

where $\phi_{ac}$ is the phase angle and can take on values from 0 to $2\pi$ rad/sec.

By starting the source at time $t = 0$ (when the switch is closed), we can set $\phi_{ac} = 0$, and so Eq. (64a) can be simplified to

$$v := V_{\text{max}} \cdot \sin(\omega \cdot t)$$

(64b)

Capacitive reactance is defined as

$$X_C := \frac{1}{2 \cdot \pi \cdot f \cdot C}$$

$$t := t \quad \frac{1}{s^3} \rightarrow \frac{t^2}{s^3} \quad \text{(correct dimensions for resistance)}$$

(65)

Using Kirchhoff’s voltage law around the circuit:

$$v = v_R + v_C$$

(66)

The current is the same around the circuit, so the instantaneous power is

$$v \cdot i = v_R \cdot i + v_C \cdot i$$

(67)
Energy conservation then requires that

\[ \int_0^t v \cdot i \, dt = \int_0^t v_R \cdot i \, dt + \int_0^t v_C \cdot i \, dt \] (68)

For an alternating electric voltage we define the time period of a cycle in terms of the angular frequency:

\[ T := \frac{2 \cdot \pi}{\omega} \quad f := \frac{1}{T} \] (69)

(If \( f = 60 \) cycles per second, \( T = 1/60 \) = seconds, so \( \omega = 120 \times \pi \) rad/sec.)

The impedance of this circuit is

\[ Z_{RC} := \sqrt{R^2 + X_C^2} \] (70)

The phase angle of this circuit is

\[ \phi_{RC} := \text{atan} \left( \frac{X_C}{R} \right) \] (71)

The steady-state current current is

\[ I := \frac{V_{\text{max}}}{Z_{RC}} \cdot \sin \left( 2 \cdot \pi \cdot f \cdot t + \phi_{RC} \right) \] (72)
The steady-state voltage is

\[ V := Z_{RC} \cdot I_{\text{max}} \cdot \sin(2 \cdot \pi \cdot f \cdot t) \]  

(73)

**Sample AC RC Circuit**

Now let's compute the results for a specific example from Ref. [21], p. 259, problem 16.14. Here,

\[ V := 250 \cdot \sin(500 \cdot t + 0) \text{ volts} \quad R := 100 \text{ ohms} \quad C := 25 \cdot 10^{-6} \text{ farads} \]

The capacitor has no initial "charge." At \( t = 0, \phi_{ac} = 0 \), the switch is closed.

\[ V_{\text{max}} := 250 \text{ volts} \quad \omega := 500 \text{ rad/sec} \quad \phi_{ac} := 0 \]

\[ T := \frac{2 \cdot \pi}{\omega} \quad T = 0.0126 \text{ sec} \quad f := \frac{1}{T} \quad f = 79.5775 \text{ cycles/sec} \]

\[ X_C := \frac{1}{2 \cdot \pi \cdot f \cdot C} \quad X_C = 80 \text{ ohms} \quad Z_{RC} := \sqrt{R^2 + X_C^2} \quad Z_{RC} = 128.0625 \text{ ohms} \]

\[ \phi_{RC} := \arctan \left( \frac{X_C}{R} \right) \quad \phi_{RC} = 0.6747 \text{ rad} \]

\[ I_{\text{max}} := \frac{V_{\text{max}}}{Z_{RC}} \quad I_{\text{max}} = 1.9522 \]
As given in the reference above, the \textit{transient} instantaneous current plus the steady-state current is:

\[
i := \exp\left(-\frac{t}{RC}\right)\left[\frac{V_{\text{max}}}{R} \cdot \sin(\phi_{\text{ac}}) - \frac{V_{\text{max}}}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}} \cdot \sin\left(\phi_{\text{ac}} + \tan^{-1}\left(\frac{1}{\omega C \cdot R}\right)\right)\right] + \frac{V_{\text{max}}}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}} \cdot \sin\left(\omega \cdot t + \phi_{\text{ac}} + \tan^{-1}\left(\frac{1}{\omega C \cdot R}\right)\right)\]
\]

amps

Figure 14. AC RC current
The expression for the voltage across the capacitor for one-quarter cycle is

\[
v_C = \frac{1}{C} \left[ \int_0^{0.25T} \exp \left( -\frac{t}{R \cdot C} \right) \frac{V_{\text{max}}}{R} \sin(\phi_{\text{ac}}) - \frac{V_{\text{max}}}{\sqrt{R^2 + \left( \frac{1}{\omega \cdot C} \right)^2}} \sin \left( \phi_{\text{ac}} + \tan \left( \frac{1}{\omega \cdot C \cdot R} \right) \right) \right] \, dt + \frac{V_{\text{max}}}{\sqrt{R^2 + \left( \frac{1}{\omega \cdot C} \right)^2}} \sin \left( (\omega) \cdot t + \phi_{\text{ac}} + \tan \left( \frac{1}{\omega \cdot C \cdot R} \right) \right) \]

\[
v_C = 132.2695 \text{ volts}
\]

The expression for energy stored at the end of the one-quarter cycle is then

\[
E_C := \frac{1}{2} \cdot C \cdot v_C^2 \quad E_C = 0.2187 \text{ joules}
\]

Note: Fig. 15 and Fig. 16 are not used in this revision of the paper and are reserved; same for Eqs. 74-77.

Also, knowing i enables us to plot the instantaneous power from the source and delivered to the resistor and capacitor.
\[ V_{\text{max}} \sin(\omega \cdot t) \left[ \exp \left( \frac{-tt}{R \cdot C} \right) \frac{V_{\text{max}}}{R} \sin(\phi_{\text{ac}}) - \frac{V_{\text{max}}}{\sqrt{R^2 + \left( \frac{1}{\omega \cdot C} \right)^2}} \sin(\phi_{\text{ac}} + \tan^{-1} \left( \frac{1}{\omega \cdot C \cdot R} \right)) \right] \]

\[ + \frac{V_{\text{max}}}{\sqrt{R^2 + \left( \frac{1}{\omega \cdot C} \right)^2}} \sin(\omega \cdot t + \phi_{\text{ac}} + \tan^{-1} \left( \frac{1}{\omega \cdot C \cdot R} \right)) \]

\[ \left[ \frac{1}{C} \right]^{tt} \left[ \exp \left( \frac{-tt}{R \cdot C} \right) \frac{V_{\text{max}}}{R} \sin(\phi_{\text{ac}}) - \frac{V_{\text{max}}}{\sqrt{R^2 + \left( \frac{1}{\omega \cdot C} \right)^2}} \sin(\phi_{\text{ac}} + \tan^{-1} \left( \frac{1}{\omega \cdot C \cdot R} \right)) \right] \cdot \frac{dt}{tt} \]

\[ + \frac{V_{\text{max}}}{\sqrt{R^2 + \left( \frac{1}{\omega \cdot C} \right)^2}} \sin(\omega \cdot (tt + \phi_{\text{ac}} + \tan^{-1} \left( \frac{1}{\omega \cdot C \cdot R} \right))) \]

Figure 17. \( p, p_R, p_C \) for AC RC Circuit
e. AC RL Circuit (Fig. 1(e))

The Reciprocal System and conventional theory are in agreement for this circuit. Therefore we can use the treatment given in Ref. [21], pp. 249-250. Applying Kirchhoff's voltage law around the circuit:

\[ V_{\text{max}} \sin(\omega t + \phi_{ac}) = R_i i + L \left( \frac{di}{dt} \right) \] (78)

In Reciprocal System space-time terms, the dimensions are

\[ \frac{t}{s^2} = \frac{t^2}{s^3} \cdot \frac{s}{t} + \frac{t^3}{s^3} \cdot \frac{s}{t} \cdot \frac{1}{t} \]

so each term has the dimensions of voltage

Also, the inductive reactance is defined as \( X_L := \frac{\omega L}{t} \) \( = \frac{2 \cdot \pi \cdot f \cdot L}{t} \), \( \frac{1}{t} \cdot \frac{t^3}{s^3} \rightarrow \frac{t^2}{s^3} \) (correct dimensions for resistance)

The complementary solution to Eq. (78) is

\[ i_c := c_0 \cdot \exp \left( -\frac{R}{L} \cdot t \right) \] (79)

The particular solution is
\[ i_p := \exp\left( -\frac{R}{L} \cdot t \right) \cdot \exp\left( \frac{R}{L} \cdot t \right) \cdot \frac{V_{\text{max}}}{L} \cdot \sin(\omega \cdot t + \phi_{ac}) \, dt \]  

(80a)

or

\[ i_p := \frac{V_{\text{max}}}{\sqrt{R^2 + \omega^2 L^2}} \cdot \sin\left( \omega \cdot t + \phi_{ac} - \arctan\left( \frac{\omega L}{R} \right) \right) \]  

(80b)

The complete solution is then

\[ i := c_0 \cdot \exp\left( -\frac{R}{L} \cdot t \right) + \frac{V_{\text{max}}}{\sqrt{R^2 + \omega^2 L^2}} \cdot \sin\left( \omega \cdot t + \phi_{ac} - \arctan\left( \frac{\omega L}{R} \right) \right) \]  

(81)

As is well known, the inductance prevents any sudden change in the current. Before the switch was closed, the current was zero, so \( i_0 = 0 \). Then at \( t = 0 \)

\[ i_0 = 0 \]  

(82a)

\[ i_0 = c_0 \cdot \exp\left( -\frac{R}{L} \cdot 0 \right) + \frac{V_{\text{max}}}{\sqrt{R^2 + \omega^2 L^2}} \cdot \sin\left( \omega \cdot 0 + \phi_{ac} - \arctan\left( \frac{\omega L}{R} \right) \right) \]  

(82b)

Solving for \( c_0 \):

\[ c_0 := \frac{-V_{\text{max}}}{\sqrt{R^2 + \omega^2 L^2}} \cdot \sin\left( \phi_{ac} - \arctan\left( \frac{\omega L}{R} \right) \right) \]  

(83)
Substituting this expression for $c_0$ into Eq. (81):

$$i := \exp\left(\frac{-R}{L} \cdot t\right) \cdot \left(\frac{-V_{\text{max}}}{\sqrt{R^2 + \omega^2 \cdot L^2}} \cdot \sin\left(\phi_{\text{ac}} - \tan\left(\frac{\omega \cdot L}{R}\right)\right)\right) + \frac{V_{\text{max}}}{\sqrt{R^2 + \omega^2 \cdot L^2}} \cdot \sin\left(\omega \cdot t + \phi_{\text{ac}} - \tan\left(\frac{\omega \cdot L}{R}\right)\right)$$

(84)

With this expression for $i$, we know the voltages for the resistor and inductor:

$$v_R := R \cdot \left[ \exp\left(\frac{-R}{L} \cdot t\right) \cdot \left(\frac{-V_{\text{max}}}{\sqrt{R^2 + \omega^2 \cdot L^2}} \cdot \sin\left(\phi_{\text{ac}} - \tan\left(\frac{\omega \cdot L}{R}\right)\right)\right) + \frac{V_{\text{max}}}{\sqrt{R^2 + \omega^2 \cdot L^2}} \cdot \sin\left(\omega \cdot t + \phi_{\text{ac}} - \tan\left(\frac{\omega \cdot L}{R}\right)\right)\right]$$

(85)

$$v_L := L \cdot \left[ \frac{d}{dt} \left[ \exp\left(\frac{-R}{L} \cdot t\right) \cdot \left(\frac{-V_{\text{max}}}{\sqrt{R^2 + \omega^2 \cdot L^2}} \cdot \sin\left(\phi_{\text{ac}} - \tan\left(\frac{\omega \cdot L}{R}\right)\right)\right) + \frac{V_{\text{max}}}{\sqrt{R^2 + \omega^2 \cdot L^2}} \cdot \sin\left(\omega \cdot t + \phi_{\text{ac}} - \tan\left(\frac{\omega \cdot L}{R}\right)\right)\right]\right]$$

(86a)

or

$$v_L := L \cdot \left[ \frac{R}{L} \cdot \exp\left(\frac{-R}{L} \cdot t\right) \cdot \frac{V_{\text{max}}}{\left(\sqrt{R^2 + \omega^2 \cdot L^2}\right)^2} \cdot \sin\left(\phi_{\text{ac}} - \tan\left(\frac{\omega \cdot L}{R}\right)\right) + \frac{V_{\text{max}}}{\left(\sqrt{R^2 + \omega^2 \cdot L^2}\right)^2} \cdot \cos\left(\omega \cdot t + \phi_{\text{ac}} - \tan\left(\frac{\omega \cdot L}{R}\right)\right) \cdot \omega\right]$$

(86b)
The instantaneous power to the circuit is

\[ P := (V_{\text{max}} \sin(\omega \cdot t + \phi_{\text{ac}})) \cdot \left[ \exp\left(-\frac{R}{L} t\right) \left(\frac{-V_{\text{max}}}{\sqrt{R^2 + \omega^2 L^2}} \sin(\phi_{\text{ac}} - \text{atan}\left(\frac{\omega L}{R}\right)) + \frac{V_{\text{max}}}{\sqrt{R^2 + \omega^2 L^2}} \sin(\omega t + \phi_{\text{ac}} - \text{atan}\left(\frac{\omega L}{R}\right))\right)\right]. \]  

(87)

\[ p_R := R \left[ \left[ \exp\left(-\frac{R}{L} t\right) \left(\frac{-V_{\text{max}}}{\sqrt{R^2 + \omega^2 L^2}} \sin(\phi_{\text{ac}} - \text{atan}\left(\frac{\omega L}{R}\right)) + \frac{V_{\text{max}}}{\sqrt{R^2 + \omega^2 L^2}} \sin(\omega t + \phi_{\text{ac}} - \text{atan}\left(\frac{\omega L}{R}\right))\right)\right]^2 \right]. \]  

(88)

\[ p_L := L \left[ \left[ \frac{R}{L} \exp\left(-\frac{R}{L} t\right) \frac{V_{\text{max}}}{\sqrt{R^2 + \omega^2 L^2}} \sin(\phi_{\text{ac}} - \text{atan}\left(\frac{\omega L}{R}\right)) + \frac{V_{\text{max}}}{\sqrt{R^2 + \omega^2 L^2}} \cos(\omega t + \phi_{\text{ac}} - \text{atan}\left(\frac{\omega L}{R}\right)) \cdot \omega \right] \exp\left(-\frac{R}{L} t\right) \frac{1}{\left(R^2 + \omega^2 L^2\right)^{\frac{1}{2}}} \right]. \]  

(89)

The energy over the first quarter cycle of the voltage is:

\[ E := \int_{\frac{T}{4}}^{\frac{3T}{4}} \left( V_{\text{max}} \sin(\omega \cdot t + \phi_{\text{ac}}) \right) \cdot \left[ \exp\left(-\frac{R}{L} t\right) \left(\frac{-V_{\text{max}}}{\sqrt{R^2 + \omega^2 L^2}} \sin(\phi_{\text{ac}} - \text{atan}\left(\frac{\omega L}{R}\right)) + \frac{V_{\text{max}}}{\sqrt{R^2 + \omega^2 L^2}} \sin(\omega t + \phi_{\text{ac}} - \text{atan}\left(\frac{\omega L}{R}\right))\right)\right] \, dt. \]  

(90)
\[ E_R := \int_0^{0.25\cdot T} R \cdot \left[ \exp\left(\frac{-R}{L} \cdot t\right) \cdot \frac{-V_{\text{max}}}{\sqrt{R^2 + \omega^2 \cdot L^2}} \cdot \sin\left(\phi_{\text{ac}} - \text{atan}\left(\frac{\omega \cdot L}{R}\right)\right) + \frac{V_{\text{max}}}{\sqrt{R^2 + \omega^2 \cdot L^2}} \cdot \sin\left(\omega \cdot t + \phi_{\text{ac}} - \text{atan}\left(\frac{\omega \cdot L}{R}\right)\right) \right] \cdot \sin\left(\omega \cdot t + \phi_{\text{ac}} - \text{atan}\left(\frac{\omega \cdot L}{R}\right)\right) dt \]

(91)

\[ E_L := \int_0^{0.25\cdot T} L \cdot \left[ \frac{R}{L} \cdot \exp\left(\frac{-R}{L} \cdot t\right) \cdot \frac{V_{\text{max}}}{1} \cdot \sin\left(\phi_{\text{ac}} - \text{atan}\left(\frac{\omega \cdot L}{R}\right)\right) + \frac{V_{\text{max}}}{1} \cdot \cos\left(\omega \cdot t + \phi_{\text{ac}} - \text{atan}\left(\frac{\omega \cdot L}{R}\right)\right) \right] \cdot \left(\frac{1}{R^2 + \omega^2 \cdot L^2}\right)^2 dt \]

(92)
Sample AC RL Circuit

We’ll use the problem (16.12) given in Ref. [21], pp. 258-259.

\[ v := 150 \cdot \sin(500 \cdot t + \phi_{ac}) \text{ volts} \quad R := 50 \text{ ohms} \quad L := .2 \text{ henries} \quad \phi_{ac} := 0 \quad \omega := 500 \text{ rad/sec} \]

\[ V_{\text{max}} := 150 \]

Here are all the plots for two periods:

\[ \exp\left(-\frac{R}{L} \cdot t\right) \left( \frac{-V_{\text{max}}}{\sqrt{R^2 + \omega^2 \cdot L^2}} \cdot \sin\left(\phi_{ac} - \text{atan}\left(\frac{\omega \cdot L}{R}\right)\right) + \frac{V_{\text{max}}}{\sqrt{R^2 + \omega^2 \cdot L^2}} \cdot \sin\left(\omega \cdot t + \phi_{ac} - \text{atan}\left(\frac{\omega \cdot L}{R}\right)\right) \right) \]

Figure 18. i for AC LR Circuit
\[ V_{\text{max}} \sin(\omega \cdot t + \phi_{\text{ac}}) \]

\[
\begin{align*}
R \left[ \exp\left(-\frac{R}{L} \cdot t\right) \right] & \left( -\frac{V_{\text{max}}}{\sqrt{R^2 + \omega^2 L^2}} \cdot \sin \left( \phi_{\text{ac}} - \text{atan} \left( \frac{\omega \cdot L}{R} \right) \right) + \frac{V_{\text{max}}}{\sqrt{R^2 + \omega^2 L^2}} \cdot \sin \left( \omega \cdot t + \phi_{\text{ac}} - \text{atan} \left( \frac{\omega \cdot L}{R} \right) \right) \right) \\
L \cdot \left[ \frac{R}{L} \exp\left(-\frac{R}{L} \cdot t\right) \right] & \frac{V_{\text{max}}}{1} \cdot \sin \left( \phi_{\text{ac}} - \text{atan} \left( \frac{\omega \cdot L}{R} \right) \right) + \frac{V_{\text{max}}}{1} \cdot \cos \left( \omega \cdot t + \phi_{\text{ac}} - \text{atan} \left( \frac{\omega \cdot L}{R} \right) \right) \cdot \omega \\
\end{align*}
\]

Figure 19. \( v, v_R, v_L \) for AC LR Circuit
\[(V_{\text{max}} \sin(\omega \cdot t + \phi_{ac})) \left[ \exp\left(\frac{-R}{L} \cdot t\right) \left(\frac{-V_{\text{max}}}{\sqrt{R^2 + \omega^2 L^2}} \cdot \sin\left(\phi_{ac} - \text{atan}\left(\frac{\omega L}{R}\right)\right) + \frac{V_{\text{max}}}{\sqrt{R^2 + \omega^2 L^2}} \cdot \sin\left(\omega \cdot t + \phi_{ac} - \text{atan}\left(\frac{\omega L}{R}\right)\right)\right) \right] \]

\[R \left[ \exp\left(\frac{-R}{L} \cdot t\right) \left(\frac{-V_{\text{max}}}{\sqrt{R^2 + \omega^2 L^2}} \cdot \sin\left(\phi_{ac} - \text{atan}\left(\frac{\omega L}{R}\right)\right) + \frac{V_{\text{max}}}{\sqrt{R^2 + \omega^2 L^2}} \cdot \sin\left(\omega \cdot t + \phi_{ac} - \text{atan}\left(\frac{\omega L}{R}\right)\right)\right)^2 \right] \]

\[L \left[ \frac{R}{L} \cdot \exp\left(\frac{-R}{L} \cdot t\right) \cdot \frac{V_{\text{max}}}{\frac{1}{R^2 + \omega^2 L^2}} \cdot \sin\left(\phi_{ac} - \text{atan}\left(\frac{\omega L}{R}\right)\right) + \frac{V_{\text{max}}}{\frac{1}{R^2 + \omega^2 L^2}} \cdot \cos\left(\omega \cdot t + \phi_{ac} - \text{atan}\left(\frac{\omega L}{R}\right)\right) \cdot \omega \right] \left[ \exp\left(\frac{-R}{L} \cdot t\right) \left(\frac{-V_{\text{max}}}{\sqrt{R^2 + \omega^2 L^2}} \cdot \sin\left(\phi_{ac} - \text{atan}\left(\frac{\omega L}{R}\right)\right) \right) \right] \]

Figure 20. \(p, p_R, p_L\) for AC LR Circuit
\[
E := \int_{0}^{0.25T} \left( V_{\text{max}} \sin(\omega t + \phi_{ac}) \right) \left[ \exp\left(\frac{-R}{L} t\right) \left( \frac{-V_{\text{max}}}{\sqrt{R^2 + \omega^2 L^2}} \sin\left(\phi_{ac} - \text{atan}\left(\frac{\omega L}{R}\right)\right) \right) + \frac{V_{\text{max}}}{\sqrt{R^2 + \omega^2 L^2}} \sin\left(\omega t + \phi_{ac} - \text{atan}\left(\frac{\omega L}{R}\right)\right) \right] \, dt
\]

\[
E = 0.1837 \text{ joules}
\]

\[
E_R := \int_{0}^{0.25T} \left( \exp\left(\frac{-R}{L} t\right) \left( \frac{-V_{\text{max}}}{\sqrt{R^2 + \omega^2 L^2}} \sin\left(\phi_{ac} - \text{atan}\left(\frac{\omega L}{R}\right)\right) \right) + \frac{V_{\text{max}}}{\sqrt{R^2 + \omega^2 L^2}} \sin\left(\omega t + \phi_{ac} - \text{atan}\left(\frac{\omega L}{R}\right)\right) \right]^2 \, dt
\]

\[
E_R = 0.0521 \text{ joules}
\]

\[
E_L := \int_{0}^{0.25T} \left[ \frac{R}{L} \exp\left(\frac{-R}{L} t\right) \left( \frac{V_{\text{max}}}{\sqrt{R^2 + \omega^2 L^2}} \sin\left(\phi_{ac} - \text{atan}\left(\frac{\omega L}{R}\right)\right) \right) + \frac{V_{\text{max}}}{\sqrt{R^2 + \omega^2 L^2}} \cos\left(\omega t + \phi_{ac} - \text{atan}\left(\frac{\omega L}{R}\right)\right) \right]^2 \, dt
\]

\[
E_L = 0.1316 \text{ joules}
\]

\[
E_R + E_L = 0.1837 \text{ in agreement with } E
\]
f. AC RLC Circuit (Fig. 1(f))

The series RLC circuit shown in Fig. 1(f) has a sinusoidal voltage applied when the switch is closed. The resulting equation, according to both conventional theory and the Reciprocal System is

\[ V_{\text{max}} \sin(\omega \cdot t + \phi_{ac}) = R \cdot i + L \cdot \left( \frac{d}{dt} i \right) + \frac{1}{C} \int i \, dt \]  \hspace{1cm} (93a)

\[ V_{\text{max}} \sin(\omega \cdot t + \phi_{ac}) = R \cdot i + L \cdot \left( \frac{d}{dt} i \right) + v_C \]  \hspace{1cm} (93b)

The instantaneous power is

\[ V_{\text{max}} \sin(\omega \cdot t + \phi_{ac}) \cdot i = R \cdot i^2 + L \cdot \left( \frac{d}{dt} i \right) \cdot i + v_C \cdot i \]  \hspace{1cm} (94)

Integrating over the first quarter of the voltage cycle gives us the energy for that period:

\[ \int_{0}^{0.25 \cdot T} V_{\text{max}} \sin(\omega \cdot t + \phi_{ac}) \cdot i \, dt = \int_{0}^{0.25 \cdot T} R \cdot i^2 \, dt + \int_{0}^{0.25 \cdot T} L \cdot \left( \frac{d}{dt} i \right) \cdot i \, dt + \int_{0}^{0.25 \cdot T} v_C \cdot i \, dt \]  \hspace{1cm} (95a)
Multiplying by the instantaneous value of the current, $i$, gives us the power.

$$\int_{0}^{.25T} V_{\text{max}} \sin(\omega t + \phi_{ac}) \cdot i \, dt = \int_{0}^{.25T} R \cdot i^2 \, dt + \int_{0}^{.25T} L \cdot \left( \frac{d}{dt} i \right) \, dt + \frac{\text{conv}iC}{C} \int_{0}^{.25T} i \, dt$$  \hspace{1cm} (95b)$$

We can now replace the capacitor term with $E_C$, which we do know for sure because we can compute the average voltage.

$$\int_{0}^{.25T} V_{\text{max}} \sin(\omega t + \phi_{ac}) \cdot i \, dt = \int_{0}^{.25T} R \cdot i^2 \, dt + \int_{0}^{.25T} L \cdot \left( \frac{d}{dt} i \right) \, dt + E_C$$  \hspace{1cm} (95c)$$

(This also gets rid of our uncertainty in the dynamic value of $v_C$.) For simplicity, we'll assume $\phi_{ac} = 0$ and we will also now assume an effective constant current, $i_{\text{eff}}$. With this, it's easy to transform the remaining terms to the correct energy expressions.

$$\frac{V_{\text{max}}}{\omega} \left[ (1 - \cos(.25 \cdot \omega)) \cdot i_{\text{eff}} \right] = R \cdot i_{\text{eff}}^2 \cdot .25 \cdot T + .5 \cdot L \cdot i_{\text{eff}}^2 + E_C$$  \hspace{1cm} (95d)$$

Solving for $i_{\text{eff}}$, we obtain:
\[
\begin{align*}
    i_{\text{eff}} & := \frac{1}{2 \cdot (R \cdot T \cdot \omega + 2 \cdot L \cdot \omega)} \left[-4 \cdot V_{\text{max}} \cdot \cos \left(\frac{1}{4} \cdot T \cdot \omega\right) + 4 \cdot V_{\text{max}} + 4 \left( V_{\text{max}}^2 \cdot \cos \left(\frac{1}{4} \cdot T \cdot \omega\right)\right)^2 - 2 \cdot V_{\text{max}}^2 \cdot \cos \left(\frac{1}{4} \cdot T \cdot \omega\right) + V_{\text{max}}^2 - 1 \right] \\
\end{align*}
\]

We already know \(v_{\text{avg}}\):

\[
    v_{\text{avg}} := \frac{4 \cdot V_{\text{max}}}{T \cdot \omega} \cdot (1 - \cos(0.25 \cdot T \cdot \omega))
\]

We know \(v_{R_{\text{avg}}}\):

\[
    v_{R_{\text{avg}}} := R \cdot i_{\text{eff}}
\]

\(v_{L_{\text{avg}}}\) is:

\[
    v_{L_{\text{avg}}} := L \cdot \frac{2 \cdot i_{\text{eff}}}{T}
\]

And so:

\[
    v_{C_{\text{avg}}} := v_{\text{avg}} - v_{R_{\text{avg}}} - v_{L_{\text{avg}}}
\]

\[
    E_C := C \cdot V_{C_{\text{avg}}}^2
\]
\[ EC := C \left( v_{avg} - v_{R\_avg} - v_{L\_avg} \right)^2 \]  
\[ (99a) \]

\[ EC := C \left[ \frac{4 \cdot V_{max}}{T \cdot \omega} \cdot (1 - \cos(0.25 \cdot T \cdot \omega)) - R \cdot i_{eff} - L \cdot \frac{2 \cdot i_{eff}}{T} \right]^2 \]
\[ (99b) \]

Putting this expression for \( EC \) in Eq. 96, we get:

\[ i_{eff} = \frac{1}{2 \cdot (R \cdot T \cdot \omega + 2 \cdot L \cdot \omega)} \left[ -4 \cdot V_{max} \cdot \cos \left( \frac{1}{4} \cdot T \cdot \omega \right) + 4 \cdot V_{max} + 4 \cdot V_{max} \cdot \cos \left( \frac{1}{4} \cdot T \cdot \omega \right) \right] - 2 \cdot V_{max} \cdot \cos \left( \frac{1}{4} \cdot T \cdot \omega \right) + V_{max}^2 \]
\[ (99c) \]

Solving \( i_{eff} \), we get:

\[ i_{eff} := \frac{16.0 \cdot C \cdot V_{max} - 16.0 \cdot C \cdot V_{max} \cdot \cos(0.25 \cdot T \cdot \omega)}{\omega \cdot T^2 + 4.0 \cdot C \cdot R \cdot \omega \cdot T + 8.0 \cdot C \cdot L \cdot \omega} \]
\[ (99d) \]
Sample AC RLC Circuit

We'll use the problem (16.42) given in Ref. [21], p 264.

\[ v := 100 \cdot \sin(250 \cdot t + \phi_{ac}) \] volts \quad V_{\text{max}} := 100 \text{ volts} \quad \phi_{ac} := 0 \quad \omega := 250 \text{ rad/sec} \]

\[ R := 5 \text{ ohms} \quad L := .1 \text{ henries} \quad C := 500 \cdot 10^{-6} \text{ farads} \quad T := \frac{2 \cdot \pi}{\omega} \quad T = 0.0251 \text{ sec} \]

First we'll find \( i_{\text{eff}} \) and then the voltages and energies and powers:

\[ i_{\text{eff}} := \frac{16.0 \cdot C \cdot V_{\text{max}} - 16.0 \cdot C \cdot V_{\text{max}} \cdot \cos(0.25 \cdot T \cdot \omega)}{\omega \cdot T^2 + 4.0 \cdot C \cdot R \cdot \omega \cdot T + 8.0 \cdot C \cdot L \cdot \omega} \quad i_{\text{eff}} = 2.4942 \text{ amps} \]

\[ V_{\text{C\_avg}} := \frac{4 \cdot V_{\text{max}}}{T \cdot \omega} \cdot (1 - \cos(0.25 \cdot T \cdot \omega)) - R \cdot i_{\text{eff}} - L \cdot \frac{2 \cdot i_{\text{eff}}}{T} \quad V_{\text{C\_avg}} = 31.3429 \text{ volts} \]

\[ E_{\text{C}} := C \cdot (V_{\text{C\_avg}})^2 \quad E_{\text{C}} = 0.4912 \text{ joules} \]

\[ V_{\text{avg}} := \frac{4 \cdot V_{\text{max}}}{T \cdot \omega} \cdot (1 - \cos(0.25 \cdot T \cdot \omega)) \quad V_{\text{avg}} = 63.662 \text{ volts} \]
\[ v_{R\text{ avg}} := R \cdot i_{\text{eff}} \quad v_{R\text{ avg}} = 12.4709 \text{ volts} \]

\[ v_{L\text{ avg}} := \frac{L \cdot 2 \cdot i_{\text{eff}}}{T} \quad v_{L\text{ avg}} = 19.8481 \]

\[ v_{R\text{ avg}} + v_{L\text{ avg}} + v_{C\text{ avg}} = 63.662 \]

\[ E_L := .5 \cdot L \cdot i_{\text{eff}}^2 \quad E_L = 0.311 \text{ joules} \]

\[ E_R := R \cdot i_{\text{eff}}^2 \cdot .25 \cdot T \quad E_R = 0.1954 \text{ joules} \]

\[ E := v_{\text{avg}} \cdot i_{\text{eff}} \cdot .25 \cdot T \quad E = 0.9977 \]

\[ E_R + E_L + E_C = 0.9977 \]

The voltages and the energies sum correctly.
The average power of this circuit is

\[ p_{\text{avg}} := v_{\text{avg}} \cdot i_{\text{eff}} \quad p_{\text{avg}} = 158.785 \text{ watts} \]

\[ p_{R_{\text{avg}}} := v_{R_{\text{avg}}} \cdot i_{\text{eff}} \quad p_{R_{\text{avg}}} = 31.1049 \text{ watts} \]

\[ p_{L_{\text{avg}}} := v_{L_{\text{avg}}} \cdot i_{\text{eff}} \quad p_{L_{\text{avg}}} = 49.505 \text{ watts} \]

\[ p_{C_{\text{avg}}} := v_{C_{\text{avg}}} \cdot i_{\text{eff}} \quad p_{C_{\text{avg}}} = 78.1751 \text{ watts} \]

\[ p_{R_{\text{avg}}} + p_{L_{\text{avg}}} + p_{C_{\text{avg}}} = 158.785 \text{ watts} \]

The powers sum correctly, of course.

But what is the instantaneous current? To compute that we will first need to compute the instantaneous average values of voltage.

The integral of the product of voltage and current over a quarter cycle is:

\[ \int_{0}^{\frac{25}{T}} v_{\text{max}} \cdot \sin(\omega \cdot t) \cdot i \, dt = \int_{0}^{\frac{25}{T}} R \cdot i^2 \, dt + \int_{0}^{\frac{25}{T}} L \cdot \left( \frac{d i}{dt} \right) \cdot i \, dt + E_{C} \]

\[ v_{\text{avg}} := \frac{v_{\text{max}}}{\omega \cdot t} \cdot (1 - \cos(\omega \cdot t)) \]

\[ v_{R_{\text{avg}}} := R \cdot \frac{i}{2} \]  \hspace{1cm} \text{(approximately)}

\[ \text{(100)} \]

\[ \text{(101)} \]
\[v_{L_{\text{avg}}} := \frac{L \cdot i}{t}\] (approximately) \hspace{1cm} (102)

\[v_{C_{\text{avg}}} := v_{\text{avg}} - v_{R_{\text{avg}}} - v_{L_{\text{avg}}}\] \hspace{1cm} (103)

\[
\int_{0}^{t} V_{\text{max}} \cdot \sin(\omega \cdot t) \cdot i \, dt = \int_{0}^{t} R \cdot i^{2} \, dt + \int_{0}^{t} L \left( \frac{d}{dt} i \right) \cdot i \, dt + C \left[ \frac{V_{\text{max}}}{\omega} \cdot (1 - \cos(\omega \cdot t)) - R \cdot \frac{i}{2} - L \cdot \frac{i}{t} \right]^{2}
\]

Solving for \(i\):

\[
i_{RLC}(V_{\text{max}}, \omega, R, L, C, t) := \frac{4 \cdot \omega^{2} \cdot t^{2} \cdot \left( 8 \cdot C \cdot L \cdot V_{\text{max}} \cdot \omega + 8 \cdot V_{\text{max}} \cdot \omega \cdot t^{2} \cdot \sin \left( \frac{\omega \cdot t}{2} \right) \right)^{2} - 8 \cdot C \cdot L \cdot V_{\text{max}} \cdot \omega \cdot \cos(\omega \cdot t) + 4 \cdot C \cdot R \cdot V_{\text{max}} \cdot \omega \cdot t - 8 \cdot \omega^{2} \cdot t^{2}}{8 \cdot \omega^{2} \cdot t^{2}}
\] \hspace{1cm} (104)
\[ i_{RLC}(V_{\text{max}}, \omega, R, L, C, t) \]
\[ \exp(-25\cdot t) \cdot (5.42 \cdot \cos(139\cdot t) - 1.89 \cdot \sin(139\cdot t)) + 5.65 \cdot \sin(250\cdot t - 73.6\, \text{deg}) \]

Now we’ll use the regular steady-state electrical engineering equations. These work with capacitance having the dimensions s\(^3\)/t.

\[ f := \frac{1}{T} \quad X_C := \frac{1}{2\cdot \pi \cdot f \cdot C} \quad X_C = 8 \, \text{ohms} \quad X_L := 2\cdot \pi \cdot f \cdot L \quad X_L = 25 \, \text{ohms} \]

\[ Z_{RLC} := \sqrt{R^2 + (X_L - X_C)^2} \quad Z_{RLC} = 17.72 \, \text{ohms} \quad \phi_{RLC} := \arctan\left(\frac{X_L - X_C}{R}\right) \quad \phi_{RLC} = 1.2847 \, \text{rad} \]

\[ I := \frac{V_{\text{max}}}{Z_{RLC}} \cdot \sin(2\cdot \pi \cdot f \cdot t - \phi_{RLC}) \quad I_{\text{max}} := \frac{V_{\text{max}}}{Z_{RLC}} \quad I_{\text{max}} = 5.6433 \quad V := Z_{RLC} \cdot I_{\text{max}} \cdot \sin(2\cdot \pi \cdot f \cdot t) \]

The graphs for the various methods of current calculation follow.
The six electrical circuits discussed in this section represent just a tiny fraction of the circuits in use; see Ref. [62] for a catalog of over 3600 circuits!
4. Dielectric Constants and Electric Susceptibilities

a. Elements

As quoted previously, from Ref. [7], Vol. 5, p. 342, "The terms 'permittivity' and 'refractive index' represent merely alternative ways of attaching numerical values to one and the same property of a dielectric medium." The question is: Why? The answer from the Reciprocal System is simple: both photons and (uncharged, massless) electrons move through the atoms and not (merely) through the interstices. The increase in space and time, due to the rotational displacements of the atoms, is the cause for both permittivity and refraction. The space-time factor is the same as that used in the electrical resistivity calculations, Ref. [53], and that used above in the index of refraction calculations. Here's the equation, repeated (from the note) for convenience:

$$n := 1 + t_r$$

$$t_r := \frac{d_{cgs}}{d_{u_{cgs}}} \left[ 0.9317 \cdot k_G \left( \frac{t_p^2 \cdot t_s}{t_{e_{mod}}} \right)^{\frac{2}{3}} \left( 1 - kr_0 \cdot \frac{1}{g} \right) \right] \left( 1 + \frac{1}{\lambda _{0\_n} \cdot F_{\nu}} \right)$$

(105a)

For noble elements—those without rotational electric displacement—the equation is obviously modified to

$$n := 1 + t_r$$

$$t_r := \frac{d_{cgs}}{d_{u_{cgs}}} \left[ 0.9317 \cdot k_G \left( \frac{t_p^2 \cdot t_s}{t_{e_{mod}}} \right)^{\frac{2}{3}} \left( 1 - kr_0 \cdot \frac{1}{g} \right) \right] \left( 1 + \frac{1}{\lambda _{0\_n} \cdot F_{\nu}} \right)$$

(105b)

(Of course, if the noble element has one or more gravitational charges—the cause for the different isotopes—then there would be an effective value of $t_{e_{mod}}$, and so Eq. (105a) would then be used.) Eq. (11c) gives the relative permittivity, or dielectric constant, if losses are excluded and if magnetic permeability can be neglected,
\[ \varepsilon_r := n^2 \]

(Ref. [52] uses the symbol K for this constant. Both symbols are common in the literature.) The electric susceptibility is, by definition, then

\[ \chi_e := \varepsilon_r - 1 \]  

Notice that our definition has nothing to do with "free" or "bound" charges as in conventional theory. The relative permittivities or dielectric constants and electric susceptibilities are usually used in regard to capacitor materials, rather than with, say, Van De Graaf generators, where we are dealing with actual electric charges. Therefore, the calculations in this section are specifically for use in describing ordinary dielectrics.

For static or low frequency calculations, we will set \( F \ell_0 / n \) to zero, otherwise we will assume that the to-and-fro cycling of an electron in the dielectric is physically similar to the transverse oscillation of a photon and set \( \ell_0/ n \) accordingly. Conventional theory makes this same assumption tacitly, right or wrong. Regardless, most of the frequency-dependent part of the dielectric constant is due to the loss term, which we are neglecting here.

Our method of attack will be to calculate the index of refraction and then the relative permittivity or dielectric constant and compare with observed data from Ref. [14], which is probably the most authoritative of the material references given at the end of this paper. We will also tabulate the electric susceptibility. For the elements, with the exception of Te, we don't have data for the different crystalline directions, so there would be no point in tabulating anisotropic values other than for Te. Differences due to physical state, solid or liquid, are not great, because the statistical average of the geometric coefficient, \( k_G \), for the liquid is close to that of the solid, as would be expected. Gases are a separate issue, and will have to be left for later treatment. The following is the table of calculations, from *Excel*:
Table II. Dielectric Constants and Electrical Susceptibilities of the Elements (No Loss)
Table Notes:

1) C, N, O, F all have values of $t_{e_{\text{mod}}} = -4$ apparently for the same reason that they have equal rotational speeds for the determination of interatomic distance, 3-3-10 or 3-3-1: the additional equivalent electric time displacement units do not change the result. Ref. [1], p.22.

2) The non-static calculations have negligible differences with the static calculations--but this is because we are neglecting the dielectric loss, which is the main frequency dependent term.

3) The only anisotropic calculation is for Te, where the ratio of the cube of the a-edge to the c-edge has been taken to multiply the normal $k_G$ value to calculate the value perpendicular to the c-edge.

4) For the resistivity calculation (and for the calculation given in Table I above) of Te, we use -6 for $t_{e_{\text{mod}}}$, whereas here we have to use -9. But the empirical data for n given in Ref. [16] and Ref. [14] do not agree either! Also, the same Ref. [14], p. 92, says that the dielectric constant of Si is 11.7, but on p. 830, it says that it's 11.0-12.0. Our calculation for Si, in Table II, gives 10.93.

5) The correlation between the calculated and observed values of $\varepsilon_r$ is very high, but obviously more work needs to be done to explain the rather wide deviations between $t_{e_{\text{mod}}}$ and $t_e$; most of this can be explained as isotopic differences, but probably not all. Also, the densities used in the calculations may be somewhat different from the actual densities used in the experiments or observations.

6) Se can be used both as a conductor and as a dielectric: for the resistivity calculations given in Ref. [53] we used the electropositive form of Se's rotational displacement; here, for the dielectric calculation, we use the electronegative form. This gives us confidence that we're on the right track.

7) On the other hand, we have found it necessary to use the electropositive form of the rotational displacement for Cl and Ar here. Clearly, more theoretical work and experimental work are necessary. Further, the loss calculations are too uncertain, at this time, to be included here.
b. Compounds

Chapter 4.4, "Dielectrics and Electrooptics," in Ref. [14] is, as stated above, probably the most authoritative reference for data on dielectric compounds, so that's what we'll use here for comparing our calculations with observations of the static dielectric constants (but using the sodium D-line to determine \( n \)), as well as Ref. [16], pp.10-246 to 10-249 for the index of refraction. A sampling of calculations follows; the Reciprocal System DataBase will have the calculations for all appropriate compounds.

**SiO\(_2\)**

\[
\begin{align*}
\text{Si:} & \quad t_p_{\text{Si}} := 3 \quad t_s_{\text{Si}} := 2 \quad t_{\text{e\_mod\_Si}} := -4 \quad t_{\text{eff\_Si}} := \left[ \frac{t_{\text{e\_mod\_Si}}}{\left(1 - t_{\text{mod\_Si}} \cdot \frac{1}{9}\right)} \right]^{\frac{1}{3}} \quad t_{\text{eff\_Si}} = 2.3295 \\
\text{kr}_{0\_\text{Si}} := 2
\end{align*}
\]

\[
\begin{align*}
\text{kr}_{\text{Si}} := \frac{1}{t_{\text{eff\_Si}}} \left(1 - \text{kr}_{0\_\text{Si}} \cdot \frac{1}{9}\right) \quad \text{kr}_{\text{Si}} = 0.3339 \\
\text{na}_{\text{Si}} := 1 \\
\text{Z}_{\text{atno\_Si}} := 14
\end{align*}
\]

**O:**

\[
\begin{align*}
\text{O:} & \quad t_p_{\text{O}} := 2 \quad t_s_{\text{O}} := 2 \quad t_{\text{e\_mod\_O}} := -2 \quad t_{\text{eff\_O}} := \left[ \frac{t_{\text{e\_mod\_O}}}{\left(1 - t_{\text{mod\_O}} \cdot \frac{1}{9}\right)} \right]^{\frac{1}{3}} \quad t_{\text{eff\_O}} = 1 \\
\text{kr}_{0\_\text{O}} := 2 \quad \text{(same as in H}_2\text{O)}
\end{align*}
\]

\[
\begin{align*}
\text{kr}_{\text{O}} := \frac{1}{t_{\text{eff\_O}}} \left(1 - \text{kr}_{0\_\text{O}} \cdot \frac{1}{9}\right) \quad \text{kr}_{\text{O}} = 0.7778 \\
\text{na}_{\text{O}} := 2 \\
\text{Z}_{\text{atno\_O}} := 8
\end{align*}
\]
\[
kr_{SiO2} := \frac{n_{a_Si}\cdot Z_{atno_Si}\cdot kr_{Si} + n_{a_O}\cdot Z_{atno_O}\cdot kr_O}{n_{a_Si}\cdot Z_{atno_Si} + n_{a_O}\cdot Z_{atno_O}} \quad kr_{SiO2} = 0.5706
\]

\[d_{SiO2} := 2.648 \quad \text{g/cm}^3 \quad \text{(from the Reciprocal System Data Base, for tridymite, to match the value from Ref. [14])}\]

Now we come to the difficult issue of the calculation of \(k_G\). Ref. [14] apparently uses the c-edge of the hexagonal unit cell here, and there are 3 molecules for this kind of SiO\(_2\) per unit cell as determined by its density.

\[V_{uc_{SiO2}} := 112.979 \quad \text{A}^3 \quad Z_{uc_{SiO2}} := 3 \quad \text{edge}_c_{SiO2} := 5.4046 \quad \text{A}\]

\[k_{G_{SiO2}} := \frac{V_{uc_{SiO2}}}{\left(\text{edge}_c_{SiO2}\right)^3 \cdot Z_{uc_{SiO2}}} \quad k_{G_{SiO2}} = 0.2386 \quad \frac{F_v}{\lambda_0\cdot n} = 0.058\]

\[n_{SiO2} := \frac{d_{SiO2}}{d_{u\cdot cg}} \left[0.9317 \cdot k_{G_{SiO2}} \cdot kr_{SiO2} \cdot \left(1 + \frac{1}{\lambda_0\cdot n} \cdot F_v\right)\right] + 1 \quad n_{SiO2} = 1.3553\]

\[\varepsilon_{r_{SiO2}} := n_{SiO2}^2 \quad \varepsilon_{r_{SiO2}} = 1.8369 \quad \chi_{e_{SiO2}} := \varepsilon_{r_{SiO2}} - 1 \quad \chi_{e_{SiO2}} = 0.8369\]
Ref. [14] says that the static \( \varepsilon_{r,\text{SiO}_2} = 3.5 \) (for the principal 11 direction) and that \( n_{\text{SiO}_2} \) (at .5461 \( \mu \)m wavelength, not too different from .5893 \( \mu \)m) is 1.46. But \( 1.46^2 = 2.1316 \). So even using the experimental value of \( n_{\text{SiO}_2} \) doesn't get us to the value of \( \varepsilon_{r,\text{SiO}_2} \) given in Ref. [14]. If we use the value of \( \varepsilon_{r,\text{SiO}_2} = 3.5 \) from Ref. [14], then \( n_{\text{SiO}_2} \) would equal 1.8708. As shown in the table below, \( n \) calculated for edge_a and edge_b is 1.473, close to the observed; the average for the three principle axes is 1.44, very close to that observed.

**CaF\(_2\)**

Ca: \( 3\text{-}2\text{-}2 \quad t_{p,\text{Ca}} := 3 \quad t_{s,\text{Ca}} := 2 \quad t_{e,\text{mod,Ca}} := 2 \quad t_{\text{eff,Ca}} := \left[ \frac{t_{e,\text{mod,Ca}}}{\left( t_{p,\text{Ca}}^2 \cdot t_{s,\text{Ca}} \right)^{\frac{1}{3}}} \right]^2 \quad t_{\text{eff,Ca}} = 0.5824 \)

\( k_{r,\text{Ca}} := 2 \)

\( k_{r,\text{Ca}} := \frac{1}{t_{\text{eff,Ca}}} \left( 1 - k_{r,\text{Ca}} \right)^{\frac{1}{9}} \quad k_{r,\text{Ca}} = 1.3355 \quad n_{a,\text{Ca}} := 1 \quad Z_{\text{atno,Ca}} := 20 \)

F: \( 2\text{-}2\text{-}(1) \quad t_{p,F} := 2 \quad t_{s,F} := 2 \quad t_{e,\text{mod,F}} := -4 \quad t_{\text{eff,F}} := \left[ \frac{t_{e,\text{mod,F}}}{\left( t_{p,F}^2 \cdot t_{s,F} \right)^{\frac{1}{3}}} \right]^2 \quad t_{\text{eff,F}} = 4 \)
\[ k_{r0_F} := 2 \]

\[ k_r_F := \frac{1}{t_{\text{eff}_F}} \left( 1 - k_{r0_F} \frac{1}{9} \right) \quad k_r_F = 0.1944 \quad n_{a_F} := 2 \quad Z_{atno_F} := 9 \]

\[ k_{r_{\text{CaF2}}} := \frac{n_{a_{Ca}} \cdot Z_{atno_{Ca}} \cdot k_{r_{Ca}} + n_{a_F} \cdot Z_{atno_F} \cdot k_{r_F}}{n_{a_{Ca}} \cdot Z_{atno_{Ca}} + n_{a_F} \cdot Z_{atno_F}} \quad k_{r_{\text{CaF2}}} = 0.795 \]

\[ d_{\text{CaF2}} := 3.179 \text{ g/cm}^3 \quad \text{(from the Reciprocal System Data Base; this matches the value from Ref. [14])} \]

The crystal unit cell is cubic, with 5.460 Å per edge. There are 4 molecules per cell, and the volume is 163.035 Å³.

\[ V_{uc_{\text{CaF2}}} := 163.035 \text{ Å}^3 \quad Z_{uc_{\text{CaF2}}} := 4 \quad \text{edge}_{\text{CaF2}} := 5.4630 \text{ Å} \]

\[ k_{G_{\text{CaF2}}} := \frac{V_{uc_{\text{CaF2}}}}{(\text{edge}_{\text{CaF2}})^{3} \cdot Z_{uc_{\text{CaF2}}}} \quad k_{G_{\text{CaF2}}} = 0.25 \]

\[ n_{\text{CaF2}} := \frac{d_{\text{CaF2}}}{d_{u_{cgs}}} \left[ 0.9317 \cdot k_{G_{\text{CaF2}}} \cdot k_{r_{\text{CaF2}}} \left( 1 + \frac{1}{\lambda_{0_n} \cdot F_v} \right) \right] + 1 \]

\[ n_{\text{CaF2}} = 1.6228 \]

\[ \varepsilon_{r_{\text{CaF2}}} := n_{\text{CaF2}}^2 \quad \varepsilon_{r_{\text{CaF2}}} = 2.6336 \quad \chi_{e_{\text{CaF2}}} := \varepsilon_{r_{\text{CaF2}}} - 1 \quad \chi_{e_{\text{CaF2}}} = 1.6336 \]
Ref. [14] says the dielectric constant is 7.4, considerably higher than our calculation. But, both Ref. [14] and Ref. [16] say that the index of refraction is 1.4388, which is somewhat lower than with our calculation. So: no conclusion can be drawn here.

**MgO**

Mg: 2-2-2  \( t_{p\_Mg} := 2 \)  \( t_{s\_Mg} := 2 \)  \( t_{e\_mod\_Mg} := 2 \)  \( t_{eff\_Mg} := \left[ \frac{t_{e\_mod\_Mg}}{1} \right]^{2} \)  \( t_{eff\_Mg} = 1 \)

\( k_{r0\_Mg} := 0 \)

\( k_{r\_Mg} := \frac{1}{t_{eff\_Mg}} \left( 1 - k_{r0\_Mg} \right) \)  \( k_{r\_Mg} = 1 \)  \( n_{a\_Mg} := 1 \)  \( Z_{atno\_Mg} := 12 \)

O: 2-2-(2)  \( t_{p\_O} := 2 \)  \( t_{s\_O} := 2 \)  \( t_{e\_mod\_O} := -4 \)  \( t_{eff\_O} := \left[ \frac{t_{e\_mod\_O}}{1} \right]^{2} \)  \( t_{eff\_O} = 4 \)

\( k_{r0\_O} := 0 \)

\( k_{r\_O} := \frac{1}{t_{eff\_O}} \left( 1 - k_{r0\_O} \right) \)  \( k_{r\_O} = 0.25 \)  \( n_{a\_O} := 1 \)  \( Z_{atno\_O} := 8 \)

\( k_{r\_MgO} := \frac{n_{a\_Mg} \cdot Z_{atno\_Mg} \cdot k_{r\_Mg} + n_{a\_O} \cdot Z_{atno\_O} \cdot k_{r\_O}}{n_{a\_Mg} \cdot Z_{atno\_Mg} + n_{a\_O} \cdot Z_{atno\_O}} \)  \( k_{r\_MgO} = 0.7 \)
\( d_{\text{MgO}} := 3.578 \text{ g/cm}^3 \) (from the Reciprocal System Data Base)

MgO is cubic, with an edge length of 4.2130 Å, and 4 molecules per cell.

\[ \nu_{\text{uc_MgO}} := 74.778 \text{ Å}^3 \quad Z_{\text{uc_MgO}} := 4 \quad \text{edge}_{\text{MgO}} := 4.2130 \text{ Å} \]

\[ k_{\text{G_MgO}} := \frac{\nu_{\text{uc_MgO}}}{(\text{edge}_{\text{MgO}})^3 \cdot Z_{\text{uc_MgO}}} \quad k_{\text{G_MgO}} = 0.25 \]

\[ n_{\text{MgO}} := \frac{d_{\text{MgO}}}{d_{\text{u_cgs}} \left[ 0.9317 \cdot k_{\text{G_MgO}} \cdot k_{\text{r_MgO}} \left( 1 + \frac{1}{\lambda_0 \cdot n} \cdot F_\nu \right) \right] + 1} \]

\( n_{\text{MgO}} = 1.6172 \)

\[ \varepsilon_{\text{r_MgO}} := n_{\text{MgO}}^2 \quad \varepsilon_{\text{r_MgO}} = 2.6155 \quad \chi_{\text{e_MgO}} := \varepsilon_{\text{r_MgO}} - 1 \quad \chi_{\text{e_MgO}} = 1.6155 \]

Ref. [14] says that \( \varepsilon_r = 9.7 \), which is way higher than our calculation. But: it also says that the refractive index is 1.7217, which is a bit higher than our calculation.
KCl

K: 3-2-1 \( t_p_K := 3 \), \( t_s_K := 2 \), \( t_{e\text{mod}}_K := 3 \)

\[ t_{eff}_K := \left[ \frac{t_{e\text{mod}}_K}{t_p_K \cdot t_s_K} \right]^2 \]

\[ t_{eff}_K = 1.3104 \]

\[ k_{r0}_K := 0 \]

\[ k_{r}_K := \frac{1}{t_{eff}_K} \left( 1 - k_{r0}_K \right) \]

\[ k_r_K = 0.7631 \]

\[ n_a_K := 1 \]

\[ Z_{atno}_K := 19 \]

Cl: 3-2-(1) \( t_p_Cl := 3 \), \( t_s_Cl := 2 \), \( t_{e\text{mod}}_Cl := -3 \)

\[ t_{eff}_Cl := \left[ \frac{t_{e\text{mod}}_Cl}{t_p_Cl \cdot t_s_Cl} \right]^2 \]

\[ t_{eff}_Cl = 1.3104 \]

\[ k_{r0}_Cl := 0 \]

\[ k_{r}_Cl := \frac{1}{t_{eff}_Cl} \left( 1 - k_{r0}_Cl \right) \]

\[ k_r_Cl = 0.7631 \]

\[ n_a_Cl := 1 \]

\[ Z_{atno}_Cl := 17 \]

\[ k_{r}_KCl := \frac{n_a_K \cdot Z_{atno}_K \cdot k_r_K + n_a_Cl \cdot Z_{atno}_Cl \cdot k_r_Cl}{n_a_K \cdot Z_{atno}_K + n_a_Cl \cdot Z_{atno}_Cl} \]

\[ k_{r}_KCl = 0.7631 \]

\[ d_{KCl} := 1.986 \text{ g/cm}^3 \]

(from the Reciprocal System Data Base)
KCl is cubic, with an edge length of 6.2929 Å, and 4 molecules per cell.

\[
V_{uc\_KCl} := 249.203 \text{ Å}^3 \quad Z_{uc\_KCl} := 4 \quad \text{edge}_{KCl} := 6.2929 \text{ Å}
\]

\[
k_{G\_KCl} := \frac{V_{uc\_KCl}}{\left(\text{edge}_{KCl}\right)^3 \cdot Z_{uc\_KCl}} \quad k_{G\_KCl} = 0.25
\]

\[
n_{KCl} := \frac{d_{KCl}}{d_{u\_cgs}} \left[.9317 \cdot k_{G\_KCl} \cdot k_{r\_KCl} \cdot \left(1 + \frac{1}{\chi_{0\_n}} \cdot F_{\nu}\right)\right] + 1
\]

\[
n_{KCl} = 1.3735
\]

\[
\varepsilon_{r\_KCl} := n_{KCl}^2 \quad \varepsilon_{r\_KCl} = 1.8865 \quad \chi_{e\_KCl} := \varepsilon_{r\_MgO} - 1 \quad \chi_{e\_KCl} = 1.6155
\]

Ref. [14] says that \( n \) is 1.4792 and \( \varepsilon_r \) is 4.6. Again, not too close to our calculations. Tables, from Excel, follow for the major binary compounds. Included are those with cubic, hexagonal, and tetragonal unit cells.
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\[(\varepsilon_{r_a}) \ (\varepsilon_{r_b}) \ (\varepsilon_{r_c})\]

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Table III. Dielectric Constants and Electrical Susceptibilities of Selected Compounds (No Loss)
The average of the ratios of observed to calculated values of the index of refraction for the c_edge and a_edge is

\[
\frac{0.9127 + 1.0780}{2} = 0.9954
\]

which seems to verify our calculations. The same value of F_r/\lambda_0 is used in each calculation (.058), and in most cases the value of k_r_0 for both elements of the compound is 2. It's important to keep in mind that we are using the edges of the unit cell to calculate the principal values of the geometric factor k_G; differences between observed and calculated values of the index of refraction and, hence, the dielectric constants and electric susceptibilities, may be due to slight inaccuracies in these lengths.

The experimental values of the dielectric constants are much more uncertain than those for the index of refraction. A table of observed values follow, from Excel, with the calculation of the effective exponents of the indexes of refraction to get the dielectric constants.

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Table IV. Calculation of Effective Exponent Relating n and \( \varepsilon_r \)
It's curious that the average *effective experimental exponent* comes out to 3.2822, when it should be precisely 2! Of course, some of the discrepancy may be due to the use of different principal directions, but still.... Again, more work is needed here.

c. Dielectric Strength

If, due to very high voltages, the dielectric breaks down, what seems to happen is this: the ("free") electrons become *negatively charged* and the atoms become *positively charged*. The electrons repel themselves to the surface of the dielectric, and the positively-charged atoms force themselves apart, rupturing the solid. Of course, this simple answer from the Reciprocal System cannot be used in conventional theory, because there the electrons and atoms are *already* charged! (Note: Some of the surface atoms become neutralized, again, from the charged electrons coming from within the solid, and also some of the newly-charged electrons leave the surface.)

Unfortunately, there do not exist experimental values of dielectric strength for most of the elements and compounds discussed above. However, Ref. [16], p. 15-44, has a table which includes NaCl, so we'll use this compound for a tentative, sample calculation.

From Eqs. (9a) and (9b) of Ref. [48] the energy required to ionize an atom-electron pair is

\[
E_{\text{I}_{\text{atom-elec}}} := 2 \cdot \hbar \cdot \frac{R}{2 \cdot \pi} \cdot \sqrt{\frac{c}{v_{\text{mag}}}} \quad \text{or} \quad E_{\text{I}_{\text{atom-elec}}} := 2 \cdot \hbar \cdot \frac{R}{2 \cdot \pi} \cdot \sqrt{\frac{c}{v_{\text{elec}}} - 1}
\]

(eV/atom-elec. (107))

(\text{where } R/2\pi \text{ is the electric rotational vibrational frequency of the charged electron; the factor 2 is due to the 2 charges created). For NaCl, the Na atoms become positively-charged, and the single "free" electron (not bound in the Reciprocal System!) becomes negatively-charged. Nothing happens to the Cl atoms, because they (ordinarily) cannot take a positive charge. For Na, } c/\sqrt{v_{\text{elec}}} = 2 \text{ (because } t_e = 1\text{), so}
\[ E_{I_{\text{Na}}_{\text{elec}}} := 4.34 \cdot \sqrt{2} - 1 \quad E_{I_{\text{Na}}_{\text{elec}}} = 4.34 \quad \text{eV/atom-elec. pair} \]

(Higher values of work function, and thus of ionization energy, are possible, though less probable.) Ref. [63], p. 537, does say that "Dielectric breakdown is caused by an enormous increase in the number of charge carriers...." Charged electrons can move through both space and time; there is less resistance in moving through the interstices than through the atoms, so these charged electrons make their way to the surface. The first group neutralizes the surface atoms; this is followed by a second group of electrons, which should make the surface, if not the whole solid, negative. The positively charged atoms now move apart, rupturing the solid.

According to the Reciprocal System Data Base, the separation of the Na atoms (at room temperature) is

\[ s_{\text{Na}} := 3.9882 \cdot 10^{-10} \, \text{m} \]

The Coulombic potential energy of two positively-charged atoms is (from Ref. [64], p. 551)

\[ U_{\text{ionic}_{\text{Na}}_{\text{pair}}} := \frac{Q_{1_{\text{C}}} \cdot Q_{2_{\text{C}}}}{4 \cdot \pi \cdot \varepsilon_0 \cdot \text{SI} \cdot s_{\text{Na}}} \quad U_{\text{ionic}_{\text{Na}}_{\text{pair}}} = 5.7867 \times 10^{-19} \quad \text{J} \quad (109) \]

For one gram-mole, there is one-half of Avogadro's number of pairs of atoms,

\[ U_{\text{ionic}_{\text{Na}}_{\text{mole}}} := U_{\text{ionic}_{\text{Na}}_{\text{pair}}} \cdot \frac{\text{Av}}{2} \cdot \frac{1}{1000} \quad U_{\text{ionic}_{\text{Na}}_{\text{mole}}} = 174.3193 \quad \text{kJ/mole} \quad (110) \]
Also, according to the Reciprocal System Data Base, the cohesive energy of solid NaCl is

\[ U_{\text{cohes}_{\text{NaCl}}_{\text{mole}}} := 184.558 \text{ kJ/mole} \]

The energy per mole to create the positive charges is half that of the ionization energy:

\[ E_{I_{\text{pos}}_{\text{mole}}} := \frac{4.34}{2} \times 1.602 \times 10^{-19} \times \frac{\text{Av}}{1000} \quad E_{I_{\text{pos}}_{\text{mole}}} = 209.4446 \text{ kJ/mole} \quad (111) \]

So we have

174.3 kJ/mole vs. 184.6 kJ/mole vs. 209.4 kJ/mole

One might think that it would be necessary for these three values to be exactly equal. Not so. Undoubtedly the energy applied to the dielectric would be uneven, so that in some "weak areas" of the solid, the Coulombic energy would be greater than the cohesive energy and the substance would rupture there and spread--hence causing the interesting patterns ("Lichtenberg figures") photographed in Von Hippel's books.

Further theoretical work and experimental work are certainly needed here, but it's time to move on to the Reciprocal System theory of magnetism.
5. Diamagnets and Magnetic Susceptibilities

a. Elements

In conventional theoretical physics, the charged electrons in an ordinary capacitor make it "polarized." In the Reciprocal System, the electrons in an ordinary capacitor are uncharged, and therefore the capacitor is not "polarized"; electric permittivity is due to the added space and time of the atoms of the dielectric. But in diamagnets and paramagnets, the theories switch sides. Here, conventional theory says that magnetic charges and poles do not really exist; ultimately it's the Coulombic electric charges of the moving electrons that are the cause of all magnetic phenomena, including that involved in magnetostatics. In the Reciprocal System, there are actual magnetic charges and these account for all magnetic phenomena, except, of course, for electromagnetism (which results from the motion of uncharged or charged electrons or other subatoms). Chapter 21 of Ref. [1] discusses electromagnetism in the Reciprocal System quite thoroughly and so there need be no further discussion of that topic here. The focus now will be on the calculation of the magnetic susceptibility of the crystalline elements and compounds, something that has not been done before in the Reciprocal System. (Larson did some work on the magnetic susceptibility of liquid organic compounds, Ref. [1], pp. 247-249.)

Each atom in the Reciprocal System has two orthogonal rotational systems. When magnetically charged, an atom takes two charges, one on each rotational system. A magnetic charge is a two-dimensional rotational vibration, whereas an electric charge is a one-dimensional rotational vibration. From Ref. [47], the physical "zero" of the rotational vibration frequency is

\[
R := \frac{R_{\nu}}{2}
\]

(here R means Rydberg frequency, not resistance)

\[
\nu_{M\_0} := \frac{R}{\pi} \text{ cycles/sec}
\]

(112)

The magnetic charge is a space displacement, hence the value of the frequency increases with each unit increase of charge:

\[
\nu_{M\_n} := \left(n_{M} + 1\right) \frac{R}{\pi} \text{ cycles/sec}
\]

(113a)
where \( n_M \) is the number of charges (same for both rotational systems). Properly speaking, what we have is a "multiple-charge" unit, rather than separate charges per se. The possible range for \( n_M \) is

\[
0 \leq n_M \leq t_p
\]  

(114)

where \( t_p \) is the principal magnetic rotational displacement of the atom. Charge saturation occurs when

\[
n_{M_{\text{sat}}} = t_p
\]  

(115)

One of the two magnetic charges on an atom may be designated the "north pole" or \( N \); the other charge would then be the "south pole" or \( S \). Monopoles do not exist for atoms and have not been observed. Subatoms, with one rotational system, may take just one charge, so it's possible to have a "monocharge." But a two-dimensional rotational vibration appears clockwise from one direction and counterclockwise from the other direction, so even here it would probably not be correct to speak of a "monopole."

Ref. [48] provides an expression for the photomagnetization energy of magnetic charges; here it is:

\[
h \cdot \nu_{\text{phot}} := h \cdot \nu_{M_n} \frac{v_{\text{mag}}}{c} \frac{B}{B_{\text{nat SI}}}
\]  

(116)

where \( v_{\text{mag}} \) is the magnetic rotational displacement (a pure number) and the other symbols have their usual meanings. For the science of magnetic materials, the most important matter property is the magnetic susceptibility. Our focus here will be on crystalline matter, rather than gases, so no further discussion of photomagnetization will be considered presently.

In what follows we will use the natural units of the Reciprocal System, as well as cgs and SI. Unfortunately, there is considerable confusion in magnetic units and dimensions in both cgs and SI. Jiles, Ref. [30], pp. 15, 40, 4, explains:
"The cgs and SI systems of magnetic units have different philosophies. The cgs system took an approach based on magnetostatics and the concept of the 'magnetic pole', while the SI system takes an electrodynamic approach to magnetism based on electric currents.

"As far as is known, neither magnetic poles nor bound currents are real in a physical sense. Instead they are both merely mathematical artifacts, or approximations, which allow calculation of magnetic fields and magnetic moments in a wide range of different situations.

"It can be shown that the description of fields due to magnetic poles or Amperian currents are mathematically equivalent so long as the calculation is made at field points which are sufficiently distant from the poles or the current sources. Therefore in general neither approach is better or worse than the other...the two formalisms are mathematically equivalent at points far from the field source."

But the magnetic charges in the Reciprocal System are physically real, and so, as stated in a previous section of this paper, the Reciprocal System is closer to the cgs system than to SI. Unfortunately, however, the H vector (the so-called "magnetic field intensity") in cgs has different dimensions than the H vector in SI. Cgs uses the same dimensions for H and B, but uses different units (oersted and gauss). (Larson discusses the issue of the H vector on pp. 224, ff., in Ref. [1].) For our purposes, it seems best to avoid H altogether and use B, the magnetic flux density, in all the equations which follow.

The internal magnetic flux density of a material is related to the external magnetic flux density by the relative permeability, \( \mu_r \):

\[
\frac{B_{\text{int}}}{B_{\text{nat\_SI}}} := \mu_r \frac{B_{\text{ext}}}{B_{\text{nat\_SI}}} \quad (\mu_r = \mu) \quad \text{(Reciprocal System)} \quad (117a)
\]

\[
B_{\text{int}} := \mu B_{\text{ext}} \quad (\mu_r = \mu) \quad \text{(cgs)} \quad (117b)
\]

(although, technically, \( \mu_r \) is not defined or used in cgs)

\[
B_{\text{int}} := \mu_r B_{\text{ext}} \quad (\mu_r := \frac{\mu}{\mu_0\_\text{SI}}) \quad \text{(SI)} \quad (117c)
\]
The relative permeability (simple permeability in cgs) is defined, as follows, in the three systems:

\[
\mu_r := 1 + \chi^r \quad \text{(Reciprocal System, dimensionless)} \quad (118a)
\]

\[
\mu_{\text{cgs}} := 1 + 4 \cdot \pi \cdot \chi_{\text{cgs}} \quad \text{(cgs, dimensionless)} \quad (118b)
\]

\[
\mu_{r\_\text{SI}} := 1 + \chi_{\text{SI}} \quad \text{(SI, dimensionless)} \quad (118c)
\]

From the above, it's seen that

\[
\chi_{\text{SI}} := 4 \cdot \pi \cdot \chi_{\text{cgs}} \quad (119)
\]

Unfortunately, the values of \( \chi \) listed in the data compilations are not the pure numbers given above. They are either mass values or molar values, computed as follows:

\[
\chi_{\text{cgs\_mass}} := \frac{\chi_{\text{cgs}}}{\rho_{\text{cgs}}} \quad \text{cm}^3/\text{g} \quad (120a)
\]

\[
\chi_{\text{SI\_mass}} := \frac{\chi_{\text{SI}}}{\rho_{\text{SI}}} \quad \text{m}^3/\text{kg} \quad (120b)
\]
\[ \chi_{\text{cgs mol}} := \chi_{\text{cgs}} \cdot \frac{V_{\text{uc}}}{Z_{\text{uc}}} \quad \text{cm}^3/\text{mol} \quad \text{(all in cgs)} \]  

(121a)

\[ \chi_{\text{SI mol}} := \chi_{\text{SI}} \cdot \frac{V_{\text{uc}}}{Z_{\text{uc}}} \quad \text{m}^3/\text{mol} \quad \text{(all in SI)} \]  

(121b)

where \( Av \) is Avogadro's number, \( V_{\text{uc}} \) is the volume of the crystal unit cell and \( Z_{\text{uc}} \) is the number of atoms in a unit cell. The CRC tabulation (Ref. [16], pp. 4-142 to 4-47) is given in \( \text{cm}^3/\text{mol} \) and so must be multiplied by \( 4\pi \) to obtain SI-defined values (but can still be expressed in cgs units). The Springer tabulation (Ref. [14], pp. 54 to 158 for the elements, lists both the mass and mole values using cgs units but with the SI definition. For Table V, which follows, the observed values of magnetic susceptibilities will be taken to be the mean value of the pure (dimensionless) values calculated from the CRC and Springer tabulations.

\[ \chi_{\text{obs Springer}} := \chi_{\text{cgs mass}} \cdot \rho_{\text{cgs}} \quad \text{(but using SI definition of } \chi, \text{ already in the Springer tabulation)} \]

\[ \chi_{\text{obs CRC}} := \chi_{\text{cgs mol}} \cdot \frac{Z_{\text{uc}}}{Av \cdot V_{\text{uc}}} \cdot 4 \cdot \pi \quad \text{(using SI definition of } \chi \text{ by multiplying CRC values by } 4\pi) \]

\[ \chi_{\text{obs}} := .5 \left( \chi_{\text{obs Springer}} + \chi_{\text{obs CRC}} \right) \quad \text{(SI and Reciprocal System definition of } \chi, \text{ dimensionless)} \]
Magnetic susceptibility is defined as the sum of the magnetic dipole moments induced per unit volume per unit external magnetic flux density. The Reciprocal System uses the Kennelly expressions for "pole strength" and "dipole moment." In SI units, the dipole moment is therefore expressed as

\[ \mu_{\text{dpm}} := n_M \cdot M_{\text{u\_weber}} \cdot \text{edge}_{\text{uc}} \] \text{weber-meter} \tag{122} \]

(\(\mu_{\text{dpm}}\) should not be confused with \(\mu_r\).) \(M_{\text{u\_weber}}\) is the Reciprocal System value of the unit magnetic charge and \(\text{edge}_{\text{uc}}\) is the length of the appropriate edge of the crystal unit cell. In non-isotropic cells, the c-edge is longest and therefore is favored in diamagnets—the greater the dipole moment, the greater the opposition to the impressed magnetic field. Therefore in what follows, the c-edge for all the diamagnetic elements will be used. (To get the theoretical value of magnetic susceptibility for \(\text{edge}_a\) or \(\text{edge}_b\), simply multiply the calculations by the ratio \(\text{edge}_a/\text{edge}_c\) or \(\text{edge}_b/\text{edge}_c\).) The "magnetic polarization" is the sum of the magnetic dipole moments divided by the appropriate volume.

\[ M_P := \frac{n_f \cdot Z_{\text{uc}}}{2} \cdot \mu_{\text{dpm}} \] \text{webers/m}^2 \tag{123} \]

where \(n_f\) is a fraction (0 to 1, rational for a unit cell, possibly irrational for bulk matter). Because there are \(Z_{\text{uc}}\) atoms in a unit cell, there can be a maximum of \(Z_{\text{uc}}/2\) dipoles in a unit cell (in which case \(n_f\) is 1). Now \(\chi\) can be expressed as

\[ \chi := \frac{M_P}{B_{\text{ext}}} \] \tag{124} \]

We can determine the saturation value for \(M_P\) without difficulty, but it's difficult to determine the value of \(B_{\text{ext}}\) at which that occurs, which then prevents us from directly calculating \(\chi\).
Fortunately we know the natural unit value of $\chi$ and $M_p$. From Larson, Ref. [1], p. 241:

"...the numerical factor relating the magnitudes of quantities differing by one scalar dimension, in terms of cgs units, is $3 \times 10^{10}$. The corresponding factor applicable to the interaction between a ferromagnetic charge and an internal magnetic charge is the square root of the product of 1 and $3 \times 10^{10}$, which amounts to $1.73 \times 10^5$. The internal magnetic effects are thus weaker than those due to ferromagnetism by about $10^5$.

In this part of this section we are using SI units, so here:

$$\chi_{u_{SI}} := \sqrt{\frac{1 \cdot \text{m}}{\text{sec}}} \cdot \frac{\text{cSI}}{\text{m} \cdot \text{sec}} \cdot \frac{1}{1} \chi_{u_{SI}} = 5.7755 \times 10^{-5}$$

$$M_{P\_nat\_t} := \frac{1 \cdot M_{u_{weber}} \cdot (s_{t\_u} \cdot 10^{-2})}{(s_{t\_u} \cdot 10^{-2})^3} \text{ webers/m}^2 \quad \text{(time region)}$$

$$M_{P\_nat\_t} = 56.561 \text{ webers/m}^2 \quad \text{(time region)}$$

Originally it was thought by workers in the Reciprocal System that the magnetic susceptibility would be proportional only to $1/t_p$ or $1/t_s$, but not both. However, assuming that the magnetic charge is on the principle magnetic rotational displacement, it must be subject to all three atomic rotational displacements—principal (or primary) magnetic, subordinate (or secondary) magnetic, and electric. Now we can form a ratio of the magnetic susceptibility to the natural value:
\[ \chi_{u_{\text{SI}}} = \frac{n_f \cdot \text{floor} \left( \frac{z_{uc}}{2} \right) \cdot n_{M} \cdot M_{u_{\text{weber}}} \cdot \text{edge}_c}{V_{uc_{\text{SI}}}} \cdot \frac{B_{ext}}{B_{ext}} \cdot \frac{1}{t_p} \cdot \frac{1}{t_s} \cdot \frac{1}{t_e} \]

But the \( B_{ext} \) cancel out, and we are finally left with

\[ \chi \equiv \chi_{u_{\text{SI}}} \left( \frac{n_f \cdot \text{floor} \left( \frac{z_{uc}}{2} \right) \cdot n_{M} \cdot M_{u_{\text{weber}}} \cdot \text{edge}_c}{V_{uc_{\text{SI}}}} \cdot \frac{1}{t_p} \cdot \frac{1}{t_s} \cdot \frac{1}{t_e} \right) \]

Using Eq. (127c) in Excel, Table V results. (\( V_{uc} \) in the table is in units of cm\(^3\) x 10\(^{-24}\) and converted to m\(^3\) in Excel; likewise, \( \text{edge}_c \) is in angstrom units and converted to m in Excel. Crystal data values come from the Reciprocal System Data Base.)
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Table V. Calculated Values of Diamagnetic Susceptibility for the Elements (Crystalline, No Loss)
Notes:

1) The diamagnetic susceptibilities are negative because $t_e$ is negative. Electronegative elements are diamagnetic, whereas electropositive elements are paramagnetic. Obviously some nominally electronegative elements use their alternative electropositive rotational displacements and so are paramagnetic, and vice versa.

2) All diamagnetic elements have $n_f = 1$.

3) Generally, the value of $n_M = t_p$, meaning that saturation has been obtained. If $n_M < t_p$, then saturation has not been obtained. It's clear that $n_M$ cancels $t_p$ or $t_s$ in many of the elements, which explains why in early work on the Reciprocal System it was thought that susceptibility was proportional to $1/t_p$ or $1/t_s$ only.

4) The correlation of the effective values of $t_p$, $t_s$, and $t_e$, with the actual rotational displacements is quite amazing!

5) The average ratio of calc/obs = 1.001, which means that the calculated and observed values are within 1.5% of each other, and therefore within the experimental error. The correlation is .996.

6) The values given above are those most probable, given the experimental conditions.

7) Except for elements like Hg, the calculations are for values of the crystal unit cell at room temperature. Other than very small changes of edge_c and $V_{uc}$ with temperature, diamagnetic susceptibility is not dependent on temperature. Crystal unit cell edge_c has been used in most of the calculations. In a few, a factor $k_{dpm}$ multiplies the value of edge_c; the maximum of this factor is 8, for Bi--this explains the remarkable diamagnetism of this element. The proof of this extended magnetic dipole length can be seen in the spiral, stair-step nature of the macroscopic crystal—which shows the misalignment of the unit cells (and allows the dipole length to go across unit cells). The diamagnetic compounds, unlike the elements, generally have $k_{dpm} > 1$.

8) Red phosphorus and gray/white tin are the forms of these elements used in the calculations.

9) Non-integer values cannot be used for $t_e$ here, because the isotopic neutrinos have no affect on atomic magnetic charges. While it's true that $t_{e\_eff}$ is not quite equal to $t_e$ for many of the elements, the series relationships are quite closely in accord: as atomic number increases in a division, $t_{e\_eff}$ decreases, as it
10) The orthogonal charge of each of the two atoms involved in a magnetic dipole has no direct effect on the magnetic susceptibility; however, they can change the dimensions of edge_a or edge_b and thus slightly change the volume of the unit cell. The dipole itself causes a small decrease of edge_c; the calculations are straightforward but beyond the scope of this paper because they involve the interatomic distance equations. See the article on "magnetostriction" in Thewlis, Ref. [7], vol. 4, pp. 480-482; the equations there are approximate macroscopic ones; to do the calculations correctly requires detailed knowledge of the atomic forces involved.

11) As should be clear, the Reciprocal System has no use for the "Bohr magneton," the "Weiss magneton," or the "nuclear magneton."

12) In diamagnets and paramagnets there is no hysteresis and therefore no energy loss. In thermodynamic terms, the process is reversible. When the horizontal electromagnet in a Gouy Balance is turned on, magnetic charges are induced in the specimen, and the diamagnet sample in the vertical test cylinder repels upward; when the electromagnet is turned off, the diamagnet sample drops back down to its equilibrium position, and vice versa for a paramagnet sample. Note that the horizontal force of the north and south poles of the electromagnet cancel, so that they cause no horizontal motion of the diamagnet or paramagnet. But magnetism is two-dimensional, according to the Reciprocal System, so vertical motion of the diamagnet or paramagnet is the result when the electromagnet is turned on. Conventional physics has no conceptual explanation for this effect!
b. Compounds

Letting $s_{dpm} =$ the average magnetic dipole length, $n_{uc} =$ net total number of atoms in a volume unit cell, $n_1 =$ net total number of atoms of element 1, and $n_2 =$ net total number of atoms of element 2, we can generalize Eq. (127c) for binary compounds:

$$
\chi := \chi_{u_SI} \frac{n_{uc}}{2} \cdot n_{M} \cdot \frac{M_{u_weber}}{s_{dpm}} \cdot \frac{V_{uc_SI}}{M_{P_{nat_t}}} \cdot \frac{1}{n_{uc}} \left[ n_1 \left( \frac{1}{t_{p1}} \cdot \frac{1}{t_{s1}} \cdot \frac{1}{t_{e1}} \right) + n_2 \left( \frac{1}{t_{p2}} \cdot \frac{1}{t_{s2}} \cdot \frac{1}{t_{e2}} \right) \right]
$$

(128)

(Further generalization to more complex compounds is obvious.) We will apply Eq. (128) to all the dielectric binary compounds previously considered which are diamagnetic. SI units are used throughout, except to convert observed values to the dimensionless value of $\chi$ for use with comparison to the calculated values. The CRC Handbook, Ref. [16], pp. 4-142 to 4-147 is used for the observed values, except for a few from Ref. [15], pp. 730, ff. Unfortunately and oddly, the Springer compilation, Ref. [14], does not have the data, a real lacuna, because it would be helpful to average the results as before.

In what follows, the following parameter equations obtain:

$$
n_f := 1 \quad n_{M} = \min(t_{p1} \cdot t_{p2})
$$

(In very weak fields, the values would begin smaller and become larger as the field increases until getting to these values.)

Also, a nominally electropositive element assumes its alternative equivalent electronegative rotational displacement. Otherwise, there are no modifications to the values of $t_p$, $t_s$, or $t_e$ from their values in the Reciprocal System periodic table.
In Eq. (128), essentially all the values are fixed, except for \( s_{dpm} \). A diamagnetic crystal will orient itself so as to maximally oppose the impressed field. This means that nature "chooses" the particular subset of all possible parallel dipole moments which has the highest average length. In many cases, this length will be a diagonal. In the case of semiconductor compounds, the lengths can actually span multiple unit cells—because of the vacancies of atoms at opposite corners; semiconductors have long-range order, not short-range order. The possible values of \( s_{dpm} \) are as follows:

\[
\begin{align*}
  s_{dpm} &= k_{dpm} \cdot s_0 \quad \text{(a multiple of shortest atomic distance, } k_{dpm} \geq 1) \quad (129a) \\
  s_{dpm} &= \frac{s_0 \cdot 1 + s_0 \cdot 2}{2} \quad \text{(an average of interatomic distances)} \quad (129b) \\
  s_{dpm} &= k_{dpm} \cdot \text{edge} \quad \text{(a multiple of one of the edges of unit cell, } k_{dpm} = (1, 2, 3)) \quad (129c) \\
  s_{dpm} &= k_{dpm} \cdot \sqrt{s_0^2 + \text{edge}^2} \quad \text{(a multiple of the diagonal formed between } s_0 \text{ and an edge)} \quad (129d)
\end{align*}
\]

In Eq. (129d), \( k_{dpm} \) can have the following values (for the compounds considered here), based on geometry of the cell:

\[
k_{dpm} = \left( \frac{1}{2}, \frac{1}{\sqrt{2}}, 1, \frac{5}{4}, \frac{7}{4}, 2, 3 \right)
\]

(129e)

As explained above, the semiconductor compounds have \( k_{dpm} \geq 1 \) for use with Eq. (129d) or (129c). Wyckoff's volumes, Ref. [65], were consulted in order to get an "intuitive feel" for how the dipoles would actually look. His lattice diagrams and packing drawings are very helpful for figuring out \( s_{dpm} \).
\[ \chi_{\text{SiO}_2} = -29.6 \times 10^{-6} \cdot 4 \cdot \pi \cdot \frac{3}{\text{Av} \cdot 112.979 \times 10^{-24}} \]

\[ \chi_{\text{obs}_{\text{SI-SiO}_2}} = -1.6394 \times 10^{-5} \]

\[ n_f := 1 \quad Z_{\text{uc}} := 3 \quad n_M := 2 \quad \text{edge}_c := 5.4046 \times 10^{-10} \text{ m} \quad V_{\text{uc-SI}} := 112.979 \times 10^{-30} \text{ m}^3 \quad n_{\text{uc}} := 9 \]

\[ \text{edge}_a := 4.9130 \times 10^{-10} \text{ m} \quad s_0 := 1.6109 \times 10^{-10} \text{ m} \]

Si: 3-2-(4) \[ n_1 := 3.1 \quad t_{p1} := 3 \quad t_{s1} := 2 \quad t_{e1} := -4 \]

O: 2-2-(2) \[ n_2 := 3.2 \quad n_2 := 6 \quad t_{p2} := 2 \quad t_{s2} := 2 \quad t_{e2} := -2 \]

\[ s_{\text{dpm}} := \text{edge}_a \]

\[ \chi_{\text{SiO}_2} := \chi_{u_{\text{SI}}} \cdot \frac{V_{\text{uc-SI}}}{M_{P_{\text{nat-t}}}} \cdot \frac{1}{n_{\text{uc}}} \left[ n_1 \cdot \left( \frac{1}{t_{p1}} \cdot \frac{1}{t_{s1}} \cdot \frac{1}{t_{e1}} \right) + n_2 \cdot \left( \frac{1}{t_{p2}} \cdot \frac{1}{t_{s2}} \cdot \frac{1}{t_{e2}} \right) \right] \]

\[ \chi_{\text{SiO}_2} = -1.6587 \times 10^{-5} \]

\[ \frac{\chi_{\text{SiO}_2}}{\chi_{\text{obs}_{\text{SI-SiO}_2}}} = 1.0118 \]

In this compound, the average dipole length = the length of edge_a.
\[
\begin{align*}
\text{CaF}_2 & \quad \chi_{\text{obs_SI_CaF}_2} := -28.0 \cdot 10^{-6} \cdot 4 \cdot \pi \cdot \frac{4}{\text{Av} \cdot 163.035 \cdot 10^{-24}} \\
& \quad \chi_{\text{obs_SI_CaF}_2} = -1.4328 \times 10^{-5}
\end{align*}
\]

\[
\begin{align*}
n_f & := 1 & Z_{\text{uc}} & := 4 & n_M & := 2 & \text{edge}_a & := 5.4360 \cdot 10^{-10} \text{ m} & V_{\text{uc_SI}} & := 163.035 \cdot 10^{-30} \text{ m}^3 & n_{\text{uc}} & := 12 \\
& \quad \text{(all edges same)} & s_0 & := 2.3655 \cdot 10^{-10} \text{ m}
\end{align*}
\]

Ca: 3-2-2 \quad n_1 := 4 \quad t_{p1} := 3 \quad t_{s1} := 3 \quad t_{e1} := -16 \quad \text{(alternative electronegative equivalent)}

F: 2-2-(1) \quad n_2 := 4.2 \quad n_2 = 8 \quad t_{p2} := 2 \quad t_{s2} := 2 \quad t_{e2} := -1

\[
s_{\text{dpm}} := s_0
\]

\[
\begin{align*}
\chi_{\text{CaF}_2} & := \chi_{\text{u_SI}} \cdot \frac{V_{\text{uc_SI}}}{M_{\text{P_nat_t}}} \cdot \frac{1}{n_{\text{uc}}} \left[ n_1 \left( \frac{1}{t_{p1}} \cdot \frac{1}{t_{s1}} \cdot \frac{1}{t_{e1}} \right) + n_2 \left( \frac{1}{t_{p2}} \cdot \frac{1}{t_{s2}} \cdot \frac{1}{t_{e2}} \right) \right]
\end{align*}
\]

\[
\chi_{\text{CaF}_2} = -1.4429 \times 10^{-5}
\]

\[
\frac{\chi_{\text{CaF}_2}}{\chi_{\text{obs_SI_CaF}_2}} = 1.007
\]

In this compound, the average dipole length = the shortest interatomic distance.
$\chi_{\text{obs_SI_MgO}} := \left(-10.2 \cdot 10^{-6}\right) \cdot 4 \cdot \pi \cdot \frac{4}{\text{Av} \cdot 74.778 \cdot 10^{-24}}$  
$\chi_{\text{obs_SI_MgO}} = -1.138 \times 10^{-5}$

$n_f := 1$  
$Z_{\text{uc}} := 4$  
$n_M := 2$  
$\text{edge}_a := 4.2130 \cdot 10^{-10}$  
$V_{\text{uc_SI}} := 74.778 \cdot 10^{-30} \text{ m}^3$  
$n_{\text{uc}} := 8$

(all edges same)

$s_{0.1} := 2.1065 \cdot 10^{-10} \text{ m}$  
$s_{0.2} := 3.6486 \cdot 10^{-10} \text{ m}$

Mg: 2-2-2  
$n_1 := 4$  
$t_{p1} := 3$  
$t_{s1} := 2$  
$t_{e1} := -6$  
(alternative electronegative equivalent)

O: 2-2-(2)  
$n_2 := 4$  
$t_{p2} := 2$  
$t_{s2} := 2$  
$t_{e2} := -2$

$s_{\text{dpm}} := \frac{s_{0.1} + s_{0.2}}{2}$  
The two interatomic distances are averaged.

$\chi_{\text{MgO}} := \frac{n_{\text{uc}} \cdot M_{\text{u_weber}} \cdot s_{\text{dpm}}}{n_f \cdot \text{floor} \left( \frac{n_{\text{uc}}}{2} \right) \cdot n_M \cdot \text{M}_{\text{P_nat_t}}} \cdot \frac{V_{\text{uc_SI}}}{\text{M}_{\text{P_nat_t}}} \cdot \frac{1}{n_{\text{uc}}} \left[ n_1 \left( \frac{1}{t_{p1}} \cdot \frac{1}{t_{s1}} \cdot \frac{1}{t_{e1}} \right) + n_2 \left( \frac{1}{t_{p2}} \cdot \frac{1}{t_{s2}} \cdot \frac{1}{t_{e2}} \right) \right]$  

$\chi_{\text{MgO}} = -1.1533 \times 10^{-5}$  
$\frac{\chi_{\text{MgO}}}{\chi_{\text{obs_SI_MgO}}} = 1.0134$

In this compound, the average dipole length = the average of two interatomic distances.
In this compound, the average dipole length = the length of edge_a.

\[
\chi_{\text{KCl}} = -1.287 \times 10^{-5}
\]

\[
\frac{\chi_{\text{KCl}}}{\chi_{\text{obs_SI_KCl}}} = 0.9908
\]
\[
\chi_{\text{NaCl}} := -30.2 \cdot 10^{-6} \cdot 4 \cdot \pi \frac{4}{\text{Av} \cdot 179.425 \cdot 10^{-24}} = -1.4043 \times 10^{-5}
\]

\[
\chi_{\text{obs \_SI \_NaCl}} := -1.3929 \times 10^{-5}
\]

\[
\chi_{\text{NaCl}} = \frac{\chi_{\text{NaCl}}}{\chi_{\text{obs \_SI \_NaCl}}} = 0.9919
\]

In this compound, the average dipole length = the diagonal formed between edge_a and s_0, reduced by 1/SQR(2).
SrF$_2$  

\[ \chi_{\text{obs}_{\text{SI}}_{\text{SrF}_2}} = -37.2 \cdot 10^{-6} \cdot 4 \cdot \pi \cdot \frac{4}{\text{Av} \cdot 194.507 \cdot 10^{-24}} \]

\[ \chi_{\text{obs}_{\text{SI}}_{\text{SrF}_2}} = -1.5956 \times 10^{-5} \]

\[ n_f := 1 \quad Z_{\text{uc}} := 4 \quad n_M := 2 \quad \text{edge}_a := 5.7940 \cdot 10^{-10} \text{ m} \]
\[ V_{\text{uc}_{\text{SI}}} := 194.507 \cdot 10^{-30} \text{ m}^3 \quad n_{\text{uc}} := 12 \]
\[ s_0 := 2.5089 \cdot 10^{-10} \text{ m} \]

Sr: 3-3-2 \quad n_1 := 4 \quad t_p_1 := 4 \quad t_s_1 := 3 \quad t_e_1 := -16 \quad \text{(alternative electronegative equivalent)}

F: 2-2-(1) \quad n_2 := 8 \quad t_p_2 := 2 \quad t_s_2 := 2 \quad t_e_2 := -1

\[ s_{\text{dpm}} := \frac{1}{2} \sqrt{s_0^2 + \text{edge}_a^2} \]

\[ \chi_{\text{SrF}_2} := \chi_{\text{u}_{\text{SI}}} \cdot \frac{V_{\text{uc}_{\text{SI}}}}{\text{MP}_{\text{nat}_{\text{t}}}} \cdot \frac{1}{n_{\text{uc}}} \left[ n_1 \left( \frac{1}{t_{p_1}} \cdot \frac{1}{t_{s_1}} \cdot \frac{1}{t_{e_1}} \right) + n_2 \left( \frac{1}{t_{p_2}} \cdot \frac{1}{t_{s_2}} \cdot \frac{1}{t_{e_2}} \right) \right] \]

\[ \chi_{\text{SrF}_2} = -1.6086 \times 10^{-5} \]

\[ \frac{\chi_{\text{SrF}_2}}{\chi_{\text{obs}_{\text{SI}}_{\text{SrF}_2}}} = 1.0081 \]

In this compound, the average dipole length = 1/2 the diagonal formed between edge_a and s_0.
In this compound, the average dipole length = 3 x edge_a.

\[ \chi_{\text{GaAs}} = -1.6366 \times 10^{-5} \]
In this compound, the average dipole length = 2 x edge_a.
\[
x_{\text{obs_SI_BeO}} := -11.9 \cdot 10^{-6} \cdot 4 \cdot \pi \cdot \frac{2}{A \cdot 27.611 \cdot 10^{-24}} \quad x_{\text{obs_SI_BeO}} = -1.7979 \times 10^{-5}
\]

\[
n_f := 1 \quad Z_{uc} := 2 \quad n_M := 2 \quad \text{edge}_a := 2.6980 \cdot 10^{-10} \text{ m} \quad \text{edge}_c := 4.3800 \cdot 10^{-10} \text{ m} \quad V_{uc_SI} := 27.611 \cdot 10^{-30} \text{ m}^3 \quad n_{uc} := 4
\]

\[
s_0 := 1.6556 \cdot 10^{-10} \text{ m}
\]

Be: \quad 2-1-2 \quad n_1 := 2 \quad t_{p1} := 2 \quad t_{s1} := 2 \quad t_{e1} := -6

O: \quad 2-2-(2) \quad n_2 := 2 \quad t_{p2} := 2 \quad t_{s2} := 2 \quad t_{e2} := -2

\[
s_{dpm} := \sqrt{s_0^2 + \text{edge}_a^2} \quad \text{(must be a semiconductor characteristic)}
\]

\[
x_{\text{BeO}} := x_{u_SI} \cdot \frac{V_{uc_SI}}{M_{\text{P_nat_t}}} \cdot \frac{n_f \sqrt{n_{uc} / 2} \cdot n_M \cdot M_{\text{u_weber}} \cdot s_{dpm}}{s_{dpm}}
\]

\[
x_{\text{BeO}} = -1.8742 \times 10^{-5} \quad \frac{x_{\text{BeO}}}{x_{\text{obs_SI_BeO}}} = 1.0424
\]

In this compound, the average dipole length = the diagonal formed between \(s_0\) and \(\text{edge}_a\).
CdS  \( \chi_{\text{obs}_SI_{-}CdS} := \frac{-50.0 \cdot 10^{-6} \cdot 4 \cdot \pi \cdot 2}{\text{Av} \cdot 99.503 \cdot 10^{-24}} \)  
\( \chi_{\text{obs}_SI_{-}CdS} = -2.0962 \times 10^{-5} \)

\( n_f := 1 \quad Z_{uc} := 2 \quad n_M := 3 \quad \text{edge}_a := 4.1364 \cdot 10^{-10} \text{m} \)

\( \text{edge}_c := 6.7152 \cdot 10^{-10} \text{m} \)

\( s_0 := 2.5182 \cdot 10^{-10} \text{m} \)

Cd: 4-3-(6)  \( n_1 := 2 \quad t_{p1} := 4 \quad t_{s1} := 3 \quad t_{e1} := -6 \)

S: 3-2-(2)  \( n_2 := 2 \quad t_{p2} := 3 \quad t_{s2} := 2 \quad t_{e2} := -2 \)

\( s_{dpm} := 3 \cdot \sqrt{s_0^2 + \text{edge}_a^2} \) (must be a semiconductor characteristic)

\[ \chi_{\text{CdS}} := \chi_{\text{u}_SI} \cdot \frac{V_{\text{uc}_SI} \cdot n_1 \cdot (1/t_{p1} - 1/t_{s1})}{M_{\text{p nat t}}} \cdot \frac{1}{n_uc} \left( 1 - \left( \frac{1}{n_1} \right) \frac{1}{t_{p1}} + \frac{1}{n_2} \frac{1}{t_{p2}} \right) \]

\[ \chi_{\text{CdS}} = -2.0885 \times 10^{-5} \]

\[ \frac{\chi_{\text{CdS}}}{\chi_{\text{obs}_SI_{-}CdS}} = 0.9963 \]

In this compound the average dipole length = 3 x the diagonal formed between \( s_0 \) and \( \text{edge}_a \).
SiC

\[ \chi_{\text{obs}_\text{SI}_\text{SiC}} := \frac{-12.8 \cdot 10^{-6} \cdot 4 \cdot \pi}{\text{Av} \cdot 82.199 \cdot 10^{-24}} \]

\[ \chi_{\text{obs}_\text{SI}_\text{SiC}} = -1.2992 \times 10^{-5} \]

\[ n_f := 1 \quad Z_{\text{uc}} := 4 \quad n_M := 2 \quad \text{edge}_a := 4.3480 \cdot 10^{-10} \text{m} \]

(all edges same)

\[ s_0 := 1.8827 \cdot 10^{-10} \text{m} \]

Si: 3-2-(4) \quad n_1 := 4 \quad t_{p1} := 3 \quad t_{s1} := 2 \quad t_{e1} := -4

C: 2-2-(4) \quad n_2 := 4 \quad t_{p2} := 2 \quad t_{s2} := 2 \quad t_{e2} := -4

\[ s_{\text{dpm}} := \frac{5}{4} \sqrt{s_0^2 + \text{edge}_a^2} \]

(must be semiconductor characteristic)

\[ \chi_{\text{SiC}} := \chi_{\text{u}_\text{SI}} \cdot \frac{V_{\text{uc}_\text{SI}}}{M_{\text{P}_\text{nat}_t}} \cdot \frac{1}{n_{\text{uc}}} \left[ n_1 \left( \frac{1}{t_{p1}} \cdot \frac{1}{t_{s1}} \cdot \frac{1}{t_{e1}} \right) + n_2 \left( \frac{1}{t_{p2}} \cdot \frac{1}{t_{s2}} \cdot \frac{1}{t_{e2}} \right) \right] \]

\[ \chi_{\text{SiC}} = -1.4723 \times 10^{-5} \]

\[ \frac{\chi_{\text{SiC}}}{\chi_{\text{obs}_\text{SI}_\text{SiC}}} = 1.1333 \]

In this compound the average dipole length = 5/4 x the diagonal formed between \( s_0 \) and \( \text{edge}_a \).
\[ \chi_{\text{obs_SI}_ZnS} = 1.3178 \times 10^{-5} \]

\[ n_f := 1 \quad Z_{uc} := 4 \quad n_M := 3 \quad \text{edge}_a := 5.4093 \times 10^{-10} m \]

\[ V_{uc_{SI}} := 158.279 \times 10^{-30} m^3 \quad n_{uc} := 8 \]

(all edges same)

\[ s_0 := 2.3423 \times 10^{-10} m \]

Zn: 3-2-(6) \quad n_1 := 4 \quad t_{p1} := 3 \quad t_{s1} := 2 \quad t_{e1} := -6

S: 3-2-(4) \quad n_2 := 4 \quad t_{p2} := 3 \quad t_{s2} := 2 \quad t_{e2} := -4

\[ s_{dpm} := \frac{7}{4} \sqrt{s_0^2 + \text{edge}_a^2} \quad \text{(must be semiconductor characteristic)} \]

\[ \chi_{ZnS} := \chi_{u_{SI}} \cdot \frac{V_{uc_{SI}}}{M_p_{nat_t}} \cdot \frac{1}{n_{uc}} \cdot \left[ n_1 \cdot \left( \frac{1}{t_{p1}} \cdot \frac{1}{t_{s1}} \cdot \frac{1}{t_{e1}} \right) + n_2 \cdot \left( \frac{1}{t_{p2}} \cdot \frac{1}{t_{s2}} \cdot \frac{1}{t_{e2}} \right) \right] \]

\[ \chi_{ZnS} = -1.3318 \times 10^{-5} \quad \frac{\chi_{ZnS}}{\chi_{\text{obs_SI}_ZnS}} = 1.0106 \]

In this compound the average dipole length = 7/4 x the diagonal formed between \( s_0 \) and \( \text{edge}_a \).
\[ \chi_{\text{obs}_\text{SI}_\text{ZnO}} = -2.4083 \times 10^{-5} \]

\[ n_f := 1 \quad Z_{\text{uc}} := 2 \quad n_M := 2 \quad \text{edge}_a := 3.2420 \times 10^{-10} \text{m} \]
\[ \text{edge}_c := 5.1760 \times 10^{-10} \text{m} \]
\[ s_0 := 1.9651 \times 10^{-10} \text{m} \]

**Zn:** 3-2-(6) \[ n_1 := 2 \quad t_{p1} := 3 \quad t_{s1} := 2 \quad t_{e1} := -6 \]

**O:** 2-2-(2) \[ n_2 := 2 \quad t_{p2} := 2 \quad t_{s2} := 2 \quad t_{e2} := -2 \]

\[ s_{\text{dpm}} := 2 \sqrt{s_0^2 + \text{edge}_a^2} \quad \text{(must be semiconductor characteristic)} \]

\[ \chi_{\text{ZnO}} := \chi_{\text{u}_\text{SI}} \cdot \frac{V_{\text{uc}_\text{SI}}}{M_{\text{p}_\text{nat}_t}} \cdot \frac{1}{n_{\text{uc}}} \left[ \frac{1}{n_1} \left( \frac{1}{t_{p1}} \cdot \frac{1}{t_{s1}} \cdot \frac{1}{t_{e1}} \right) + \frac{1}{n_2} \left( \frac{1}{t_{p2}} \cdot \frac{1}{t_{s2}} \cdot \frac{1}{t_{e2}} \right) \right] \]

\[ \chi_{\text{ZnO}} = -2.4116 \times 10^{-5} \quad \frac{\chi_{\text{ZnO}}}{\chi_{\text{obs}_\text{SI}_\text{ZnO}}} = 1.0014 \]

In this compound the average dipole length = \( 2 \times \) the diagonal formed between \( s_0 \) and \( \text{edge}_a \).
\[ \chi_{\text{obs_SI}_\text{GaN}} = -2.5633 \times 10^{-5} \]

\[ n_f := 1 \quad Z_{\text{uc}} := 2 \quad n_M := 2 \quad \text{edge}_a := 3.1800 \times 10^{-10} \text{m} \quad V_{\text{uc_SI}} := 45.242 \times 10^{-30} \text{m}^3 \quad n_{\text{uc}} := 4 \]

\[ \text{edge}_c := 5.1660 \times 10^{-10} \text{m} \]

\[ s_0 := 1.9373 \times 10^{-10} \text{m} \]

\[
\begin{align*}
\text{Ga:} & \quad 3-3-(5) \quad n_1 := 2 \quad t_{p1} := 3 \quad t_{s1} := 3 \quad t_{e1} := -5 \\
\text{N:} & \quad 2-2-(3) \quad n_2 := 2 \quad t_{p2} := 2 \quad t_{s2} := 2 \quad t_{e2} := -3 
\end{align*}
\]

\[ s_{\text{dpm}} := 3 \sqrt{s_0^2 + \text{edge}_a^2} \quad \text{(must be semiconductor characteristic)} \]

\[
\chi_{\text{GaN}} = \frac{\chi_{\text{u_SI}} \cdot V_{\text{uc_SI}}}{M_{\text{P_nat_t}}} \cdot \frac{1}{n_{\text{uc}}} \left[ n_1 \left( \frac{1}{t_{p1}} \cdot \frac{1}{t_{s1}} \cdot \frac{1}{t_{e1}} \right) + n_2 \left( \frac{1}{t_{p2}} \cdot \frac{1}{t_{s2}} \cdot \frac{1}{t_{e2}} \right) \right]
\]

\[ \frac{\chi_{\text{GaN}}}{\chi_{\text{obs_SI}_\text{GaN}}} = 0.9973 \]

In this compound the average dipole length = 3 x the diagonal formed between \( s_0 \) and \( \text{edge}_a \).
The average of the ratios of calculated to observed values of $\chi$ is:

\[
\frac{\chi_{\text{SiO}_2}}{\chi_{\text{obs}_\text{SI}_{\text{SiO}_2}}} + \frac{\chi_{\text{CaF}_2}}{\chi_{\text{obs}_\text{SI}_{\text{CaF}_2}}} + \frac{\chi_{\text{MgO}}}{\chi_{\text{obs}_\text{SI}_{\text{MgO}}}} + \frac{\chi_{\text{KCl}}}{\chi_{\text{obs}_\text{SI}_{\text{KCl}}}} + \frac{\chi_{\text{NaCl}}}{\chi_{\text{obs}_\text{SI}_{\text{NaCl}}}} + \frac{\chi_{\text{SrF}_2}}{\chi_{\text{obs}_\text{SI}_{\text{SrF}_2}}} \\
+ \frac{\chi_{\text{GaAs}}}{\chi_{\text{obs}_\text{SI}_{\text{GaAs}}}} + \frac{\chi_{\text{GaP}}}{\chi_{\text{obs}_\text{SI}_{\text{GaP}}}} + \frac{\chi_{\text{BeO}}}{\chi_{\text{obs}_\text{SI}_{\text{BeO}}}} + \frac{\chi_{\text{CdS}}}{\chi_{\text{obs}_\text{SI}_{\text{CdS}}}} + \frac{\chi_{\text{SiC}}}{\chi_{\text{obs}_\text{SI}_{\text{SiC}}}} + \frac{\chi_{\text{ZnS}}}{\chi_{\text{obs}_\text{SI}_{\text{ZnS}}}} \\
+ \frac{\chi_{\text{ZnO}}}{\chi_{\text{obs}_\text{SI}_{\text{ZnO}}}} + \frac{\chi_{\text{GaN}}}{\chi_{\text{obs}_\text{SI}_{\text{GaN}}}} = 1.019
\]

Therefore, we may conclude that the calculations agree with the observed values to within the experimental error.

**Note:** Superconductors have a diamagnetic susceptibility of -1 (Ref. 30, p. 104), meaning that $\mu_r = 0$, and this is easily explained by the Reciprocal System. The massless, uncharged electrons (space units) comprising the electric current have rotational displacement 1-0-(1), which shows that the one-dimensional electric displacement is -1. This one-dimensional motion combined with the one-dimensional translation provides two-dimensional electromagnetism in opposition to the impressed field--diamagnetism--and there is no reduction factor. No magnetic charges are induced in the matter of the conductor.
6. Paramagnets and Magnetic Susceptibilities

a. Elements

Previous to this paper there has been little work done on the calculation of the magnetic susceptibilities of the solid paramagnetic substances; therefore the following is somewhat provisional. In the first edition of Ref. [2], p. 84, Larson says:

"From this new theoretical viewpoint it would appear that there should be a class of paramagnetic substances corresponding to the diamagnetic group with relatively small positive susceptibilities independent of temperature. When we examine the experimental susceptibilities of the electropositive elements this is just what we find....these small positive susceptibilities show little or no temperature variation....Where any temperature variation does exist it is usually an increase in the susceptibility at the higher temperatures, ..., and suggests that secondary causes may be responsible.

"The remaining electropositive elements have much higher susceptibilities which are strongly temperature-dependent, and while there is a very large difference between the susceptibilities of the ferromagnetic elements and the other elements of this group, it would seem that they all belong in the class of inductive paramagnetics." Chapter 26 of the Russian data compilation, Ref. [15], illustrates these conclusions graphically.

We will call the temperature-independent paramagnets Class I and the temperature-dependent paramagnets Class II. Some of the Class II elements, like Cr, Mn, and V, can become ferromagnetic under some circumstances. The rare earth elements will be considered to be "low-end" ferromagnets, rather than as "high-end paramagnets" and will be treated as such in section 7 of this paper.

Eq. (127c) is modified for electropositive elements by inverting (naturally!) the rotational displacements terms:

\[
\chi := \chi_{u\_SI} \left( \begin{array}{c}
\frac{n_f}{2} Z_{UC} - n_M M_{u\_weber} e_{edgec} \\
\frac{V_{uc\_SI}}{M_{P\_nat\_t}} - \frac{t_p}{1} \frac{t_s}{1} \frac{t_e}{1}
\end{array} \right)
\]

Class I Paramagnets (130)
Here, $t_e$ is always positive, and so the susceptibilities are always positive. This equation applies to the lower elements of the various paramagnetic elements in each group in the following table, Table VI. For the higher elements of each such group, Eq. (130) needs a temperature term added (with $w = $ atomic mass, $T = $ temperature, $T_{t_u} = $ natural unit of temperature for the solid state). Provisionally:

\[
\chi := \chi_{u\_SI} \left( \frac{n_f \cdot Z_{uc}}{2} \cdot nM \cdot M_{u\_weber} \cdot \frac{edge_c}{V_{uc\_SI}} \right) \cdot \frac{t_p}{1} \cdot \frac{t_s}{1} \cdot \frac{t_e}{1} + \chi_{u\_SI} \cdot n_f \cdot Z_{uc} \cdot w \cdot \frac{T_{t_u}}{T} \cdot \frac{1}{V_{uc\_SI}} \cdot \frac{1}{\left( s_{t_u} \cdot 10^{-2} \right)^3}
\]

(131)

Class II Paramagnets

The second term is properly non-dimensional, like the first. Note that the dimensions of $w/T$ are

\[
[t^2/s^2] = [t^3/s^3]/[t/s]
\]

(132)

These are the dimensions of magnetic charge. What appears to be happening is this:

1) Unlike the situation with gases, the solid inter-atomic forces and thermal vibrations initially prevent most of the atoms from obtaining internal magnetic charges, but some do.

2) The induced internal magnetic charges then cause the magnetic susceptibility given by Eq. (130) or the first term of Eq. (131). $n_f$ increases from 0 to 1 and $n_M$ increases from 0 to $t_p$, with temperature (up to a certain limit).

3) Thermal energy now produces translational vibrations of the atoms with magnetic charges, which creates an additional magnetic effect, similar to the effect of moving charged electrons in a magnetic field. (Incidentally, this motion should induce a small electric current in a nearby electrical conductor, although this might be hard to detect.) The magnetic effect is proportional to the mass of the atom and inversely proportional to the temperature—the lower the temperature, the less the disorienting effect of the thermal motion.
In the electropositive elements, the thermal motion is translational vibration in space, just as the magnetic charge is rotational vibration in space; therefore the effects are additive. This is in contrast with the electronegative elements, in which the thermal motion is in time. Hence the diamagnets do not have a temperature term in their susceptibility equation.

The only "adjustable" parameter in Eq. (130) and Eq. (131) is $n_f$, the fraction of the crystal unit cells which are magnetized and oriented in the direction of the magnetic field. Whereas in the diamagnetic elements, $n_f$ is normally equal to 1, this is not the case with the paramagnetic elements--because of the stronger solid inter-atomic forces and the spatial thermal vibrations. For use in Table VI, we will therefore use the "best-fit" values--these should be considered to be predictions, to be verified by experiment later. For paramagnetic gases, we would expect the value of $\chi$ to be roughly $1/n_f$ times that of the solid values. Unfortunately, empirical data verifying this deduction are hard to come by--none of the references have data comparing the values of $\chi$ for the same paramagnetic substances in the solid, liquid, and gas states.

However, the literature does refer to the paramagnetic metals as being "feebly paramagnetic." Both the classical theory from Curie and Weiss and the Quantum Mechanics theories of Van Vleck and his associates focus mostly on the paramagnetic gases. Van Vleck spends just six pages out of 384 on the topic of solid metallic paramagnetism (Ref. [22], pp. 347-353). Terms such as "quenching" and "exchange demagnetization" and "Pauli paramagnetism" are bandied about. The "standard" Curie-Weiss Law is expressed as

$$\chi = \frac{C}{T + \Delta}$$

where $C$ is the Curie constant and $\Delta$ is the Weiss constant. But Van Vleck says (Ref. [22], p. 303), "Whole pages could be devoted to recording the values of $C$ and $\Delta$ reported by different investigators, not always in overly good accord with each other." So, Van Vleck doesn't devote any pages to this data--it's that unreliable! (A search on the Internet has revealed that the situation hasn't changed in the intervening years.) Furthermore, negative values are sometimes given for $\Delta$! We have to conclude that the "modern" theory of magnetic susceptibility is not very tenable.

In the following table, $T$ has been set to room temperature, 293.15 K, for the Class II Paramagnets. $n_M$ has been set to $t_p$ at this temperature. Other data are in the same units as the previous table.
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**Avg. n_f** | **0.0471**

**Table VI. Susceptibilities of Paramagnetic Elements (Crystalline, No Loss)**
Table Notes:

1) The column labeled "T_flag" indicates whether the paramagnet is Class I ("N") or Class II ("Y").

2) The column labeled T_factor shows the results for the second term of Eq. (131).

3) Notice that some nominally electronegative elements assume their alternative electropositive rotations and so have a positive magnetic susceptibility.

4) As with the diamagnetic elements, the two sources for observed magnetic susceptibilities are the CRC compilation and the Springer compilation, averaged.

5) Edge_c of the crystal unit cell has been used for all calculations, rather than edge_a, and there is no need for k_dpm.

6) The crystal unit cell values come from the Reciprocal System Data Base (which in turn makes use of the data in Ref. [58] and Ref. [65]).

7) The average ratio of observed to calculated values = 1, with a correlation of 1.000--due to the "best-fit" values of \( n_f \). The average value of \( n_f = 0.0471 \). So only approx. 5% of the atoms have the induced magnetic charges.

8) No parameter other than \( n_f \) has been "manipulated." The \( n_f \) values should be considered as predictions. Presumably, the gas values will be roughly \( 1/n_f \) or approximately 21 times the values given here.

9) Eq. (131) should work down to \( T = T_1 \), the temperature at the end of the first specific heat line segment (which begins at \( T_0 \), which is itself \( 1/2 \times T_{sc} \), the superconducting temperature). See Ref. [1], pp. 84-86.
Note 1: The Special Case of Oxygen

At first glance, both the conventional theory and the Reciprocal System would categorize oxygen as diamagnetic, not paramagnetic. Conventional theory says that only atoms with unpaired electrons or an "odd number" of electrons can be paramagnetic. But oxygen is claimed to have 16 electrons, an even number. As for the Reciprocal System, oxygen has the atomic rotational displacements 2-2-2(2) and should therefore be diamagnetic. Even Larson says (Ref. [1], p. 242), "Just why oxygen is the element in the negative division in which the polarity reversal takes place is not yet known."

The alternative equivalent electropositive rotational displacements of oxygen are: 2-1-6. Here is the calculation for solid oxygen (cubic-FCC) at $T = 54$ K, assuming 83% complete saturation (and data from Reciprocal System Data Base):

$$
n_f := .83 \quad Z_{uc} := 16 \quad V_{uc\_SI} := 318.612 \cdot 10^{-30} \quad m^3 \quad edge_c := 6.830 \cdot 10^{-10} \quad m \quad t_p := 2 \quad t_s := 1 \quad t_e := 6
$$

$$
n_M := 2 \quad T := 54 \quad K \quad w := 15.9994
$$

(all edges same)

$$
\chi_O := \chi_{u\_SI} \left( \frac{Z_{uc}}{n_f \cdot n_M \cdot M_{u\_weber} \cdot edge_c} \cdot \frac{V_{uc\_SI}}{M_{p\_nat\_t}} \cdot \frac{t_p \cdot t_s \cdot t_e}{1 \cdot 1 \cdot 1} \right) + \chi_{u\_SI} \cdot n_f \cdot Z_{uc} \cdot \frac{w}{1} \cdot \frac{T_{t\_u}}{T} \cdot \frac{1}{V_{uc\_SI}} \left( s_{t\_u} \cdot 10^{-2} \right)^3
$$

$$
\chi_O = 0.0107
$$

The observed value comes from the CRC handbook (Ref. [16], p. 4-145), and converted to a pure $\chi$ and SI:

$$
\chi_{obs\_O} := .0102 \cdot \frac{Z_{uc}}{Av \cdot V_{uc\_SI} \cdot 10^6} \cdot 4 \cdot \pi \quad \chi_{obs\_O} = 0.0107
$$
The Springer compilation has the same value. At complete saturation, \( n_f = 1 \), and \( \chi_0 = 0.0907 \). This should be testable with a strong enough field. As it stands, we do not have a means of verifying that \( n_f = 0.83 \) for the experimental setup(s), and so it will have to be treated as a prediction.

**Note 2: Antiparamagnetic or "Antiferromagnetic" Substances**

If the magnetic dipoles in a crystal unit cell are oriented in opposite directions, then the first term of Eq. (131) cancels out, leaving just the second term:

\[
\chi := \chi_{u_SI} \cdot n_f \cdot Z_{uc} \cdot \frac{w}{T} \cdot \frac{T_{t_u} \cdot 1}{V_{uc_SI}} \cdot \frac{1}{\left( s_{t_u} \cdot 10^{-2} \right)^3}
\]

Unfortunately, the magnetochemists, like Selwood (Ref. [27], Chapter 13, pp. 326-350), define as “antiferromagnetic” substances which have the following magnetic susceptibility vs. temperature curve (p.327, Fig. 108): the first leg is a linear increase of susceptibility with temperature up to a maximum (the Neel point); the second leg is paramagnetic, with susceptibility decreasing as \( C/T \) (Curie) or \( C/(T+\Delta) \) (Curie-Weiss).

The Reciprocal System interpretation is: the first leg is not temperature dependent—it’s just the first term of Eq. (131), but with \( n_f \) increasing from 0 to its maximum and with \( n_M \) increasing from 0 to its maximum (therefore, the dipoles are *not* neutralized). At the Neel point, the second term kicks in, but \( n_f \) and \( n_M \) remain constant (at the maximum values), apparently. One would think that all paramagnetic elements would have these two legs. Note: this interpretation is provisional, simply because we don’t have enough empirical data to make a final determination; most of the tests do not go below 100 K, so the first leg is not seen. It’s possible that the Neel point coincides with \( T_1 \), the end of the first segment of the specific heat curve (Ref. [1], p. 86).
b. Compounds

As Larson shows in the second edition of Ref. [2], pp. 219-284, all chemical compounds are combinations of electropositive and electronegative components. In Ref. [1], p. 242, he says, "The presence of any significant amount of motion in time (space displacement) in a molecular structure prevents establishment of the positive magnetic orientation. All compounds, except those that are ferromagnetic, or heavily weighted with paramagnetic elements, are therefore diamagnetic. The overwhelming preference for diamagnetism in the compounds is probably what led to the currently accepted hypothesis of a universal diamagnetism." Perusal of the table of magnetic susceptibilities of the elements and inorganic compounds in the CRC Handbook (Ref. [16], pp. 4-142 to 4-147) confirms that diamagnetic compounds far outnumber paramagnetic compounds. However, there are a sufficient number (five) to make an attempt at a comparison of theory with observation.
In this compound the average dipole length = edge_a. Selwood, Ref. [27], p. 332, says that this compound is "antiferromagnetic" with a "sharp antiferromagnetic Curie point at -151°C" and follows the Curie-Weiss law with "5.9 Bohr magnetons and a Weiss constant of about 610°."
Using the conventional equations in the literature (which are semi-empirical and semi-theoretical):

Given: Bohr magneton \( \mu_B := 5.9 \)  
Weiss constant \( \Delta := 610 \)  
(empirically determined)

Then:

Weiss magneton \( \mu_W := \mu_B \cdot 4.95 \) \( \mu_W = 29.205 \)

Curie Constant on molar basis \( C_M := \frac{\mu_W^2}{14.06^2} \) \( C_M = 4.3146 \)

\[ \chi_{\text{MnO_conv}} \frac{C_M}{T + \Delta} \cdot 4 \cdot \pi \cdot \frac{Z_{uc}}{\text{Av} \cdot V_{uc SI} \cdot 10^6} \] \( \chi_{\text{MnO_conv}} = 0.0044 \)  
(which does agree with observation at room temperature)

For the graphs below, the starting point will be at the Neel point \((-151 + 273.15 = 122.15 \text{ K})\):
Neither Selwood nor Van Vleck nor Bhatnagar nor Stoner provides a theoretical calculation for $\mu_B$ or $\Delta$ for MnO; it seems odd that $\Delta$ should be so large and why $\mu_B$ should be 5.9. In the Reciprocal System, $n_M = 2$, an integer, for MnO. On the other hand, it’s not clear why we have $n_f = .34$; however, from $T_{0\_MnO}$ to 122.15 K, $n_f$ probably goes from 0 to .34 and $n_M$ goes from 0 to 1 to $t_p$, which accounts for the increase in susceptibility up to 122.15 K. The peak Reciprocal System susceptibility value for MnO is .0082 at the supposed Curie [Neel] point (but we don't have the empirical data to confirm it).
\[
\chi_{\text{CrCl}_2} \chi_{\text{obs}_\text{SI}_\text{CrCl}_2} := \frac{7230 \cdot 10^{-6} \cdot 4 \cdot \pi \cdot 2}{\text{Av} \cdot 138.026 \cdot 10^{-24}} \]
\[
\chi_{\text{obs}_\text{SI}_\text{CrCl}_2} = 0.0022
\]

\[n_f := 0.25 \quad Z_{\text{uc}} := 2 \quad n_M := 2 \quad \text{edge}_a := 5.9740 \cdot 10^{-10} \text{ m} \]
\[\text{edge}_b := 6.6240 \cdot 10^{-10} \text{ m} \]
\[\text{edge}_c := 3.4880 \cdot 10^{-10} \text{ m} \]

(by iteration)
\[s_0 := 2.9169 \cdot 10^{-10} \text{ m} \]
\[w := 122.9020 \quad T := 293.15 \text{ K} \]

\[
\chi_{\text{CrCl}_2} = 0.0022 \quad \frac{\chi_{\text{CrCl}_2}}{\chi_{\text{obs}_\text{SI}_\text{CrCl}_2}} = 1.0028
\]

\[
\begin{align*}
n_f &= 0.25 & Z_{\text{uc}} &= 2 & n_M &= 2 & \text{edge}_a &= 5.9740 \cdot 10^{-10} \text{ m} \\
\text{edge}_b &= 6.6240 \cdot 10^{-10} \text{ m} & \text{edge}_c &= 3.4880 \cdot 10^{-10} \text{ m} \\
\text{edge}_a &= 5.9740 \cdot 10^{-10} \text{ m} & \text{edge}_b &= 6.6240 \cdot 10^{-10} \text{ m} & \text{edge}_c &= 3.4880 \cdot 10^{-10} \text{ m} \\
\end{align*}
\]

Cr: 3-2-6 \quad n_1 := 2 \quad t_{p1} := 3 \quad t_{s1} := 2 \quad t_{e1} := 6

Cl: 3-2-(1) \quad n_2 := 4 \quad t_{p2} := 2 \quad t_{s2} := 2 \quad t_{e2} := 7

(orthorhombic crystal cell)

\[
\begin{align*}
\chi_{\text{CrCl}_2} &= \chi_{\text{u}_\text{SI}} \cdot \frac{V_{\text{uc}_\text{SI}}}{M_{\text{P}_\text{nat}_t}} \cdot \frac{1}{n_{\text{uc}}} \cdot \left[ n_1 \cdot \left( \frac{t_{p1}}{1} \cdot \frac{t_{s1}}{1} \cdot \frac{t_{e1}}{1} \right) + n_2 \cdot \left( \frac{t_{p2}}{1} \cdot \frac{t_{s2}}{1} \cdot \frac{t_{e2}}{1} \right) \right] \\
&\quad + \chi_{\text{u}_\text{SI}} \cdot n_f \cdot Z_{\text{uc}} \cdot \frac{w}{1} \cdot \frac{T_{t_u}}{T} \cdot \frac{1}{V_{\text{uc}_\text{SI}}} \cdot \left[ \left( \frac{t_{t_u}}{10^{-2}} \right)^3 \right]
\end{align*}
\]

\[
\frac{\chi_{\text{CrCl}_2}}{\chi_{\text{obs}_\text{SI}_\text{CrCl}_2}} = 1.0028
\]
In this compound the average dipole length = edge_b (longest side). Selwood, Ref. [27], p. 332, says that this compound is "antiferromagnetic" and that "the susceptibility is a complicated function of temperature." No parameters for the Curie-Weiss law are provided. Van Vleck (Ref. [22], p. 304) covers CrCl$_3$, not CrCl$_2$. Stoner says (Ref. [28], p. 325) that the effective value of the Bohr magneton for this compound is 4.81, for use in the Curie (not Curie-Weiss) law. Here:

\[
\chi_{M\text{-CrCl}_2\text{\_conv}} := \frac{4.81^2}{2.839^2} \cdot \frac{1}{T} \quad \chi_{M\text{-CrCl}_2\text{\_conv}} = 0.0098 \quad \text{(based on pp. 282, 325 of Stoner)}
\]

\[
\chi_{\text{CrCl}_2\text{\_conv}} := \chi_{M\text{-CrCl}_2\text{\_conv}} \cdot \frac{4 \cdot \pi \cdot Z_{uc}}{\text{Av} \cdot V_{uc\_SI} \cdot 10^6} \quad \chi_{\text{CrCl}_2\text{\_conv}} = 0.003
\]

This is quite a bit larger than the currently accepted observed value.
\[
\frac{4.81^2}{2.839^2} \cdot \frac{1}{TT} \cdot 4\pi \cdot \frac{Z_{uc}}{Av \cdot V_{uc_SI} \cdot 10^6}
\]

\[
\chi_{u_SI} = \frac{n_f \cdot \text{floor} \left( \frac{n_{uc}}{2} \right) \cdot n_M \cdot M_u \cdot \text{weber} \cdot \text{sdpm}}{V_{uc_SI} \cdot M_P_{nat_t}} 
+ \frac{x_{u_SI} \cdot n_f \cdot Z_{uc} \cdot w \cdot T_{t_u} \cdot 1}{TT} \cdot \frac{1}{V_{uc_SI}} \left[ \left( s_{t_u} \cdot 10^{-2} \right)^3 \right]
\]

Figure 23. Paramagnetic Susceptibility of CrCl2 (Reciprocal System curve is in blue; conventional theory is in red)
\[
\chi_{\text{CuO}} \equiv \chi_{\text{obs_SI CuO}} := \frac{238 \cdot 10^{-6} \cdot 4 \cdot \pi}{\text{Av} \cdot 79.940 \cdot 10^{-24}} \quad \chi_{\text{obs_SI CuO}} = 0.00025 \\
\]

\( n_f := 0.013 \) \( Z_{uc} := 4 \) \( n_M := 2 \) \( \text{edge}_a := 4.6530 \cdot 10^{-10} \) m \( \text{edge}_b := 3.4100 \cdot 10^{-10} \) m \( \text{edge}_c := 5.1080 \cdot 10^{-10} \) m \\
(by iteration)

\( s_0 := 1.9472 \cdot 10^{-10} \) m \( w := 79.5450 \) \( T := 293.15 \) K

Cu: \( 3-3-(7) \) \( n_1 := 4 \) \( t_{p1} := 3 \) \( t_{s1} := 2 \) \( t_{e1} := 11 \) (alternative electropositive equivalent)

O: \( 2-2-(2) \) \( n_2 := 4 \) \( t_{p2} := 2 \) \( t_{s2} := 1 \) \( t_{e2} := 6 \) (alternative electropositive equivalent)

\( s_{dpm} := \text{edge}_c \) (longest side) \( T := 293.15 \) K

\[
\chi_{\text{CuO}} := \chi_{\text{u_SI}} \cdot \frac{n_f \cdot \text{floor} \left( \frac{n_{uc}}{2} \right) \cdot n_{M} \cdot \mu_{\text{weber}} \cdot s_{dpm}}{V_{\text{uc_SI}}} \cdot \frac{1}{n_{uc}} \left[ n_1 \left( \frac{t_{p1}}{1} \cdot \frac{t_{s1}}{1} \cdot \frac{t_{e1}}{1} \right) + n_2 \left( \frac{t_{p2}}{1} \cdot \frac{t_{s2}}{1} \cdot \frac{t_{e2}}{1} \right) \right] + \chi_{\text{u_SI}} \cdot n_f \cdot Z_{uc} \cdot \frac{\mu \cdot T_t \cdot u}{T} \cdot \frac{1}{V_{\text{uc_SI}}} \left[ \frac{1}{\left( s_{t \cdot u} \cdot 10^{-2} \right)^3} \right]
\]

\[
\chi_{\text{CuO}} = 0.00026 \\
\frac{\chi_{\text{CuO}}}{\chi_{\text{obs_SI CuO}}} = 1.0304
\]
Selwood (Ref. [27], p. 335 says, "Cupric oxide has a very low paramagnetism. There seems little doubt that it exhibits antiferromagnetism, with a Curie [Neel] point at about 150° C." He does not supply values for the Curie or Curie-Weiss laws. And neither do Bhatnagar, Stoner, or Van Vleck. Presumably, at

\[
T := 150 + 273.15 \quad T = 423.15 \text{ K} \quad n_f := 1 \quad \text{(maximum value, might be lower)}
\]

\[
\chi_{\text{CuO sat}} := \chi_{\text{u SI}} \cdot \frac{n_{f \text{ floor}}}{2} \cdot n_{M \text{ u weber sdpm}} \frac{V_{\text{uc SI}}}{M_{P \text{ nat t}}} \cdot \frac{1}{n_{uc}} \left[ n_1 \left( \frac{t_{p1}}{1} \cdot \frac{t_{s1}}{1} \cdot \frac{t_{e1}}{1} \right) + n_2 \left( \frac{t_{p2}}{1} \cdot \frac{t_{s2}}{1} \cdot \frac{t_{e2}}{1} \right) \right] + \chi_{\text{u SI}} \cdot n_f \cdot Z_{\text{uc}} \cdot \frac{w}{w} \cdot \frac{T_{t \text{ u}}}{T} \cdot \frac{1}{V_{\text{uc SI}}} \left[ \frac{s_{t \text{ u}}}{10^{-2}} \right]^3
\]

\[
\chi_{\text{CuO sat}} = 0.0166
\]

The references do not supply an observed value of \( \chi_{\text{CuO sat}} \) at this temperature.
\( \text{Tm}_2\text{O}_3 \quad \chi_{\text{obs SI Tm}_2\text{O}_3} := \frac{51444 \cdot 10^{-6} \cdot 4 \cdot \pi \cdot 16}{\text{Av} \cdot 1153.661 \cdot 10^{-24}} \quad \chi_{\text{obs SI Tm}_2\text{O}_3} = 0.01488 \)

\( n_f := 0.36 \quad Z_{\text{uc}} := 16 \quad n_M := 2 \quad \text{edge}_c := 10.4880 \cdot 10^{-10} \quad m \quad V_{\text{uc SI}} := 1153.661 \cdot 10^{-30} \quad m^3 \quad n_{\text{uc}} := 80 \)

(by iteration) (all edges same)

\( s_0 := 2.2462 \cdot 10^{-10} \quad m \quad w := 385.8660 \quad T := 293.15 \quad \text{K} \)

Tm: 4-3-15 \( n_1 := 32 \quad t_{p1} := 4 \quad t_{s1} := 3 \quad t_{e1} := 15 \)

O: 2-2-(2) \( n_2 := 48 \quad t_{p2} := 2 \quad t_{s2} := 1 \quad t_{e2} := 6 \quad \) (alternative electropositive equivalent)

sdpm := edge_c \quad T := 293.15 \quad \text{K}

\[
\begin{align*}
\chi_{\text{Tm}_2\text{O}_3} := & \chi_{\text{u SI}} \cdot \frac{V_{\text{uc SI}}}{M_{\text{P nat t}}} \cdot \frac{1}{n_{\text{uc}}} \left[ n_1 \cdot \left( \frac{t_{p1}}{1} \cdot \frac{t_{s1}}{1} \cdot \frac{t_{e1}}{1} \right) + n_2 \cdot \left( \frac{t_{p2}}{1} \cdot \frac{t_{s2}}{1} \cdot \frac{t_{e2}}{1} \right) \right] \\
& + \chi_{\text{u SI}} \cdot n_f \cdot Z_{\text{uc}} \cdot \frac{w}{1} \cdot \frac{T_{\text{t u}}}{T} \cdot \frac{1}{V_{\text{uc SI}}} \cdot \frac{1}{\left( s_{t u} \cdot 10^{-2} \right)^3} \\
\end{align*}
\]

\( \chi_{\text{Tm}_2\text{O}_3} = 0.01497 \quad \frac{\chi_{\text{Tm}_2\text{O}_3}}{\chi_{\text{obs SI Tm}_2\text{O}_3}} = 1.0058 \)

None of the references have empirical values for the Curie or Curie-Weiss laws.
\[
\text{Cr}_2\text{O}_3 \quad \chi_{\text{obs_SI Cr2O3}} := \frac{1960 \cdot 10^{-6}}{4 \cdot \pi} \cdot \frac{6}{\text{Av} \cdot 288.716 \cdot 10^{-24}} \quad \chi_{\text{obs_SI Cr2O3}} = 0.00085
\]

\[
n_f := 0.037 \quad Z_{\text{uc}} := 6 \quad n_M := 2 \quad \text{edge}_a := 4.9540 \cdot 10^{-10} \text{ m} \quad V_{\text{uc_SI}} := 288.716 \cdot 10^{-30} \text{ m}^3 \quad n_{\text{uc}} := 30
\]

(by iteration)

\[
\text{edge}_c := 13.5840 \cdot 10^{-10} \text{ m}
\]

\[
s_0 := 2.0130 \cdot 10^{-10} \text{ m} \quad w := 151.9900 \quad T := 293.15 \text{ K}
\]

\begin{align*}
\text{Cr:} & \quad 3-2-6 \quad n_1 := 12 \quad t_{p1} := 3 \quad t_{s1} := 2 \quad t_{e1} := 6 \\
\text{O:} & \quad 2-2-(2) \quad n_2 := 18 \quad t_{p2} := 2 \quad t_{s2} := 1 \quad t_{e2} := 6 \quad \text{(alternative electropositive equivalent)}
\end{align*}

\[
s_{\text{dpm}} := \text{edge}_c \quad T := 293.15 \text{ K}
\]

\[
\chi_{\text{Cr2O3}} := \chi_u_{\text{SI}} \cdot \frac{n_f \cdot Z_{\text{uc}} \cdot w \cdot T_{t_u} \cdot V_{\text{uc_SI}}}{M_{\text{P nat t}}} \cdot \frac{1}{n_{\text{uc}}} \cdot \left[ n_1 \left( \frac{t_{p1} \cdot t_{s1} \cdot t_{e1}}{1 \cdot 1 \cdot 1} \right) + n_2 \left( \frac{t_{p2} \cdot t_{s2} \cdot t_{e2}}{1 \cdot 1 \cdot 1} \right) \right] \ldots
\]

\[
\chi_{\text{Cr2O3}} = 0.00084
\]

\[
\frac{\chi_{\text{Cr2O3}}}{\chi_{\text{obs_SI Cr2O3}}} = 0.9937
\]
According to Selwood (Ref. [27], p. 330): "Chromia shows a very weakly defined antiferromagnetic Curie [Neel] point in the neighborhood of 45 to 55° C; the susceptibility at liquid air temperature is definitely lower than at room temperature. Above the Curie [Neel] point, it has an approximately normal moment of about 3.8 Bohr magnetons, but a Weiss constant of well over 300°."

Using the conventional equations in the literature (which are semi-empirical and semi-theoretical):

Given: Bohr magneton $\mu_B := 3.8$  Weiss constant $\Delta := 305$  ("well over 300")

Then:

- Weiss magneton $\mu_W := \mu_B \cdot 4.95$  $\mu_W = 18.81$  $T := 50 + 273.15$  $T = 323.15$
- Curie Constant on molar basis $C_M := \frac{\mu_W^2}{14.06^2}$  $C_M = 1.7898$
- $\chi_{\text{MnO_conv}} := \frac{C_M}{T + \Delta} \cdot 4 \cdot \pi \cdot \frac{Z_{\text{uc}}}{A_v \cdot V_{\text{uc_SI}} \cdot 10^6}$  $\chi_{\text{MnO_conv}} = 0.0012$

For the graphs below, the starting point will be at the Neel point (50 + 273.15 = 323.15 K); also, by iteration, at that temperature:

$n_f := .056$
\[
\frac{C_M}{TT+\Delta} \cdot 4\cdot \pi \cdot \frac{Z_{uc}}{Av \cdot V_{uc\_SI} \cdot 10^6}
\]

\[
\begin{align*}
\chi_{u\_SI} + \frac{V_{uc\_SI}}{M_{P\_nat\_t}} \cdot \frac{1}{TT} \cdot \frac{1}{V_{uc\_SI}} \cdot \left( \frac{1}{s_{T\_u}} \cdot 10^{-2} \right)^3 \\
\cdot \left( \frac{1}{n_{uc} \cdot \left( \frac{t_{p1}}{1} \cdot \frac{t_{s1}}{1} \cdot \frac{t_{e1}}{1} \right)^2} + n_{2} \cdot \left( \frac{t_{p2}}{1} \cdot \frac{t_{s2}}{1} \cdot \frac{t_{e2}}{1} \right) \right)
\end{align*}
\]

**Figure 24.** Paramagnetic Susceptibility of \( \text{Cr}_2\text{O}_3 \) (Reciprocal System curve is in blue; conventional theory is in red; starting point is Selwood’s supposed Curie point, 323.15 K)

Neither Selwood nor Van Vleck nor Bhatnagar nor Stoner provides a theoretical calculation for \( \mu_B \) or \( \Delta \) for \( \text{Cr}_2\text{O}_3 \); it seems odd that \( \Delta \) should be so large and why \( \mu_B \) should be 3.8. In the Reciprocal System, \( n_M = 2 \), an integer, for \( \text{Cr}_2\text{O}_3 \). On the other hand, it’s not clear why we have \( n_f = .056 \) (at and above the Neel point); however, from \( T_{n, \text{Cr}_2\text{O}_3} \) to 323.15 K, \( n_f \) probably goes from 0 to .056 and \( n_M \) goes from 0 to \( t_p \), which accounts for the increase in susceptibility up to 323.15 K. The peak Reciprocal System susceptibility value for \( \text{Cr}_2\text{O}_3 \) is .0012 at the supposed Neel point (but we don’t have the empirical data to confirm it).
That's about it for solid paramagnetic compounds--as explained above, practically all compounds are diamagnetic. Four of these paramagnetic compounds contain oxygen, itself oddly paramagnetic, which probably explains why these compounds are paramagnetic. The other compound contains Cl--it's interesting that we needed to use the electropositive rotations of this element for the dielectric constant calculation, as well.
Supplement: The Stern-Gerlach Experiment

Before leaving the discussion of diamagnetism and paramagnetism, it's important to consider the Stern-Gerlach experiment. Here we are dealing with individual atoms in the vapor state, rather than with atoms in a crystal unit cell of the solid state. The experiment is commonly claimed to have disproved Classical Mechanics and proved Quantum Mechanics; while we can agree that it disproved Classical Mechanics, because the results show that magnetic effects are quantized rather than continuous, we cannot agree that it has proved Quantum Mechanics, because the Reciprocal System is also consistent with the results, as will be shown.

A diagram of the experiment is shown in Figure 25 (adapted from Selwood, Ref. [27], p. 139, Fig. 64). Other references for the experiment include Ref. [18], pp. 215-218; Ref. [24], pp. 114-146; Ref. [25], pp. 100-102; Ref. [26], pp. 178-182; Ref. [28], pp. 213-228; Ref. [29], pp. 12-20; Ref. [30], pp. 278-280; and Ref. [66], pp. 120-123. Magnetic moments for 20 elements were obtained using the methods of Stern and Gerlach over the 1920's and 1930's. After that, experimental physicists switched to resonance techniques, and focused on "nuclear" moments rather than on total atomic moments, as described by Ramsey in Ref. [25].

A small quantity of the substance under investigation is placed in a 200 watt, 2.5 amp electric filament cylindrical oven, measuring 20 mm long and 7 mm diameter. The oven aperture is a slit, and there are two additional collimating slits, exactly aligned by telescope. The electromagnet pole pieces (the first being a wedge having a 70° angle and the second being a channel with a height of 3-4 mm and a depth of 3 cm, the point of the wedge being 2.5 mm from the center of the opening of the channel) are usually 60 to 100 mm long, and the usual inhomogeneous field gradient is in the range of 60 tesla/m (measured, and presumably even across the height of the front of the channel); a homogeneous field would not cause any deflection of the atoms. The detector is usually of the condensation type, either glass or metal. The oven temperature must be high enough to vaporize (sublimate) a sufficient number of atoms of the substance to obtain visible traces on the detector. The interaction of the inhomogeneous field with the atoms' magnetic moments deflects the atoms. From the measured separation of the traces the magnetic moment of the atoms can be inferred.

Whereas, the Quantum Mechanics interpretation of the experiment is that it reveals the permanent magnetic moment (if any) of an atom, the Reciprocal System interpretation is that the magnetic field induces two magnetic charges (or two multiple charge units) in the atoms—one north and one south.

If we assume that the stronger pole face (the wedge) is north and the weaker pole face (the channel) is south, an atom with it's south pole to the wedge side will be pulled toward the wedge, whereas an atom with its north pole to the wedge side will be pushed to the channel side, and vice versa, by Coulomb's magnetic law. Hence there will be two traces, if all atoms have the same value of $n_M$. If the atoms have 2 or 3 or 4 different values of $n_M$, then there will be twice that number of traces displayed. Regardless, by probability, half of the atoms will deflect one way and half the other way.
Figure 25. Diagram of Stern-Gerlach Apparatus
Now we'll compute the Reciprocal System natural unit of *atomic magnetic dipole moment*, which involves *both* magnetic vibration and thermal vibration. As Larson discusses in Ref. [1], pp. 59-60, there are three natural units of temperature in the Reciprocal System:

\[
T_u := 7.20423 \cdot 10^{12} \text{ K} \quad \text{(time-space region, for gases, including electron gases)}
\]

\[
T_{v_u} := 3.5978 \cdot 10^9 \text{ K} \quad \text{(borderline between time-space region and time region, for vapors)}
\]

\[
T_{t_u} := 510.8 \text{ K} \quad \text{(time region, for solids and liquids)}
\]

For the electrical resistivity calculations, involving the high speed electron gas, we used \( T_u \) (see Ref. [53]). For the solid state paramagnetic calculations, we used \( T_{t_u} \). Now for the Stern-Gerlach experiment calculations, we must use \( T_{v_u} \) as the appropriate natural unit.

The length of the atomic magnetic dipole moment is simply the *diameter of the atom*. This size, in the Reciprocal System, corresponds to the diameter of the "nucleus" in conventional theory. It's calculated in the Reciprocal System Data Base (and depends on the atomic magnetic rotational speeds, rather than directly on the rotational displacements), and the values have already been used in Ref. [53]. The natural unit for atomic diameter is \( 3.359 \times 10^{-15} \text{ m} \). Elements below Na, which have incomplete rotational dimensions, have diameters in the range of \( 2 \times 10^{-15} \text{ m} \).

At vapor temperatures, no atom can be in a ferromagnetic state, so the reduction factor previously used for the natural unit of susceptibility must be used for the natural unit of atomic magnetic dipole moment. Putting all this together we have

\[
D_u := 3.359 \cdot 10^{-15} \text{ m} \quad n_M := 1 \quad (134)
\]

\[
H_{\text{adpm}_u} := \frac{\sqrt{1 \cdot \frac{m}{\text{sec}}} \cdot \frac{T_{v_u}}{n_M \cdot M_u \cdot \text{weber} \cdot D_u}}{\sqrt{c \cdot \frac{m}{\text{sec}}} \cdot \frac{1}{1}}
\]

\[
H_{\text{adpm}_u} = 9.3175 \times 10^{-37} \text{ weber-meter} \quad (135)
\]
**Equation:**

\[
\mu_{\text{adpm}} := \mu_{\text{adpm}} \cdot \frac{TV_u}{T} \cdot \frac{nM}{1} \cdot \frac{D}{Du} \text{ weber-meter} \quad (136)
\]

For silver, which was used in the original experiment of 1921, the parameters are:

- \( D_{Ag} := 1.277 \times 10^{-14} \text{ m} \) (from Reciprocal System Data Base)
- \( T_{Ag} := 1000 \text{ K} \) (This is somewhat of an estimate, because in many cases the experimenters didn't really know what the vapor temperature was! Fraser uses this value in his example calculation, Ref. [24], p. 116. This is below the melting point, so there must have been sublimation.)
- \( n_{M_Ag} := 1 \) (This value, for silver, 4-3-(7), can theoretically range from 1 to 4.)

\[
\mu_{\text{adpm}}_{Ag} := \mu_{\text{adpm}} \cdot \frac{TV_u}{T_{Ag}} \cdot \frac{n_{M_Ag}}{1} \cdot \frac{D_{Ag}}{Du} \mu_{\text{adpm}}_{Ag} = 1.2744 \times 10^{-29} \text{ weber-meter for this value of } n_M
\]

The Reciprocal System uses the *Kennelly* magnetic system, whereas Quantum Mechanics uses the *Sommerfeld* magnetic system. It's evident from Jiles, Ref. [30], p. 16, Table 1.2, that we can convert from the Kennelly system to the Sommerfeld system by dividing the Kennelly value by the SI permeability of free space. So:

\[
\mu_{\text{adpm}}_{Ag_{som}} := \frac{\mu_{\text{adpm}}_{Ag}}{\mu_{0_{SI}}} \mu_{\text{adpm}}_{Ag_{som}} = 1.0142 \times 10^{-23} \text{ joule/tesla}
\]

In conventional base units:

\[
\text{weber-m} \times \frac{1}{\mu_{0_{SI}}} = \text{joule/tesla}
\]

\[
\frac{m^3}{\text{coul-sec}} \cdot \frac{\text{coul}^2}{\text{kg} \cdot \text{m}} \rightarrow \frac{\text{coul} \cdot \text{m}^2}{\text{sec}} \quad (\text{checks})
\]
In space-time terms:

\[
\frac{t^2 s^4}{s t^3} \rightarrow \frac{s^3}{t}
\]

(checks)

In Quantum Mechanics, the Bohr magneton has the value

\[
\mu_{B_{SI}} := 9.2741 \cdot 10^{-24} \text{ joule/tesla}
\]

Therefore, in terms of the Bohr magneton,

\[
\frac{\mu_{\text{adpm}_{\text{Ag}_{som}}}}{\mu_{B_{SI}}} = 1.0935
\]

From Quantum Mechanics, the predicted value for the experiment with silver is +/- 1 Bohr magneton, due to the up/down spin of the alleged one unpaired electron (and zero total orbital angular momentum). The experimental error has been estimated to be 10% (Ref. [24], p.138). Both the Reciprocal System and Quantum Mechanics therefore agree with the experimental value. However, the Reciprocal System predicts that if much stronger fields could be used, we could get 2, 3, or even 4 times this value. Thus, it would be desirable for this or a similar experiment to be run with much stronger fields in order to distinguish between the two theories. Both the Reciprocal System and Quantum Mechanics predict changes in atomic magnetic moment with temperature, but with opposite effects, so that would be another test.

Table VII, from Excel, shows the calculations for the 20 elements which have been investigated, together with the observed results, in Bohr magnetons, for convenience. Three of these, Co, N, and Bi have observed values which are too uncertain to be included in the observation column. The shaded cells in the table show the most probably value of \( n_M \). The T column values are estimates obtained (as best as we can) from the references listed above. Eight of the zero values are somewhat dubious, because the Maxwellian distribution of velocities smudged the traces so badly that determination of the separation of the traces was difficult. For the remainder of the 17 elements, the Reciprocal System is in agreement with the experimental values.
<table>
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<tr>
<th>Element</th>
<th>At. No.</th>
<th>Rot. Displ.</th>
<th>n_M max</th>
<th>D (fm)</th>
<th>T_mp (K)</th>
<th>T (K)</th>
<th>mu if n_M=0</th>
<th>mu if n_M=1</th>
<th>mu if n_M=2</th>
<th>mu if n_M=3</th>
<th>mu if n_M=4</th>
<th>mu_obs</th>
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<td>1.000</td>
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<td>3.426</td>
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<td>0.796</td>
<td>1.194</td>
<td>1.000</td>
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<td>1800</td>
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<td>700</td>
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<td>3.411</td>
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<td>3.411</td>
<td>5.117</td>
<td>6.823</td>
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<td></td>
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</table>

Table VII. Stern-Gerlach Experiment Calculations and Observations

The only difficulty here for the Reciprocal System is the claimed atomic magnetic moment of Tl, 1/3 \( \mu_B \). Given that the nearby elements of Hg and Pb have zero moment (no induced magnetic charges), it seems rather odd that Tl would have a non-zero value. Quantum Mechanics explains this value as consistent with spectroscopy. More precise experiments should be conducted to resolve this issue.
Sample Calculation of Trace Separation for Silver

based on Ref. [66], p. 123; Ref. [29], pp. 14-15

\[ m_{\text{Ag}} := 1.79 \times 10^{-25} \text{ kg} \]  
(mass of individual silver atom)

\[ T_{\text{Ag}} := 1000 \text{ K} \]  
(estimated temperature of vapor)

\[ v_{\text{Ag}} := \sqrt{\frac{3.5 \cdot k_B \cdot T_{\text{Ag}}}{m_{\text{Ag}}}} \quad v_{\text{Ag}} = 519.4539 \text{ m/sec} \]  
(effective mean velocity according to Stern average energy \( = (3k_B T + 4k_B T) / 2 \))

\[ \text{dBdz} := 60 \text{ tesla/m} \]  
(magnetic field gradient in z-direction of inhomogeneous field)

\[ F_z := \frac{\mu_{\text{adpm}}_{\text{Ag}}}{\mu_{0 \_SI}} \cdot \text{dBdz} \quad F_z = 6.085 \times 10^{-22} \text{ N} \]  
(z-force on atom causing deflection in \textit{perpendicular} direction, x)

\[ y := .1 \text{ m} \]  
(length of path in y-direction through magnet poles)

\[ s_d := \frac{1}{2} \cdot \frac{1}{m_{\text{Ag}}} \cdot \frac{\mu_{\text{adpm}}_{\text{Ag}}}{\mu_{0 \_SI}} \cdot \text{dBdz} \cdot \frac{y^2}{v_{\text{Ag}}^2} \]  
(deflection from center line, in x-direction)

\[ s_d = 6.2992 \times 10^{-5} \text{ m} \]

\[ s_{\text{tot \_sep}} := 2 \cdot s_d \quad s_{\text{tot \_sep}} = 0.000126 \text{ m} \]  
(total separation of traces)
Ref. [67] is a YouTube video showing the Stern-Gerlach experiment, but with three sets of magnets. Quantum Mechanics fails to give the observed result for this situation, whereas the Reciprocal System does. My comment on this YouTube video is:

"Quantum Mechanics is not the only game in town. A competing theory called the Reciprocal System can explain this gentleman's video, as follows. The neutral atoms (silver, hydrogen, whatever) in the experiment do not initially have a magnetic moment or charge at all. Upon passing through the first magnet, each atom is induced to have two charges (one N and one S). Consequently, the beam splits. Upon leaving the first magnet, the induction ends, the charges revert to thermal energy, etc..." [YouTube does not provide sufficient comment space for a full explanation, so it’s implied.]

Obviously, the same result occurs for the second set and third set--the beam becomes charged (with two poles) and splits in two, each time! If there’s no gap between the sets of magnets, then the charges do not revert back to thermal energy. Either way, the beam is charged (with two poles) and it must split because, by probability, half are oriented in one direction, and half in the other direction! This demonstrates that the Reciprocal System is correct, and Quantum Mechanics is not. However, the conventional theoreticians refuse to accept this verdict and invoke the Heisenberg Uncertainty Principle, which is what they always do when they find themselves in a tight spot.

Oh, and by the way, silver is diamagnetic in the solid state!
7. Ferromagnets and Magnetic Hysteresis Curves

a. Elements

Quantum Mechanics has numerous theories attempting to explain ferromagnetism—but none of them work. Jiles (Ref. [30], pp. 291-321) reviews these attempts and concludes: "It seems therefore that although the Heisenberg model is a useful concept the interactions between electrons in real solids are probably not the simple direct Heisenberg exchange."....[Also] "The drawback of the intinerant electron theory is that it is extremely difficult to make fundamental calculations based on it." [Also] "Therefore although the whole approach of Heitler-London, Heisenberg and Bethe still provides a useful conceptual framework for discussing the magnetic interactions of electrons, the method seems to be ultimately inadequate and we await a better description which can give more accurate values...from first principles.

The waiting is now over. The Reciprocal System is a unified, general theory and so it can be applied to any aspect of the physical world, including ferromagnetism.

Quoting Larson (Ref. [1], pp. 215-216):

"For an explanation in terms of the theory of the universe of motion, we need to consider the nature of the atomic motion. If a two-dimensional positive [time] rotational vibration is added to the three-dimensional combination of motions that constitutes the atom it modifies the magnitudes of those motions, and the product is not the same atom with a magnetic charge, it is an atom of a different kind [an isotope]. A magnetic charge, as a distinct entity, can exist only where an atom is so constituted that there is a portion of the atomic structure that can vibrate two-dimensionally independently of the main body of the atom. The requirements are met, so far as the magnetic rotation is concerned, where this rotation is asymmetric, that is, there are n displacement units in one of the two magnetic dimensions [t₃] and n+1 in the other [tₚ].

"On this basis, the symmetrical B group of elements, which have magnetic rotations 1-1, 2-2, 3-3, and 4-4 are excluded. While the magnetic charge has no third dimension, the electric rotation [t₃] with which it is associated in the three-dimensional motion of the atom must be independent of that associated with the remainder of the atom. The electric rotational displacement must therefore exceed 7, so that one complete unit (7 displacement units plus an initial unit level) can stay with the main body of the magnetic rotation, while the excess applies to the magnetic charge. Furthermore, the electric displacement must be positive, as the reference system cannot accommodate two different
negative displacements (motion in time, [space displacements]), in the same atomic structure. The electronegative divisions (III and IV) are thus totally excluded. The effect of all these exclusions is to confine the magnetic charges to Division II elements of Groups 3A and 4A."

The ferromagnetic elements are listed in Table VIII, together with the properties we will now calculate. These properties include the saturation magnetic flux density at 0 K, $B_s$; the magnetic remanence, $B_r$; the external coercive magnetic flux density, $B_{ext-c}$; and the Curie temperature, $T_c$.

1) saturation magnetic flux density at 0 K, $B_s$

The equations for the susceptibilities given previously for diamagnets and paramagnets do not apply to ferromagnets, because the latter have hysteresis curves, which means that a given external flux density can result in different internal flux densities and hence permeabilities and susceptibilities. Importantly, the reduction factor, $\chi_{u_{\text{SI}}}$ does not apply. However, we can adapt the numerator of Eq. (127a) to calculate the saturation magnetic flux density at 0 K.

$$B_{s_{\text{calc}}} := I_M \frac{n_f \cdot \text{floor} \left( \frac{Z_{uc}}{2} \right) \cdot n_M \cdot M_{u_{\text{weber}}} \cdot \text{edge}_{uc}}{V_{uc_{\text{SI}}}} \text{ tesla} \quad (137a)$$

or

$$B_{s_{\text{calc}}} := I_M \frac{n_f \cdot \text{floor} \left( \frac{Z_{uc}}{2} \right) \cdot n_M \cdot M_{u_{\text{weber}}} \cdot s_0}{V_{uc_{\text{SI}}}} \text{ tesla} \quad (137b)$$

depending on whether the magnetic dipole is along an edge of the unit cell (typically edge_c, if the unit cell is not cubic) or between the atoms the shortest distance apart ($s_0$). $I_M$ is the "magnetic transmission ratio" -- this is similar to the interregional ratio, $I_R$, and the rotational ratio, $I_{d0}$, as described in other Reciprocal System books and papers.
\[ I_R := \frac{1}{156.4444} \quad I_R = 0.0064 \] (used in the atomic force repulsion equation, time region)

\[ I_{d0} := 2.1475 \cdot 10^9 \] (used in the denominator of the natural gravitational force equation, time-space region)

Magnetic flux density has the space-time dimensions

\[ \frac{t^2}{s^4} \]

This can be considered to be the square of the space-time dimensions of one-dimensional force:

\[ \left( \frac{t}{s^2} \right)^2 \]

So, in a sense, the magnetic flux density is a \textit{two-dimensional force or "field intensity."} (This is another one of the reasons why we have no need to use the H-vector of conventional magnetic physics. Also, note that magnetomotive force has the dimensions \(t^2/s^3\), which means that it's really a two-dimensional "potential.") Like the other forces discussed above, there must be a "transmission ratio" or factor in the equation for ferromagnetic \(B_s\). For one dimension, there are three possible orthogonal directions, so the starting ratio is 1/3. But the magnetic flux density is two-dimensional, so this must be squared: \((1/3)^2 = 1/9\). And: a magnetic dipole includes \textit{two charges}, so we finally get for the magnetic transmission ratio:

\[ I_M := \left( \frac{1}{3} \right)^2 \cdot \left( \frac{1}{3} \right)^2 \quad I_M = 0.01235 \] (138)

The detailed calculations follow for each ferromagnetic element. The crystal data come from the Reciprocal System Data Base. The observed values are from the ferromagnetic sections of Ref. [12], Ref. [15], and Ref. [30].
Fe 3-2-8 BCC  
\[ n_M_{\text{Fe}} := 3 \quad Z_{\text{uc}} := 2 \quad n_f := 1 \quad V_{\text{uc SI}} := 23.258 \cdot 10^{-30} \text{ m}^3 \]

\[ \text{edge}_{c_{\text{Fe}}} := \frac{1}{3} V_{\text{uc SI}} \]
\[ \text{edge}_{c_{\text{Fe}}} = 2.8545 \times 10^{-10} \text{ m} \quad \text{(all edges same, 0 K)} \]
\[ s_0_{\text{Fe}} := 2.4825 \cdot 10^{-10} \text{ m} \]

\[ B_{s_{\text{Fe calc}}} := l_M \cdot \frac{n_f \cdot \text{floor} \left( \frac{Z_{\text{uc}}}{2} \right) \cdot n_M_{\text{Fe}} \cdot M_u_{\text{weber}} \cdot \text{edge}_{c_{\text{Fe}}}}{V_{\text{uc SI}}} \]
\[ B_{s_{\text{Fe calc}}} = 2.1832 \quad B_{s_{\text{Fe obs}}} := 2.187 \quad \text{(tesla)} \quad \text{(99.95 \% pure)} \]

Co 3-2-9 FCC  
\[ n_M_{\text{Co}} := 3 \quad Z_{\text{uc}} := 4 \quad n_f := .9 \quad V_{\text{uc SI}} := 42.637 \cdot 10^{-30} \text{ m}^3 \]

\[ \text{edge}_{c_{\text{Co}}} := \frac{1}{3} V_{\text{uc SI}} \]
\[ \text{edge}_{c_{\text{Co}}} = 4.0496 \cdot 10^{-10} \text{ m} \]
\[ s_0_{\text{Co}} := 2.4123 \cdot 10^{-10} \text{ m} \]

\[ B_{s_{\text{Co calc}}} := l_M \cdot \frac{n_f \cdot \text{floor} \left( \frac{Z_{\text{uc}}}{2} \right) \cdot n_M_{\text{Co}} \cdot M_u_{\text{weber}} \cdot s_0_{\text{Co}}}{V_{\text{uc SI}}} \]
\[ B_{s_{\text{Co calc}}} = 1.8116 \quad B_{s_{\text{Co obs}}} := 1.797 \quad \text{(tesla)} \quad \text{(99 \% pure)} \]

(The HCP version of Co gives similar results).
Nickel's nominal rotational displacements are 3-3-(8); so probably only half have the alternative equivalent positive rotational displacements 3-2-10 which can take a ferromagnetic charge. The maximum value of $n_M$ is 2, rather than 3.

$$\text{edge}_c = 3.5175 \times 10^{-10} \quad \text{(all edges)}$$

$$s_0_{\text{Ni}} := 2.5061 \times 10^{-10}$$

(1/2 face diagonal in unit cell, confirms illus. in Ref. [30], p. 229))

$$B_{s_{\text{Ni}}_{\text{calc}}} := \frac{l_M \cdot \left(\frac{Z_{\text{uc}}}{2}\right) \cdot n_{M_{\text{Ni}}} \cdot \text{M}_{\text{weber}} \cdot s_{0_{\text{Ni}}}}{V_{\text{uc}_\text{SI}}}$$

$$B_{s_{\text{Ni}}_{\text{calc}}} = 0.6829 \quad \text{tesla}$$

$$B_{s_{\text{Ni}}_{\text{obs}}} := .653$$

(99% pure)

(The HCP version of Ni gives similar results.)
Rare Earth Elements

All of the rare earth elements have four magnetic charges and are HCP, but the axial ratio (\text{edge}_c/\text{edge}_a) is considerably less than 1.633, which means that the atoms in the same basal plane are usually a little farther apart than those immediately above and below (Wychoff, Ref. [65], p. 10); so, approximately, 50% of the time, the dipole length is \text{edge}_c, and 50% of the time the dipole length is 2 \times \text{edge}_c, because of the alternate misalignment. The average dipole length is then 1.5 \times \text{edge}_c. According to Martin (Ref. [23], p. 76), Tb, Dy, Ho, and Er have "helical anti-ferromagnetism" or "helical ferromagnetism"—this means that the angle of the magnetic moment changes by some angle \( \theta \) with each unit cell basal plane. From the standpoint of the Reciprocal System, this effect seems to be captured by the 1.5 factor for dipole length of these elements. In all cases, except for Tm, the value of \( n_f = 1 \). Tm has a reduction to .9. On the basis of Larson's statement above, elements Sm, Eu, and Yb should be ferromagnetic, but they are anti-ferromagnetic or paramagnetic, unless in compounds (like V, Cr, and Mn). Ce and Nd are also anti-ferromagnetic, except in compounds. Sm is treated as a paramagnet in this paper; Eu is on the fence.

\begin{align*}
\text{Gd} & \quad 4\text{-}3\text{-}10 \quad \text{HCP} \quad n_{M_{\text{Gd}}} := 4 \quad Z_{\text{uc}} := 2 \quad n_f := .8 \quad V_{\text{uc}_\text{SI}} := 65.778 \cdot 10^{-30} \quad \text{m}^3 \\
& \quad \text{edge}_a := 3.4195 \cdot 10^{-10} \quad \text{edge}_{c_{\text{Gd}}} := 5.6255 \cdot 10^{-10} \quad s_0 := 3.5708 \cdot 10^{-10} \quad \text{m} \\
& \quad B_{s_{\text{Gd}_\text{calc}}} := \frac{l_{\text{M}} \cdot n_f \cdot \text{floor} \left( \frac{Z_{\text{uc}}}{2} \right) \cdot n_{M_{\text{Gd}}} \cdot M_{\text{u Weber}} \cdot \text{edge}_{c_{\text{Gd}}}^3}{2 \cdot V_{\text{uc}_\text{SI}}} \quad B_{s_{\text{Gd}_\text{calc}}} = 2.4341 \quad B_{s_{\text{Gd}_\text{obs}}} := 2.488 \quad (\text{tesla})
\end{align*}
\[
\text{Tb} \quad 4-3-11 \quad \text{HCP} \quad n_{M_{\text{Tb}}} := 4 \quad Z_{uc} := 2 \quad n_f := 1 \quad V_{uc_{\text{SI}}} := 62.736 \cdot 10^{-30} \quad \text{m}^3
\]

\[
\text{edge}_a := 3.4001 \cdot 10^{-10} \quad \text{edge}_{c_{\text{Tb}}} := 5.4268 \cdot 10^{-10} \quad s_0 := 3.5028 \cdot 10^{-10} \quad \text{m}
\]

\[
B_{s_{\text{Tb}}_{\text{calc}}} := \frac{l_M \cdot n_f \cdot n_{M_{\text{Tb}}} \cdot M_{\text{u_weber}} \cdot \text{edge}_{c_{\text{Tb}}}^3}{2 \cdot V_{uc_{\text{SI}}}} = 3.0775 \quad B_{s_{\text{Tb}}_{\text{obs}}} := 3.407 \quad \text{(tesla)}
\]

\[
\text{Dy} \quad 4-3-12 \quad \text{HCP} \quad n_{M_{\text{Dy}}} := 4 \quad Z_{uc} := 2 \quad n_f := 1 \quad V_{uc_{\text{SI}}} := 62.972 \cdot 10^{-30} \quad \text{m}^3
\]

\[
\text{edge}_a := 3.3654 \cdot 10^{-10} \quad \text{edge}_{c_{\text{Dy}}} := 5.56 \cdot 10^{-10} \quad s_0 := 3.5089 \cdot 10^{-10} \quad \text{m}
\]

\[
B_{s_{\text{Dy}}_{\text{calc}}} := \frac{l_M \cdot n_f \cdot n_{M_{\text{Dy}}} \cdot M_{\text{u_weber}} \cdot \text{edge}_{c_{\text{Dy}}}^3}{2 \cdot V_{uc_{\text{SI}}}} = 3.1412 \quad B_{s_{\text{Dy}}_{\text{obs}}} := 3.768
\]

\[
\text{Ho} \quad 4-3-13 \quad \text{HCP} \quad n_{M_{\text{Ho}}} := 4 \quad Z_{uc} := 2 \quad n_f := 1 \quad V_{uc_{\text{SI}}} := 60.043 \cdot 10^{-30} \quad \text{m}^3
\]

\[
\text{edge}_a := 3.3589 \cdot 10^{-10} \quad \text{edge}_{c_{\text{Ho}}} := 5.3247 \cdot 10^{-10} \quad s_0 := 3.4570 \cdot 10^{-10} \quad \text{m}
\]

\[
B_{s_{\text{Ho}}_{\text{calc}}} := \frac{l_M \cdot n_f \cdot n_{M_{\text{Ho}}} \cdot M_{\text{u_weber}} \cdot \text{edge}_{c_{\text{Ho}}}^3}{2 \cdot V_{uc_{\text{SI}}}} = 3.155 \quad B_{s_{\text{Ho}}_{\text{obs}}} := 3.909 \quad \text{(tesla)}
\]
\( \textbf{Er} \) 4-3-14 HCP \( n_{M-Er} := 4 \quad Z_{uc} := 2 \quad n_f := 1 \quad V_{uc\_Sl} := 60.848 \cdot 10^{-30} \quad \text{m}^3 \)

\[ \text{edge}_a := 3.3409 \cdot 10^{-10} \quad \text{edge}_{c\_Er} := 5.4516 \cdot 10^{-10} \quad s_0 := 3.4628 \cdot 10^{-10} \quad \text{m} \]

\[ B_{s\_Er\_calc} := I_M \cdot \frac{n_f \cdot \text{floor} \left( \frac{Z_{uc}}{2} \right) \cdot n_{M\_Er} \cdot \text{Mu\_weber} \cdot \text{edge}_{c\_Er}^3}{V_{uc\_Sl}} \]

\[ B_{s\_Er\_calc} = 3.1875 \quad B_{s\_Er\_obs} := 3.439 \quad (\text{tesla}) \]

\( \textbf{Tm} \) 4-3-15 HCP \( n_{M\_Tm} := 4 \quad Z_{uc} := 2 \quad n_f := .9 \quad V_{uc\_Sl} := 58.161 \cdot 10^{-30} \quad \text{m}^3 \)

\[ \text{edge}_a := 3.3350 \cdot 10^{-10} \quad \text{edge}_{c\_Tm} := 5.2294 \cdot 10^{-10} \quad s_0 := 3.4080 \cdot 10^{-10} \quad \text{m} \]

\[ B_{s\_Tm\_calc} := I_M \cdot \frac{n_f \cdot \text{floor} \left( \frac{Z_{uc}}{2} \right) \cdot n_{M\_Tm} \cdot \text{Mu\_weber} \cdot \text{edge}_{c\_Tm}^3}{V_{uc\_Sl}} \]

\[ B_{s\_Tm\_calc} = 2.8789 \quad B_{s\_Tm\_obs} := 2.884 \quad (\text{tesla}) \]
The average ratio of calculated to observed for all of the ferromagnetic elements is

\[
\frac{B_{s\_Fe\_calc}}{B_{s\_Fe\_obs}} + \frac{B_{s\_Co\_calc}}{B_{s\_Co\_obs}} + \frac{B_{s\_Ni\_calc}}{B_{s\_Ni\_obs}} + \frac{B_{s\_Gd\_calc}}{B_{s\_Gd\_obs}} + \frac{B_{s\_Tb\_calc}}{B_{s\_Tb\_obs}} + \frac{B_{s\_Dy\_calc}}{B_{s\_Dy\_obs}} + \frac{B_{s\_Ho\_calc}}{B_{s\_Ho\_obs}} + \frac{B_{s\_Er\_calc}}{B_{s\_Er\_obs}} + \frac{B_{s\_Tm\_calc}}{B_{s\_Tm\_obs}} = 0.9444
\]

Note that our calculations are based on 100% pure crystal unit cells without imperfections. Actual experimental specimens undoubtedly have some impurities and crystal defects, and so that would account for the fairly small average difference between calculated and observed values.
2) magnetic remanence

When the external magnetic flux density goes to zero, the ferromagnet loses some of its magnetism—one or two ferromagnetic charges revert back to thermal energy, leaving the atom with just one or two charges. Also the dipole lengths often shorten to the smallest interatomic distance, \( s_0 \). Temperature seems to have little effect on remanence, and so all calculations here are for room temperature parameters, using Eq. (137a) or (137b).

\[
\text{Fe BCC} \quad n_{M_r,Fe} := 2 \quad Z_{uc} := 2 \quad n_f := 1 \quad V_{uc,SI} := 23.556 \cdot 10^{-30} \quad m^3
\]

\[
\frac{1}{3} \text{edge}_c := V_{uc,SI} \quad \text{edge}_c = 2.8666 \times 10^{-10} \quad s_0 := 2.4886 \cdot 10^{-10} \quad m
\]

\[
B_{r,Fe,calc} := \frac{l_{M} \cdot n_f \cdot \text{floor} \left( \frac{Z_{uc}}{2} \right) \cdot n_{M_r,Fe} \cdot M_{\text{weber}} \cdot s_0}{V_{uc,SI}} \quad B_{r,Fe,calc} = 1.2529 \quad B_{r,Fe,obs} := 1.30 \quad (\text{tesla}) \quad (99.8\% \text{ pure})
\]

\[
\text{Co FCC} \quad n_{M_r,Co} := 1 \quad Z_{uc} := 4 \quad n_f := .9 \quad V_{uc,SI} := 43.238 \cdot 10^{-30} \quad m^3
\]

\[
\frac{1}{3} \text{edge}_c := V_{uc,SI} \quad \text{edge}_c = 3.5098 \times 10^{-10} \quad s_0 := 2.4235 \cdot 10^{-10} \quad m
\]

\[
B_{r,Co,calc} := \frac{l_{M} \cdot n_f \cdot \text{floor} \left( \frac{Z_{uc}}{2} \right) \cdot n_{M_r,Co} \cdot M_{\text{weber}} \cdot s_0}{V_{uc,SI}} \quad B_{r,Co,calc} = 0.5982 \quad B_{r,Co,obs} := .5 \quad (\text{tesla}) \quad (99 \% \text{ pure})
\]

(The HCP version of Co gives similar results).
\( \text{Ni} \) FCC \( n_{M_{r}\_Ni} := 1 \) \( Z_{uc} := 4 \) \( n_f := .5 \) \( V_{uc\_SI} := 43.122 \cdot 10^{-30} \) m\(^3\)

\[
\text{edge}_c := \frac{1}{3} V_{uc\_SI}
\]

\[
\text{edge}_c = 3.5067 \times 10^{-10} \quad s_0 := 2.5178 \cdot 10^{-10} \quad \text{m}
\]

\[
B_{r\_Ni\_calc} := M \cdot \frac{n_f \cdot \text{floor} \left( \frac{Z_{uc}}{2} \right) \cdot n_{M_{r}\_Ni} \cdot M_{u\_weber} \cdot s_0}{V_{uc\_SI}}
\]

\[
B_{r\_Ni\_calc} = 0.3462 \quad B_{r\_Ni\_obs} := .4
\]

(tesla)

\[
\frac{B_{r\_Fe\_calc}}{B_{r\_Fe\_obs}} + \frac{B_{r\_Co\_calc}}{B_{r\_Co\_obs}} + \frac{B_{r\_Ni\_calc}}{B_{r\_Ni\_obs}} = 1.0086
\]

Certainly within the experimental accuracy.

Data for the remanence of rare earth elements are not available. Prediction: their values for \( n_{M_{r}} \) would be 1 or 2, or possibly 3.
3) **external magnetic coercive flux density, \( B_{\text{ext,c}} \)**

The external magnetic coercive flux density (that required to reduce the induction of a magnetized material to zero) depends on the impurities or "foreign" elements present, domain walls, magnetostriction, strain, and heat treatment. It is a secondary property and is determined by the form of the hysteresis curves, which themselves are based on \( B_s, B_r, \) and \( \mu_{\text{r,avg}} \). Other ferromagnetic theories cannot make use of this, because they do not have the correct characterization of the curves.

4) **Curie temperature and magnetic saturation as a function of Curie temperature**

At the Curie temperature, \( T_c \), a ferromagnet converts to a paramagnet. \( T_c \) is the key variable for the curve of saturation magnetic flux density vs. temperature, and as with other such Reciprocal System equations, it must therefore be in the denominator. Whenever the key variable is in the denominator for a time region function, it must be squared. This means that the temperature, \( T \), must be in the numerator, and must also be squared for dimensional balance. At \( T = 0 \), the magnetic flux density is at its maximum value, \( B_s \) and so the factor \( T^2/T_c^2 \) must be subtracted from 1. Thus we obtain

\[
\frac{B_{S,T}}{B_s} = \left[ 1 - \left( \frac{T^2}{T_c^2} \right) \right]^n
\]

Now the question is: what should be the value of the exponent \( n \)? Most likely, \( n \) could be 1, 1/2, 1/3, or 1/4. Let \( x = T/T_c \). The graphs follow.
Based on the experimental data (Ref. [30], p. 231, p. 308), it's clear that $n = 1/4$. This is the same value heuristically derived by Larson, Ref. [1], p. 251. Now we turn to the calculation of $T_c$. 

Figure 26. Variation of Magnetic Saturation, $B_{s,T}/B_s$ with temperature, $T/T_c$
The energy of a magnetic dipole is simply

\[ E_{\text{M\_dipole}} := \frac{n_M \cdot M_{\text{u\_weber}} \cdot n_M \cdot M_{\text{u\_weber}}}{4 \cdot \pi \cdot \mu_0_{\text{SI}} \cdot \mu_r_0 \cdot \text{edge\_uc}} \cdot \text{conv\_joule\_to\_eV} \ \text{eV} \]  

or

\[ E_{\text{M\_dipole}} := \frac{n_M \cdot M_{\text{u\_weber}} \cdot n_M \cdot M_{\text{u\_weber}}}{4 \cdot \pi \cdot \mu_0_{\text{SI}} \cdot \mu_r_0 \cdot s_0} \cdot \text{conv\_joule\_to\_eV} \ \text{eV} \]  

depending on whether the dipole is along the edge of a unit cell or the shortest length between two atoms.

The Reciprocal System Data Base has very precise calculations of enthalpy for solid matter as a function of temperature. But a good approximation for the thermal energy of a solid atom is simply

\[ E_{\text{th}} := 3 \cdot k_B_{\text{ev}} \cdot T \]  

(141)

When \( E_{\text{th\_atom}} = E_{\text{M\_dipole}} \), the ferromagnet converts to a paramagnet. Therefore,

\[ T_c := \left( \frac{n_M \cdot M_{\text{u\_weber}} \cdot n_M \cdot M_{\text{u\_weber}}}{4 \cdot \pi \cdot \mu_0_{\text{SI}} \cdot \mu_r_0 \cdot \text{edge\_uc}} \cdot \text{conv\_joule\_to\_eV} \right) \cdot \frac{1}{3 \cdot k_B_{\text{ev}}} \]  

(142a)
or
\[
T_c := \frac{n_M \cdot M_{u\text{-weber}} \cdot n_M \cdot M_{u\text{-weber}}}{4 \cdot \pi \cdot \mu_0 \cdot \mu_r \cdot 0 \cdot s_0} \cdot \text{convjoulestoev}
\]

\[
\frac{3 \cdot k_B \cdot \text{ev}}{3k_B \cdot \text{ev}}
\]

(142b)

Of course, there is a probability distribution of temperatures in solids, and so some atoms will convert before others; that's why the transition from ferromagnetism to paramagnetism takes place over a large zone of temperatures centered on \( T_c \).

**Fe**

\[
E_{M\_Fe} := \frac{n_{M\_Fe} \cdot M_{u\text{-weber}} \cdot n_{M\_Fe} \cdot M_{u\text{-weber}}}{4 \cdot \pi \cdot \mu_0 \cdot \mu_r \cdot 0 \cdot \text{edge}_c \_Fe} \cdot \text{convjoulestoev}
\]

\[
E_{M\_Fe} = 0.2875 \quad \text{eV}
\]

\[
T_{c\_Fe\_calc} := \frac{E_{M\_Fe}}{3 \cdot k_B \cdot \text{ev}}\quad T_{c\_Fe\_calc} = 1112.1074 \quad T_{c\_Fe\_obs} := 1043.15 \quad \text{K}
\]

**Co**

\[
E_{M\_Co} := \frac{n_{M\_Co} \cdot M_{u\text{-weber}} \cdot n_{M\_Co} \cdot M_{u\text{-weber}}}{4 \cdot \pi \cdot \mu_0 \cdot \mu_r \cdot 0 \cdot \text{edge}_c \_Co} \cdot \text{convjoulestoev}
\]

\[
E_{M\_Co} = 0.3402 \quad \text{eV}
\]

\[
T_{c\_Co\_calc} := \frac{E_{M\_Co}}{3 \cdot k_B \cdot \text{ev}}\quad T_{c\_Co\_calc} = 1315.9505 \quad T_{c\_Co\_obs} := 1403.15 \quad \text{K}
\]
Ni

\[ E_{M_{\text{Ni}}} := \frac{n_{\text{M}_{\text{Ni}}} \cdot M_{\text{u}_\text{weber}} \cdot n_{\text{M}_{\text{Ni}}} \cdot M_{\text{u}_\text{weber}}}{4 \cdot \pi \cdot \mu_0 \cdot \mu_r \cdot \mu_0_{\text{SI}}} \cdot \text{conv joule to eV} \quad E_{M_{\text{Ni}}} = 0.1455 \quad \text{eV} \]

\[ T_{c_{\text{Nicalc}}} := \frac{E_{M_{\text{Ni}}}}{3 \cdot k_B \cdot \text{ev}} \quad T_{c_{\text{Ni calc}}} = 562.9761 \quad \text{K} \]

\[ T_{c_{\text{Niobs}}} := 631.15 \quad \text{K} \]

There is considerable uncertainty in the observed values of \( T_c \) for the rare earth elements; nonetheless, we'll make the attempt at a comparison of theory to experiment. Also, note that these elements have the 3/2 factor applied to their unit cell edge lengths.
Gd

\[ n_{M_{\text{Gd}}} := 3 \quad \text{apparently Gd loses a unit of magnetic charge prior to converting to a paramagnet} \]

\[ E_{M_{\text{Gd}}} := \frac{n_{M_{\text{Gd}}} \cdot M_{\text{u}_\text{weber}} \cdot M_{\text{u}_\text{weber}}}{4 \cdot \pi \cdot \mu_0 \cdot \mu_r \cdot \text{edge}_c_{\text{Gd}} \cdot \frac{3}{2}} \cdot \text{convjoulestoev} \quad E_{M_{\text{Gd}}} = 0.0973 \quad \text{eV} \]

\[ T_{c_{\text{Gd}}_{\text{calc}}} := \frac{E_{M_{\text{Gd}}}}{3 \cdot k_B_{\text{ev}}} \quad T_{c_{\text{Gd}}_{\text{calc}}} = 376.1997 \quad T_{c_{\text{Gd}}_{\text{obs}}} := 293 \quad \text{K} \]

Tb

\[ n_{M_{\text{Tb}}} := 2 \quad \text{apparently Tb loses two units of magnetic charge prior to converting to a paramagnet} \]

\[ E_{M_{\text{Tb}}} := \frac{n_{M_{\text{Tb}}} \cdot M_{\text{u}_\text{weber}} \cdot M_{\text{u}_\text{weber}}}{4 \cdot \pi \cdot \mu_0 \cdot \mu_r \cdot \text{edge}_c_{\text{Tb}} \cdot \frac{3}{2}} \cdot \text{convjoulestoev} \quad E_{M_{\text{Tb}}} = 0.0448 \quad \text{eV} \]

\[ T_{c_{\text{Tb}}_{\text{calc}}} := \frac{E_{M_{\text{Tb}}}}{3 \cdot k_B_{\text{ev}}} \quad T_{c_{\text{Tb}}_{\text{calc}}} = 173.3218 \quad T_{c_{\text{Tb}}_{\text{obs}}} := 220 \quad \text{K} \]
Dy

\[ n_{M_{\text{Dy}}} := 1 \] apparently Dy loses three units of magnetic charge prior to converting to a paramagnet

\[ E_{M_{\text{Dy}}} := \frac{n_{M_{\text{Dy}}} \cdot M_{\text{u weber}} \cdot n_{M_{\text{Dy}}} \cdot M_{\text{u weber}}}{4 \cdot \pi \cdot \mu_0 \cdot \mu_r \cdot \text{edge}^3_{c_{\text{Dy}}}} \cdot \text{conv joule to ev} \quad E_{M_{\text{Dy}}} = 0.0109 \text{ eV} \]

\[ T_{c_{\text{Dy calc}}} := \frac{E_{M_{\text{Dy}}}}{3 \cdot k_B \cdot \text{ev}} \quad T_{c_{\text{Dy calc}}} = 42.2924 \quad T_{c_{\text{Dy obs}}} := 87 \text{ K} \]

Ho

\[ n_{M_{\text{Ho}}} := 1 \] apparently Ho loses three units of magnetic charge prior to converting to a paramagnet

\[ E_{M_{\text{Ho}}} := \frac{n_{M_{\text{Ho}}} \cdot M_{\text{u weber}} \cdot n_{M_{\text{Ho}}} \cdot M_{\text{u weber}}}{4 \cdot \pi \cdot \mu_0 \cdot \mu_r \cdot \text{edge}^3_{c_{\text{Ho}}}} \cdot \text{conv joule to ev} \quad E_{M_{\text{Ho}}} = 0.0114 \text{ eV} \]

\[ T_{c_{\text{Ho calc}}} := \frac{E_{M_{\text{Ho}}}}{3 \cdot k_B \cdot \text{ev}} \quad T_{c_{\text{Ho calc}}} = 44.1613 \quad T_{c_{\text{Ho obs}}} := 20 \text{ K} \]

Er

\[ n_{M_{\text{Er}}} := 1 \] apparently Er loses three units of magnetic charge prior to converting to a paramagnet

\[ E_{M_{\text{Er}}} := \frac{n_{M_{\text{Er}}} \cdot M_{\text{u weber}} \cdot n_{M_{\text{Er}}} \cdot M_{\text{u weber}}}{4 \cdot \pi \cdot \mu_0 \cdot \mu_r \cdot \text{edge}^3_{c_{\text{Er}}}} \cdot \text{conv joule to ev} \quad E_{M_{\text{Er}}} = 0.0112 \text{ eV} \]
\[ T_{c\_Er\_calc} := \frac{E_{M\_Er}}{3 \cdot k_{B\_ev}} \quad T_{c\_Er\_calc} = 43.1333 \quad T_{c\_Er\_obs} := 32 \quad K \]

**Tm**

\[ n_{M\_Tm} := 1 \]

apparently Tm loses three units of magnetic charge prior to converting to a paramagnet

\[ E_{M\_Tm} := \frac{n_{M\_Tm} \cdot M_{\text{u\_weber}} \cdot n_{M\_Tm} \cdot M_{\text{u\_weber}}}{4 \cdot \pi \cdot \mu_0 \cdot \mu_r \cdot \text{magnetic field}_{\text{c\_Tm}} \cdot 3} \cdot \text{conv\_joule\_to\_ev} \]

\[ E_{M\_Tm} = 0.0116 \quad \text{eV} \]

\[ T_{c\_Tm\_calc} := \frac{E_{M\_Tm}}{3 \cdot k_{B\_ev}} \quad T_{c\_Tm\_calc} = 44.9661 \quad T_{c\_Tm\_obs} := 32 \quad K \]

Note that for the last four elements the average observed value is:

\[ \frac{T_{c\_Dy\_obs} + T_{c\_Ho\_obs} + T_{c\_Er\_obs} + T_{c\_Tm\_obs}}{4} = 42.75 \quad K \]

For these elements, the average calculated value is:

\[ \frac{T_{c\_Dy\_calc} + T_{c\_Ho\_calc} + T_{c\_Er\_calc} + T_{c\_Tm\_calc}}{4} = 43.6383 \quad K \]

For all the ferromagnetic elements:
So 10% off, on average. Of course actual solid atoms have energy a bit different from $3 \times k_B \times T$. Also, there are impurities and crystal defects. Nonetheless, our calculations seem to be close enough to confirm the theory, particularly as there is a great amount of experimental uncertainty regarding the rare earth elements. And keep in mind that the experimenters may be measuring the temperature at which essentially all atoms have converted--by the probability distribution of temperatures in solids, this bulk temperature could be considerably higher than $T_c$. 

\[
\frac{T_{c\, Fe\, calc}}{T_{c\, Fe\, obs}} + \frac{T_{c\, Co\, calc}}{T_{c\, Co\, obs}} + \frac{T_{c\, Ni\, calc}}{T_{c\, Ni\, obs}} + \frac{T_{c\, Gd\, calc}}{T_{c\, Gd\, obs}} \ldots \\
\frac{T_{c\, Tb\, calc}}{T_{c\, Tb\, obs}} + \frac{T_{c\, Dy\, calc}}{T_{c\, Dy\, obs}} \ldots \\
\frac{T_{c\, Ho\, calc}}{T_{c\, Ho\, obs}} + \frac{T_{c\, Er\, calc}}{T_{c\, Er\, obs}} + \frac{T_{c\, Tm\, calc}}{T_{c\, Tm\, obs}} = 1.0415
\]
Now we will calculate the value of the magnetic saturation flux density (in tesla) at room temperature (300 K here) for the ferromagnetic elements other than the rare earth elements—for these an appropriate temperature between 0 and $T_c$ will be used.

$$
Bs_{Fe\_300} := Bs_{Fe\_calc} \left( 1 - \left( \frac{300}{T_{c\_Fe\_calc}} \right)^2 \right)^{\frac{1}{4}}
$$

$$
Bs_{Fe\_300} = 2.1423
$$

$$
Bs_{Fe\_300\_obs} := 2.15 \quad \text{(Ref. [30], p. 165)}
$$

$$
Bs_{Co\_300} := Bs_{Co\_calc} \left( 1 - \left( \frac{300}{T_{c\_Co\_calc}} \right)^2 \right)^{\frac{1}{4}}
$$

$$
Bs_{Co\_300} = 1.7876
$$

$$
Bs_{Co\_300\_obs} := 1.759 \quad \text{(Ref. [30], p. 165)}
$$

$$
Bs_{Ni\_300} := Bs_{Ni\_calc} \left( 1 - \left( \frac{300}{T_{c\_Ni\_calc}} \right)^2 \right)^{\frac{1}{4}}
$$

$$
Bs_{Ni\_300} = 0.6282
$$

$$
Bs_{Ni\_300\_obs} := 0.603 \quad \text{(Ref. [30], p. 165)}
$$

$$
Bs_{Gd\_160} := Bs_{Gd\_calc} \left( 1 - \left( \frac{160}{T_{c\_Gd\_calc}} \right)^2 \right)^{\frac{1}{4}}
$$

$$
Bs_{Gd\_160} = 2.3157
$$

$$
Bs_{Gd\_160\_obs} := 2.194 \quad \text{(Ref. [15], p. 766)}
$$

$$
Bs_{Tb\_100} := Bs_{Tb\_calc} \left( 1 - \left( \frac{100}{T_{c\_Tb\_calc}} \right)^2 \right)^{\frac{1}{4}}
$$

$$
Bs_{Tb\_100} = 2.7813
$$

$$
Bs_{Tb\_100\_obs} := 3.17 \quad \text{(Ref. [15], p. 766)}
$$
\[ B_{s,Dy\_30} := B_{s,Dy\_calc} \left[ 1 - \left( \frac{20}{T_{c,Dy\_calc}} \right)^2 \right]^{\frac{1}{4}} \]

\[ B_{s,Dy\_30} = 2.9486 \]

\[ B_{s,Dy\_30\_obs} := 3.498 \quad \text{Ref. [15], p. 766) } \]

\[ B_{s,Ho\_10} := B_{s,Ho\_calc} \left[ 1 - \left( \frac{10}{T_{c,Ho\_calc}} \right)^2 \right]^{\frac{1}{4}} \]

\[ B_{s,Ho\_10} = 3.1138 \]

\[ B_{s,Ho\_10\_obs} := 3.739 \quad \text{(Ref. [15], p. 767)} \]

\[ B_{s,Er\_18} := B_{s,Er\_calc} \left[ 1 - \left( \frac{18}{T_{c,Er\_calc}} \right)^2 \right]^{\frac{1}{4}} \]

\[ B_{s,Er\_18} = 3.0386 \]

\[ B_{s,Er\_18\_obs} := 2.981 \quad \text{(Ref. [15], p. 767)} \]

\[ B_{s,Tm\_20} := B_{s,Tm\_calc} \left[ 1 - \left( \frac{20}{T_{c,Tm\_calc}} \right)^2 \right]^{\frac{1}{4}} \]

\[ B_{s,Tm\_20} = 2.7246 \]

\[ B_{s,Tm\_20\_obs} := 2.787 \quad \text{(Ref. [15], p. 767)} \]

Note: The values from Ref. [15] are in graph form expressed in units of \( A \times m^2 /kg = \text{emu/g} \). They have been converted to tesla by multiplying the value taken from the graph by the density in g/cm\(^3\) and by \(4\pi \times 10^{-4}\).
\[
\frac{B_{s,Fe\_300}}{B_{s,Fe\_300\_obs}} + \frac{B_{s,Ni\_300}}{B_{s,Ni\_300\_obs}} + \frac{B_{s,Co\_300}}{B_{s,Co\_300\_obs}} + \frac{B_{s,Gd\_160}}{B_{s,Gd\_160\_obs}} + \frac{B_{s,Tb\_100}}{B_{s,Tb\_100\_obs}} + \frac{B_{s,Dy\_30}}{B_{s,Dy\_30\_obs}} + \frac{B_{s,Ho\_10}}{B_{s,Ho\_10\_obs}} + \frac{B_{s,Er\_18}}{B_{s,Er\_18\_obs}} + \frac{B_{s,Tm\_20}}{B_{s,Tm\_20\_obs}} = 0.9622
\]

Therefore the calculations are within 5% of the observed values, which is quite probably within the experimental uncertainty, especially for the rare earth elements. The expression used in conventional physics works quite well for Fe, Co, and Ni, but does not work very well for the rare earth elements. See the figure on p. 99, Vol. 3, of Ref. [7]. In a sense, this confirms both the calculation of $T_c$ and the use of the 1/4 exponent.
5) magnetic hysteresis curves

According to Jiles, Ref. [30], p. 148:

"When a magnetic field is applied to a demagnetized ferromagnetic material the changes in magnetic induction B when traced on the B, H plane generate the initial magnetization curve. At low fields the first domain process occurs which is a growth of domains which are aligned favorably with respect to the field according to a minimization of the field energy ... and a consequent reduction in size of domains which are aligned in directions opposing the field....

"At moderate field strengths a second mechanism becomes significant; this is domain rotation, in which the atomic magnetic moments within an unfavorably aligned domain overcome the anisotropy energy and suddenly rotate from their original direction of magnetization into one of the crystallographic 'easy' axes which is nearest to the field direction.

"The final domain process which occurs at high fields is coherent rotation. In this process the magnetic moments, which are all aligned along the preferred magnetic crystallographic easy axes lying close to the the field direction, are gradually rotated into the field direction as the magnitude of the field is increased. This results in a single-domain sample."

For solid state properties, conventional theoretical physicists use rather complicated statistical functions like the Fermi-Dirac distribution to describe the motion of their charged electrons—which they mistakenly use to explain valence, atomic-molecular bonding, spectroscopy, electric currents, and magnetism. This is in contrast with the Reciprocal System, where we commonly use the Gaussian or Normal distribution to describe the solid state properties. It's rather obvious, from inspection, that the ferromagnetic hysteresis curves are cumulative probability distributions representing the fraction of unit cells fully magnetized in the direction of the external field (and reference system), and so the error function, erf, applies. (This function is somewhat misnamed—it's simply an integration of the Gaussian function.) The external flux density, $B_{ext}$, must be divided by its maximum value, $B_{ext_{max}}$, to obtain an input variable ranging between -1 and 1:

$$B_{\text{ext\_ratio}} := \frac{B_{\text{ext}}}{B_{\text{ext\_max}}}$$  \hspace{1cm} (143)
Then, with two statistical parameters, $\alpha$ and $\beta$, the argument for the error function will be

$$\frac{B_{\text{ext\_ratio}} + \alpha}{\beta}$$

With this, the \textit{internal} magnetic flux density can be expressed as

$$B_{\text{int}} := \text{erf}\left(\frac{B_{\text{ext\_ratio}} + \alpha}{\beta}\right) \cdot B_{S\_T}$$  \hspace{1cm} (144)$$

where $B_{S\_T} =$ the saturation internal magnetic flux density at the temperature $T$. For the hysteresis plots which follow, the abscissa ($B_{\text{ext\_ratio}}$) will range from -1 to +1. For proper scaling (and to visually show $\mu_r$), the ordinate \textit{must be scaled by} $B_{\text{ext\_max}}$, as well.

The value of the statistical parameter $\alpha$ can be expressed in terms of $B_{\text{ext\_max}}$, as follows:

At $B_{\text{ext}} = B_{\text{ext\_c}}$ (the external flux density at the coercivity point, where the demagnetizing curve crosses from positive to negative $B_{\text{int}} = 0$, and therefore we can solve for $\alpha$.

$$B_{\text{ext\_ratio}} := \frac{B_{\text{ext\_c}}}{B_{\text{ext\_max}}} \hspace{1cm} \text{at point of coercivity}$$

$$0 = \text{erf}\left(\frac{B_{\text{ext\_c}} + \alpha}{B_{\text{ext\_max}}}\right) \cdot B_{S\_T}$$
\[ \alpha := \frac{-B_{ext\_c}}{B_{ext\_max}} \]  

(145)

Data for ferromagnetic materials usually include the values of \( B_{s\_T} \), \( B_r \) (the remanent induction—the internal flux density \( B_{ext} = 0 \)), and \( \mu_{r\_avg} \) (the average permeability in the second quadrant between \( B_{ext} = 0 \) and \( B_c \)). Note: some of the tables give \( \mu_{r\_max} \), for the first quadrant; but because the magnetizing and demagnetizing curves are mostly parallel, one can use the value of \( \mu_{r\_max} \), at least as a starting point for iteration, for \( \mu_{r\_avg} \).

Now we will compute the hysteresis plots for the three main ferromagnetic materials, Fe, Co, Ni. Note: we will use the observed values of \( B_{s\_T} \), \( B_r \), and \( \mu_{r\_avg} \), as input because our purpose here is to show the validity of the use of the erf to describe the hysteresis curves. The previously calculated values of \( B_{s\_T} \), \( B_r \) could, of course, be used but would hardly change the results. Keep in mind, as well, that even a small percentage of impurities have a big impact on hysteresis. With \( \mu_{r\_avg} \) given, it's easy to see that

\[ B_{ext\_c} := -\frac{B_r}{\mu_{r\_avg}} \]  

(146)

(Or, if \( B_{ext\_c} \) is more certain, then we can get \( \mu_{r\_avg} \) from this equation.) In what follows, for economy, we will simply use \( B_s \) to represent \( B_{s\_T} \). It's understood that we're computing the properties at room temperature.

To find the hysteresis loss—the area enclosed by the two curves—we subtract the area below the magnetizing curve from the area below the demagnetizing curve (dividing by \( \mu_{0\_SI} \) to get the correct units):

\[
W_H := \int_{-1}^{1} \left( \text{erf} \left( \frac{B_{ext\_ratio} + \alpha}{\beta} \right) \cdot B_s \cdot \frac{B_{ext\_max}}{\mu_{0\_SI}} - \text{erf} \left( \frac{B_{ext\_ratio} - \alpha}{\beta} \right) \cdot B_s \cdot \frac{B_{ext\_max}}{\mu_{0\_SI}} \right) dB_{ext\_ratio}
\]  

(147)
**99.95% Pure Iron**  Date from Ref. [30], p. 113 and Ref. [12], p. 4-142 and Ref. [16], p. 12-108, averaged

- $B_S := 2.15$ tesla (internal magnetic flux density saturation)
- $B_r := 1.30$ tesla (internal remanent magnetic flux density)
- $\mu_{r\text{avg}} := 220000$ (avg. permeability, 2nd quad.–the reference max. values range from 180000 to 350000)
- $B_{\text{ext\_c}} := \frac{-B_r}{\mu_{r\text{avg}}}$ tesla $B_{\text{ext\_c}} = -5.9091 \times 10^{-6}$ (external magnetic flux density at coercivity point)
- $W_{H\text{_Fe\_obs\_99.95}} := 40 \ J/m^3$ (reference values range from 30 to 60)

There are two equations in two unknowns, $\beta$ and $B_{\text{ext\_max}}$, plus we would like to minimize the error in computing $W_H$.

Starting Values: $B_{\text{ext\_max}} := 2 \times 10^{-4}$ tesla $\beta := .5$

$TOL := .000001$ $CTOL := .000001$

Given

$$\mu_{r\text{avg}} = \frac{\text{erf}\left(\frac{-B_{\text{ext\_c}}}{B_{\text{ext\_max}}/\beta}\right) \cdot B_S}{0 - B_{\text{ext\_c}}} - 0$$

$$B_r = \text{erf}\left(\frac{-B_{\text{ext\_c}}}{B_{\text{ext\_max}}/\beta}\right) \cdot B_S$$

$$\int_{-1}^{1} \text{erf}\left[\frac{B_{\text{ext\_ratio}} + \left(-\frac{B_{\text{ext\_c}}}{B_{\text{ext\_max}}/\beta}\right)}{\beta}\right] \cdot B_S \cdot \frac{B_{\text{ext\_max}}}{\mu_0\text{SI}} - \text{erf}\left[\frac{B_{\text{ext\_ratio}} - \left(-\frac{B_{\text{ext\_c}}}{B_{\text{ext\_max}}/\beta}\right)}{\beta}\right] \cdot B_S \cdot \frac{B_{\text{ext\_max}}}{\mu_0\text{SI}} \ dB_{\text{ext\_ratio}} = W_{H\text{_Fe\_o}}$$
\[
\left( \frac{B_{\text{ext\_max}}}{\beta} \right) := \text{Find}\left( \frac{B_{\text{ext\_max}}}{\beta} \right) \\
B_{\text{ext\_max}} = 1.9583 \times 10^{-5} \quad \alpha := \frac{-B_{\text{ext\_c}}}{B_{\text{ext\_max}}} \quad \alpha = 0.3017
\]
\[
\beta = 0.5021
\]
\[
\mu_{r_{\text{calc}}} := \frac{\text{erf}\left( \frac{\alpha}{\beta} \right) \cdot B_s - 0}{0 - B_{\text{ext\_c}}} \quad \mu_{r_{\text{calc}}} = 2.2 \times 10^5 \quad B_{r_{\text{calc}}} := \text{erf}\left( \frac{\alpha}{\beta} \right) \cdot B_s \quad B_{r_{\text{calc}}} = 1.3 \text{ tesla}
\]
\[
\frac{\mu_{r_{\text{calc}}}}{\mu_{r_{\text{avg}}}} = 1 \quad \frac{B_{r_{\text{calc}}}}{B_r} = 1
\]
\[
W_{H_{\text{Fe\_99.95}}} := \int_{-1}^{1} \text{erf}\left( \frac{B_{\text{ext\_ratio}} + \alpha}{\beta} \right) \cdot B_s \cdot \frac{B_{\text{ext\_max}}}{\mu_0_{\text{SI}}} - \text{erf}\left( \frac{B_{\text{ext\_ratio}} - \alpha}{\beta} \right) \cdot B_s \cdot \frac{B_{\text{ext\_max}}}{\mu_0_{\text{SI}}} \cdot dB_{\text{ext\_ratio}}
\]
\[
W_{H_{\text{Fe\_99.95}}} = 40 \text{ J/m}^3
\]
Figure 27. Hysteresis Curves for 99.95 % Iron

\[
B_{r,Fe} := B_r \quad B_{\text{ext}_c,Fe} := B_{\text{ext}_c}
\]  
(for use in the section on compounds and alloys)
99.8 % Pure Iron

Date from Ref. [30], p. 113, Ref. [12], p. 4-142, Ref. [69], p. 739

\( B_s := 2.14 \) tesla \hspace{1cm} \text{(internal magnetic flux density saturation)}

\( B_r := .77 \) tesla \hspace{1cm} \text{(internal remanent magnetic flux density)}

\( \mu_{r_{avg}} := 9000 \) \hspace{1cm} \text{(avg. permeability for 2nd quadrant, adjusted to get} \ B_{ext_{c}} \text{correct)}

\( B_{ext_{c}} := \frac{-B_r}{\mu_{r_{avg}}} \) tesla \hspace{0.5cm} \( B_{ext_{c}} = -8.5556 \times 10^{-5} \) \hspace{1cm} \text{(external magnetic flux density at coercivity point)}

\( W_{H_{Fe\_99.8}} := 500 \) J/m\(^3\) \hspace{1cm} \text{(Ref. [30], p. 113)}

Starting Values: \( B_{ext\_max} := 1 \cdot 10^{-3} \) tesla \hspace{0.5cm} \( \beta := .5 \)

TOL := .000001 \hspace{1cm} CTOL := .000001

Given

\[
\mu_{r_{avg}} = \frac{erf\left(\frac{-B_{ext_{c}}}{B_{ext\_max}}\right)}{\frac{\beta}{0 - B_{ext_{c}}}} \cdot B_s - 0
\]

\[
B_r = erf\left(\frac{-B_{ext_{c}}}{B_{ext\_max}}\right) \cdot B_s
\]

\[
\int_{-1}^{1} \left[ erf\left(\frac{B_{ext\_ratio} + \alpha}{\beta}\right) \cdot B_s \cdot \frac{B_{ext\_max}}{\mu_{0\_SI}} - erf\left(\frac{B_{ext\_ratio} - \alpha}{\beta}\right) \cdot B_s \cdot \frac{B_{ext\_max}}{\mu_{0\_SI}} \right] dB_{ext\_ratio} = W_{H_{Fe\_99.8}}
\]
\[
\left( \frac{B_{\text{ext\_max}}}{\beta} \right) := \text{Find}(B_{\text{ext\_max}}, \beta) \quad B_{\text{ext\_max}} = 0.0003 \quad \beta = 0.9183 \quad \alpha := \frac{-B_{\text{ext\_c}}}{B_{\text{ext\_max}}} \quad \alpha = 0.3035
\]
\[
\mu_r_{\text{calc}} := \frac{\text{erf} \left( \frac{\alpha}{\beta} \right) \cdot B_s - 0}{0 - B_{\text{ext\_c}}} \quad \mu_r_{\text{calc}} = 9000 \quad B_{r\text{\_calc}} := \text{erf} \left( \frac{\alpha}{\beta} \right) \cdot B_s \quad B_{r\text{\_calc}} = 0.77 \text{ tesla}
\]
\[
\frac{\mu_r_{\text{calc}}}{\mu_r_{\text{avg}}} = 1 \quad \frac{B_{r\text{\_calc}}}{B_r} = 1
\]
\[
W_{H\text{\_Fe\_99.8}} := \int_{-1}^{1} \frac{\text{erf} \left( \frac{B_{\text{ext\_ratio}} + \alpha}{\beta} \right) \cdot B_s \cdot B_{\text{ext\_max}}}{\mu_0\text{\_SI}} - \text{erf} \left( \frac{B_{\text{ext\_ratio}} - \alpha}{\beta} \right) \cdot B_s \cdot B_{\text{ext\_max}}}{\mu_0\text{\_SI}} \ dB_{\text{ext\_ratio}}
\]
\[
W_{H\text{\_Fe\_99.8}} = 502.8717 \text{ J/m}^3
\]
Figure 28. Hysteresis Curves for 99.8 % Pure Iron
**99% Pure Cobalt**  
Date from Ref. [12], p. 4-142 and Ref. [30], p. 165

\[ B_s := 1.759 \text{ tesla} \quad \text{(internal magnetic flux density saturation)} \]

\[ B_r := .5 \text{ tesla} \quad \text{(internal remanent magnetic flux density)} \]

\[ \mu_{r\_avg} := 250 \quad \text{(avg. permeability in 2nd quadrant)} \]

\[ B_{ext\_c} := \frac{-B_r}{\mu_{r\_avg}} \text{ tesla} \quad B_{ext\_c} = -0.002 \quad \text{(external magnetic flux density at coercivity point)} \]

The references do not have a value for \( W_{H\_Co\_obs} \)

**Starting Values:**

\[ B_{ext\_max} := 4 \cdot 10^{-3} \text{ tesla} \quad \beta := .13 \]

\[ TOL := .000001 \quad CTOL := .000001 \]

**Given**

\[
\mu_{r\_avg} = \frac{\text{erf}\left(\frac{-B_{ext\_c}}{B_{ext\_max}}\right) \cdot B_s}{0 - B_{ext\_c}}
\]

\[
B_r = \text{erf}\left(\frac{-B_{ext\_c}}{B_{ext\_max}}\right) \cdot B_s
\]

\[
\left(\frac{B_{ext\_max}}{\beta}\right) := \text{Find}(B_{ext\_max}, \beta) \quad B_{ext\_max} = 0.0241 \quad \beta = 0.3227 \quad \alpha := \frac{-B_{ext\_c}}{B_{ext\_max}} \quad \alpha = 0.0831
\]
\[
\mu_{r\_calc} := \frac{\text{erf}\left(\frac{\alpha}{\beta}\right) \cdot B_S - 0}{0 - B_{\text{ext\_c}}} \\
\mu_{r\_calc} = 250 \\
B_{r\_calc} := \text{erf}\left(\frac{\alpha}{\beta}\right) \cdot B_S \\
B_{r\_calc} = 0.5 \text{ tesla}
\]

\[
\frac{\mu_{r\_calc}}{\mu_{r\_avg}} = 1 \\
\frac{B_{r\_calc}}{B_r} = 1
\]

Figure 29. Hysteresis Curves for 99% Pure Cobalt
\[ \begin{align*}
W_{HCo} &:= \int_{-1}^{1} \operatorname{erf}\left(\frac{B_{\text{ext\_ratio}} + \alpha}{\beta}\right) \cdot B_s \cdot \frac{B_{\text{ext\_max}}}{\mu_0 SI} - \operatorname{erf}\left(\frac{B_{\text{ext\_ratio}} - \alpha}{\beta}\right) \cdot B_s \cdot \frac{B_{\text{ext\_max}}}{\mu_0 SI} \cdot dB_{\text{ext\_ratio}} \\
W_{HCo} &= 11197.9458 \text{ J/m}^3 \quad \text{(a prediction)}
\end{align*} \]

\[ \begin{align*}
B_{rCo} &:= B_r \quad B_{\text{ext\_c\_Co}} &:= B_{\text{ext\_c}} \quad \text{(for use in the section on compounds and alloys)}
\end{align*} \]
**99% Pure Nickel**  
Date from Ref. [12], p. 4-142 and Ref. [30], p. 165

\[ B_s := 0.603 \text{ tesla} \quad \text{(internal magnetic flux density saturation)} \]

\[ B_r := 0.4 \text{ tesla} \quad \text{(internal remanent magnetic flux density)} \]

\[ \mu_{r_{\text{avg}}} := 600 \quad \text{(avg. permeability, considered to be approximately the same as the max. given in the data)} \]

\[ B_{\text{ext}_c} := \frac{-B_r}{\mu_{r_{\text{avg}}}} \text{ tesla} \quad B_{\text{ext}_c} = -0.0007 \quad \text{(external magnetic flux density at coercivity point)} \]

The references do not have a value for \( W_{H_{\text{Ni}}_{\text{obs}}} \)

Starting Values:  
\[ B_{\text{ext}_{\text{max}}} := 5 \cdot 10^{-3} \text{ tesla} \quad \beta := 0.8 \]

\[ TOL := 0.00001 \quad CTOL := 0.00001 \]

Given

\[ \mu_{r_{\text{avg}}} = \text{erf} \left( \frac{-B_{\text{ext}_c}}{\frac{B_{\text{ext}_{\text{max}}}}{\beta}} \cdot B_s - 0 \right) \]

\[ B_r = \text{erf} \left( \frac{-B_{\text{ext}_c}}{\frac{B_{\text{ext}_{\text{max}}}}{\beta}} \right) \cdot B_s \]

\[
\begin{cases}
  B_{\text{ext}_{\text{max}}} := \text{Find} \left( B_{\text{ext}_{\text{max}}} ; \beta \right) & B_{\text{ext}_{\text{max}}} = 0.0026 \\
  \beta = 0.3803 & \alpha := \frac{-B_{\text{ext}_c}}{B_{\text{ext}_{\text{max}}}} \\
  \alpha = 0.2584 & \end{cases}
\]
\[ \mu_{r_{\text{calc}}} := \frac{\text{erf} \left( \frac{\alpha}{\beta} \right) \cdot B_S - 0}{0 - B_{\text{ext}_c}} \]

\[ \mu_{r_{\text{calc}}} = 600 \]

\[ B_{r_{\text{calc}}} := \text{erf} \left( \frac{\alpha}{\beta} \right) \cdot B_S \]

\[ B_{r_{\text{calc}}} = 0.4 \text{ tesla} \]

\[ \frac{\mu_{r_{\text{calc}}}}{\mu_{r_{\text{avg}}}} = 1 \]

\[ \frac{B_{r_{\text{calc}}}}{B_r} = 1 \]

Figure 30. Hysteresis Curves for 99% Pure Nickel
\[ W_{H_{\text{Ni}}} := \int_{-1}^{1} \text{erf} \left( \frac{B_{\text{ext\_ratio}} + \alpha}{\beta} \right) \cdot B_s \cdot \frac{B_{\text{ext\_max}}}{\mu_0 \text{SI}} \cdot \text{erf} \left( \frac{B_{\text{ext\_ratio}} - \alpha}{\beta} \right) \cdot B_s \cdot \frac{B_{\text{ext\_max}}}{\mu_0 \text{SI}} \, \text{dB}_{\text{ext\_ratio}} \]

\[ W_{H_{\text{Ni}}} = 1278.4381 \, \text{J/m}^3 \quad \text{(a prediction)} \]

\[ B_{r_{\text{Ni}}} := B_r \quad B_{\text{ext\_c\_Ni}} := B_{\text{ext\_c}} \quad \text{(for use in the section on compounds and alloys)} \]

The curves calculated above look very similar to the experimental hysteresis curves given in the physics literature, and therefore we can consider the \text{erf} function to be confirmed for this property. Unfortunately there are no readily available curves for the rare earth elements, so there wouldn't be much point to computing the theoretical curves for these elements. It's also rather disconcerting that there are no values for the hysteresis loss of Co and Ni ferromagnets--so our calculations for these elements are predictions.
Note: Initial Magnetization Curve

A simple sigmoid probability curve, like the following, can represent, approximately, the initial magnetization curve:

\[
\frac{B_{\text{int}}}{B_s} = \frac{1}{1 + e^{-\left(\frac{B_{\text{ext}}}{B_{\text{ext max}}} - \alpha_i\right) / \beta_i}}
\]

(148)

where \( \alpha_i \) and \( \beta_i \) are the relevant statistical parameters. Let \( \alpha_i := .3 \) \( \beta_i := .08 \)

![Graph of initial magnetization curve](image)

Figure 31. Initial Magnetization Curve (Generic)

This curve agrees, qualitatively, with the curves in the literature (like Ref. [18], p. 215, Fig. 29.3) but there isn't sufficient empirical data to set up a detailed comparison of theory with experiment.
6) paramagnetic behavior above the Curie temperature

The paramagnetic equation for ferromagnetic elements above the Curie point is slightly modified from the previous equation for paramagnets, Eq. (131): the Curie temperature must be subtracted from the applied temperature.

\[
\chi := \chi_{u_SI} \left( \frac{Z_{uc} n_f Z_{uc} M_{u weber}^\text{edgec}}{n_M} \cdot \frac{t_p t_s t_e}{1 1 1} \right) + \chi_{u_SI} n_f Z_{uc} \cdot \frac{w}{1} \cdot \frac{T_{t_u}}{T - T_c} \cdot \frac{1}{V_{uc_SI}} \left[ \left( s_{t_u} 10^{-2} \right)^3 \right]
\]

\[
(149)
\]

**Fe** BCC

\( n_f = 1 \) for Fe when ferromagnetic or paramagnetic.

\[
\begin{align*}
n_f & := 1 \\
Z_{uc} & := 2 \\
n_M & := 3 \\
\text{edgec} & := 2.9196 \cdot 10^{-10} \\
V_{uc_SI} & := 24.866 \cdot 10^{-30}
\end{align*}
\]

\[
\begin{align*}
t_p & := 3 \\
t_s & := 2 \\
t_e & := 8 \\
T_{c Fe\_obs} & = 1043.15 \\
w & := 55.8470
\end{align*}
\]

According to Stoner (Ref. [28], p. 379), the (empirical) Curie-Weiss equation for iron above the Curie point is

\[
\chi := \frac{0.0395 \cdot \rho_{Fe} \cdot 4 \cdot \pi}{T - T_{c Fe}}
\]

\[
\rho_{Fe} := 7.481 \quad \text{(assumed approximately constant)}
\]

We'll use the observed, rather than the calculated value, of \( T_c \) for the following graphs (because the empirical curves are based on it).
The theoretical curve and the empirical curve have the same shape.
\[ n_f = 0.5 \text{ for Co when paramagnetic} \]

\[ n_f := 0.5 \quad n_M := 3 \quad Z_{uc} := 4 \quad \text{edge}_c := 3.6193 \cdot 10^{-10} \quad s_0 := 2.4991 \cdot 10^{-10} \]

\[ V_{uc_SI} := 47.4104 \cdot 10^{-30} \]

\[ t_p := 3 \quad t_s := 2 \quad t_e := 9 \quad w := 58.9332 \]

\[ T_{c,Co_obs} = 1403.15 \quad \rho_{Co} := 8.116 \]

\[
\frac{0.0217 \cdot \rho_{Co} \cdot 4 \cdot \pi}{(TT - T_{c Co_obs})}

\chi_{u_SI} \quad \left( \frac{Z_{uc}}{n_f} \cdot \frac{n_M \cdot M_{u weber}}{s_0} \right) \cdot \left( \frac{V_{uc_SI}}{M_{p nat t}} \cdot \frac{t_p \cdot t_s \cdot t_e}{1 \cdot 1 \cdot 1} \right) + \chi_{u_SI} \cdot n_f \cdot Z_{uc} \cdot \frac{w}{1} \left( \frac{T_{t u}}{TT - T_{c Co_obs}} \right) \cdot \frac{1}{V_{uc_SI}} \left( \frac{s_{t u} \cdot 10^{-2}}{3} \right) \]

Figure 33. Cobalt Susceptibility Above \( T_c \)
Ni  FCC

\( n_f = .5 \) for Ni when ferromagnetic, \( n_f = .25 \) when paramagnetic.

\( n_f := .25 \quad Z_{uc} := 2 \quad n_M := 2 \quad edge_c := 3.4904 \cdot 10^{-10} \quad s_0 := 2.5382 \cdot 10^{-10} \)

\( V_{uc SI} := 43.5232 \cdot 10^{-30} \)

\( t_p := 3 \quad t_s := 2 \quad t_e := 10 \quad w := 58.6900 \)

\( T_{c_{Ni obs}} = 631.15 \quad \rho_{Ni} := 7.357 \)

\[
\chi_{u SI} \left( \frac{\frac{Z_{uc}}{2} \cdot n_M \cdot M_{u weber} \cdot s_0}{V_{uc SI} \cdot M_{P nat t}} \right) + \chi_{u SI} \cdot n_f \cdot Z_{uc} \cdot \left( \frac{T_{t u}}{T_T - T_{c_{Ni obs}}} \right) \cdot \frac{1}{V_{uc SI} \left( s_{t u} \cdot 10^{-2} \right)^{3}}
\]

Again, the curves are very close.

Figure 34. Nickel Susceptibility Above \( T_c \)
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</table>

Table VIII. Summary of Properties of Ferromagnetic Elements

Note: edge_a, edge_c, s0 in units of 10^{-10} m; Vuc in units of 10^{-30} m^3; Bs_calc, Bs_obs, Br_calc, Br_obs, Bs_T_calc, Bs_T_obs, all in units of tesla (webers/m^2)
b. Compounds and Alloys

Von Hippel explains (Ref. [20], p. v.) that "...molecular engineering [is] the building of materials to order. We now begin to design materials with properties prescribed for the purpose in hand." By adding various non-ferromagnetic elements to a ferromagnetic material we can obtain compounds and alloys with desired values of external magnetic flux density coercivity and internal magnetic flux density and remanence. Generally, adding non-ferromagnetic elements reduces the magnetic remanence ($B_r$) and magnetic saturation ($B_s$) of the pure material, while increasing the coercivity ($B_{ext.c}$). Essentially, the non-ferromagnetic elements block, to some degree, the magnetic dipole moments of the ferromagnetic atoms, and increase the energy required to demagnetize the pure material.

If, after the addition of non-ferromagnetic atoms and after various kinds of heat and cold treatment of the resulting alloy, we could know the exact placement of the atoms in a crystal unit cell, we could calculate—using the previously-given equations—the new values of $B_r$ and $B_s$. But this is usually not the case—we don't know precisely how many of the "foreign" atoms are in the way of the magnetic dipoles, and how many are not. So, what we will do is define two new constants, $n_r$ and $k_c$.

$$n_r := \text{factor of remanence due to foreign atoms} \quad (\text{usually} < 1, \text{but not always})$$

$$k_c := \text{factor of coercivity due to foreign atoms} \quad (\text{usually} > 1, \text{but not always})$$

$n_r$ and $k_c$ will be given for each compound/alloy which follows. Of course, the base values for $B_r$ and $B_{ext.c}$ will be those of the predominant ferromagnetic material. Using the erf equations previously given we will calculate the hysteresis loss and $BH_{max}$ values (commonly tabulated by commercial vendors) and compare with such empirical values as are available.
**78 Permalloy**  
Date from Ref. [12], p. 4-142 and Ref. [30], p. 130

- **$B_s$** := 1.07 tesla (internal magnetic flux density saturation)
- **$B_r$** := .4 tesla (internal remanent magnetic flux density)
- **$\mu_{r,avg}$** := 77000 (avg. permeability, 2nd quadrant)

\[
B_{ext,c} := \frac{-B_r}{\mu_{r,avg}} \text{ tesla} \quad B_{ext,c} = -5.1948 \times 10^{-6} \quad \text{(external magnetic flux density at coercivity point)}
\]

**$W_{H,78Permalloy_{obs}}$** := 20  
(Ref. [30], p. 113)

Starting Values:  
- **$B_{ext,max}$** := $1 \cdot 10^{-3}$ tesla  
- **$\beta$** := .5

- **TOL** := .000001  
- **CTOL** := .000001

Given

\[
\frac{-B_{ext,c}}{B_{ext,max} \beta} \cdot B_s = 0
\]

\[
\mu_{r,avg} = \frac{-B_{ext,c}}{0 - B_{ext,c}}
\]

\[
B_r = \text{erf} \left( \frac{-B_{ext,c}}{B_{ext,max} \beta} \right) \cdot B_s
\]

\[
\int_{-1}^{1} \text{erf} \left( \frac{B_{ext\_ratio} + \alpha}{\beta} \right) \cdot B_s \cdot \frac{B_{ext\_max}}{\mu_0\_SI} - \text{erf} \left( \frac{B_{ext\_ratio} - \alpha}{\beta} \right) \cdot B_s \cdot \frac{B_{ext\_max}}{\mu_0\_SI} dB_{ext\_ratio} = W_{H,78Permalloy_{obs}}
\]
\[ B_{\text{ext\_max}}^\beta := \text{Find}(B_{\text{ext\_max}}^\beta, \beta) \]

\[ B_{\text{ext\_max}} = 2.3567 \times 10^{-5} \]

\[ \alpha := \frac{-B_{\text{ext\_c}}}{B_{\text{ext\_max}}} \]
\[ \alpha = 0.2204 \]

\[ \beta = 0.64 \]

\[ \mu_r_{\text{calc}} := \frac{\text{erf}(\frac{\alpha}{\beta}) \cdot B_s - 0}{0 - B_{\text{ext\_c}}} \]
\[ \mu_r_{\text{calc}} = 77000 \]

\[ B_{r_{\text{calc}}} := \text{erf}(\frac{\alpha}{\beta}) \cdot B_s \]
\[ B_{r_{\text{calc}}} = 0.4 \text{ tesla} \]

\[ \frac{\mu_r_{\text{calc}}}{\mu_r_{\text{avg}}} = 1 \]
\[ \frac{B_{r_{\text{calc}}}}{B_r} = 1 \]

\[ W_{\text{H\_78Permalloy}} := \int_{-1}^{1} \text{erf} \left( \frac{B_{\text{ext\_ratio}} + \alpha}{\beta} \right) \cdot B_s \cdot \frac{B_{\text{ext\_max}}}{\mu_0 \text{SI}} - \text{erf} \left( \frac{B_{\text{ext\_ratio}} - \alpha}{\beta} \right) \cdot B_s \cdot \frac{B_{\text{ext\_max}}}{\mu_0 \text{SI}} \text{dB}_{\text{ext\_ratio}} \]

\[ W_{\text{H\_78Permalloy}} = 17.1037 \text{ J/m}^3 \]
78 Permalloy is 78% Ni, 22% Fe.

\[ n_{r,\text{78 Permalloy}} := \frac{B_r}{B_{r,\text{Ni}}} \quad n_{r,\text{78 Permalloy}} = 1 \]

(no reduction because, of course, Fe is ferromagnetic!)
**k\textsubscript{c\_78Permalloy}** := \( \frac{B_{\text{ext\_c}}}{B_{\text{ext\_c\_Ni}}} \) \quad k\textsubscript{c\_78Permalloy} = 0.0078 \\ 

Now we'll compute |BH\textsubscript{max}|, the *maximum energy product*. The maximum value of "B x H" occurs at .5 x B\textsubscript{ext\_c}.

\[
BH_{\text{max}} := \left| \text{erf} \left( \frac{0.5 \cdot B_{\text{ext\_c}}}{B_{\text{ext\_max}}} \right) + \alpha \right| \cdot B_s \cdot \frac{0.5 \cdot B_{\text{ext\_c}}}{\mu_0 \text{SI}} \quad \text{J/m}^3
\]

But \( \alpha := \frac{-B_{\text{ext\_c}}}{B_{\text{ext\_max}}} \) so

\[
BH_{\text{max}} := \left| \text{erf} \left( \frac{-0.5 \cdot B_{\text{ext\_c}}}{B_{\text{ext\_max}}} \right) \right| \cdot B_s \cdot \frac{0.5 \cdot B_{\text{ext\_c}}}{\mu_0 \text{SI}} \quad \text{J/m}^3
\]

BH\textsubscript{max} = 0.4256 \quad \text{J/m}^3 \quad BH\textsubscript{max\_78permalloy} := BH\textsubscript{max}

Unfortunately, none of the references given at the end of this paper provide an experimental value.
**Silicon Iron (3% Si) Oriented**

Date from Ref. [16]. p. 12-108—there are many other sources, all in disagreement.

\( B_S := 2.01 \) tesla (internal magnetic flux density saturation)

\( B_r := 1.0 \) tesla (internal remanent magnetic flux density)

\( \mu_{r\_avg} := 40000 \) (avg. permeability, considered to be approximately the same as the max. given in the data)

\( B_{ext\_c} := \frac{-B_r}{\mu_{r\_avg}} \) tesla \( B_{ext\_c} = -2.5 \times 10^{-5} \) (external magnetic flux density at coercivity point)

\( W_{H\_SiFe\_obs} := 140 \)

Starting Values:

\( B_{ext\_max} := 6 \cdot 10^{-5} \) tesla \( \beta := .8 \)

\( TOL := .000001 \) \( CTOL := .000001 \)

Given

\[
\mu_{r\_avg} = \frac{\text{erf} \left( \frac{B_{ext\_c}}{B_{ext\_max}} \frac{B_s}{\beta} \right)}{0 - B_{ext\_c}} B_s - 0
\]

\[
B_r = \text{erf} \left( \frac{-B_{ext\_c}}{B_{ext\_max}} \frac{B_s}{\beta} \right) B_s
\]

\[
1 \int \text{erf} \left[ \frac{B_{ext\_ratio} + \left( \frac{-B_{ext\_c}}{B_{ext\_max}} \right)}{\beta} \right] B_s \frac{B_{ext\_max}}{\mu_{0\_SI}} - \text{erf} \left[ \frac{B_{ext\_ratio} - \left( \frac{-B_{ext\_c}}{B_{ext\_max}} \right)}{\beta} \right] B_s \frac{B_{ext\_max}}{\mu_{0\_SI}} dB_{ext\_ratio} = W_{H\_SiFe}
\]
\( \left( \frac{B_{\text{ext\_max}}}{\beta} \right) := \text{Find}(B_{\text{ext\_max}}, \beta) \quad B_{\text{ext\_max}} = 6.1403 \times 10^{-5} \quad \alpha := \frac{B_{\text{ext\_c}}}{B_{\text{ext\_max}}} \quad \alpha = 0.4071 \)

\[ \beta = 0.8586 \]

\[
\mu_{r_{\text{calc}}} := \frac{\text{erf}\left(\frac{\alpha}{\beta}\right) \cdot B_s - 0}{0 - B_{\text{ext\_c}}} \quad \mu_{r_{\text{calc}}} = 40000 \quad B_{r_{\text{calc}}} := \text{erf}\left(\frac{\alpha}{\beta}\right) \cdot B_s \quad B_{r_{\text{calc}}} = 1 \text{ tesla} \]

\[ \frac{\mu_{r_{\text{calc}}}}{\mu_{r_{avg}}} = 1 \quad \frac{B_{r_{\text{calc}}}}{B_r} = 1 \]

\[
W_{H_{\text{SiFe}}} := \int_{-1}^{1} \text{erf}\left(\frac{B_{\text{ext\_ratio}} + \alpha}{\beta} \right) \cdot B_s \cdot \frac{B_{\text{ext\_max}}}{\mu_{0_{\text{Sl}}}} - \text{erf}\left(\frac{B_{\text{ext\_ratio}} - \alpha}{\beta} \right) \cdot B_s \cdot \frac{B_{\text{ext\_max}}}{\mu_{0_{\text{Sl}}}} \text{dB}_{\text{ext\_ratio}} \]

\[ W_{H_{\text{SiFe}}} = 140 \quad \text{J/m}^3 \]
Figure 36. Hysteresis Curves for Oriented Silicon Iron
Silicon iron is 96% Fe, 4% Si.

\[ n_{r_{\text{SiFe}}} := \frac{B_r}{B_{r_{\text{Fe}}}} \quad \text{with} \quad n_{r_{\text{SiFe}}} = 0.7692 \]

\[ k_{c_{\text{SiFe}}} := \frac{B_{\text{ext}_c}}{B_{\text{ext}_c_{\text{Fe}}}} \quad \text{with} \quad k_{c_{\text{SiFe}}} = 4.2308 \]

\[ B_{H_{\text{max}_{\text{SiFe}}}} := \text{erf} \left( \frac{-0.5 \cdot B_{\text{ext}_c}}{B_{\text{ext}_\text{max}}} \right) \cdot B_S \cdot \frac{0.5 \cdot B_{\text{ext}_c}}{\mu_0 \cdot \mu_{\text{SI}}} \quad \text{with} \quad B_{H_{\text{max}_{\text{SiFe}}}} = 4.5301 \text{ J/m}^3 \]

The references do not provide an experimental value.

Ref. [68] provides data for commercial magnets, including the maximum energy product, so we'll select a few of these for comparison with the theory; unfortunately Ref. [68] does not have values for \( B_S \). Ref. [13], [30], [31], and [32] also have tables of data—but usually one or more necessary values are missing, which means that approximations have to be made.
Alnico 5 Anisotropic Cast

\[ B_S := 1.5 \text{ tesla} \quad \text{(internal magnetic flux density saturation) (approx. from Ref. [31], pp. 156-158)} \]

\[ B_r := 1.280 \text{ tesla} \quad \text{(internal remanent magnetic flux density)} \]

\[ \mu_{r\_avg} := 4 \quad \text{(average permeability, approximated from Ref. [31], pp. 156-158)} \]

\[ B_{\text{Hmax\_Alnico5\_obs}} := 40000 \quad \text{(Ref. [30], p. 377)} \]

\[ B_{\text{ext\_c}} := \frac{-B_r}{\mu_{r\_avg}} \text{ tesla} \quad B_{\text{ext\_c}} = -0.32 \quad \text{(external magnetic flux density at coercivity point)} \]

Starting Values: \[ B_{\text{ext\_max}} := 3 \text{ tesla} \quad \beta := .5 \]

\[ \text{TOL} := .000001 \quad \text{CTOL} := .000001 \]

Given

\[ B_r = \text{erf} \left( \frac{-B_{\text{ext\_c}}}{B_{\text{ext\_max}}} \right) \cdot B_S \]

\[ \mu_{r\_avg} = \frac{\text{erf} \left( \frac{-B_{\text{ext\_c}}}{B_{\text{ext\_max}}} / \beta \right) \cdot B_S}{0 - B_{\text{ext\_c}}} \]

\[ \text{erf} \left( -0.5 \cdot \frac{B_{\text{ext\_c}}}{B_{\text{ext\_max}}} \right) \cdot B_S \cdot \frac{0.5 \cdot B_{\text{ext\_c}}}{\mu_{0\_SI}} = B_{\text{Hmax\_Alnico5\_obs}} \]
\[
\begin{align*}
B_{\text{ext\_max}}^\beta & := \text{Find}(B_{\text{ext\_max}}, \beta) \\
\alpha & := \frac{-B_{\text{ext\_c}}}{B_{\text{ext\_max}}} \\
\beta & = 0.366
\end{align*}
\]

\[
\begin{align*}
\mu_r_{\text{calc}} := & \frac{\text{erf}(\frac{\alpha}{\beta}) \cdot B_s - 0}{0 - B_{\text{ext\_c}}} \\
\mu_r_{\text{calc}} & = 4 \\
B_r_{\text{calc}} := & \text{erf}(\frac{\alpha}{\beta}) \cdot B_s \\
B_r_{\text{calc}} & = 1.28
\end{align*}
\]

\[
\begin{align*}
\frac{\mu_r_{\text{calc}}}{\mu_r_{\text{avg}}} & = 1 \\
\frac{B_r_{\text{calc}}}{B_r} & = 1
\end{align*}
\]

\[
\begin{align*}
B_{\text{H\_max\_Alnico5}} & := \text{erf}\left(\frac{-0.5 \cdot B_{\text{ext\_c}}}{B_{\text{ext\_max}}} \cdot B_s \cdot \frac{0.5 \cdot B_{\text{ext\_c}}}{\mu_0 SI}\right) \\
B_{\text{H\_max\_Alnico5}} & = 40000 \text{ J/m}^3
\end{align*}
\]

\[
\begin{align*}
W_{\text{H\_Alnico5}} & := \int_{-1}^{1} \text{erf}\left(\frac{B_{\text{ext\_ratio}} + \alpha}{\beta} \right) \cdot B_s \cdot B_{\text{ext\_max}} \cdot \frac{B_{\text{ext\_max}}}{\mu_0 SI} \text{dB}_{\text{ext\_ratio}} - \text{erf}\left(\frac{B_{\text{ext\_ratio}} - \alpha}{\beta} \right) \cdot B_s \cdot B_{\text{ext\_max}} \cdot \frac{B_{\text{ext\_max}}}{\mu_0 SI}
\end{align*}
\]

\[
W_{\text{H\_Alnico5}} = 1.5251 \times 10^6 \text{ J/m}^3 \quad \text{(prediction)}
\]

The references do not provide an experimental value--but clearly one would not use this alloy in a transformer!
Figure 37. Hysteresis Curves for Alnico 5
Alnico 5 is 51% Fe, 8% Al, 14% Ni, 24% Co, and 3% Cu.

\[ n_{r_{\text{Alnico5}}} := \frac{B_r}{B_{r_{\text{Fe}}}} \]
\[ n_{r_{\text{Alnico5}}} = 0.9846 \]  
(clearly with Ni and Co, there's not much reduction)

\[ k_{c_{\text{Alnico5}}} := \frac{B_{\text{ext}_c}}{B_{\text{ext}_c_{\text{Fe}}}} \]
\[ k_{c_{\text{Alnico5}}} = 54153.8462 \]  
(wow)
Platinum Cobalt

\[ B_s := 1.759 \text{ tesla} \]  
(\text{internal magnetic flux density saturation}) (assumed to be the same as Co)

\[ B_r := .645 \text{ tesla} \]  
(\text{internal remanent magnetic flux density})

\[ \mu_{r_{\text{avg}}} := 1.2 \]  
(average permeability, from Ref. [13], p. 5-165)

\[ B_{\text{ext}_c} := \frac{-B_r}{\mu_{r_{\text{avg}}}} \text{ tesla} \]  
\[ B_{\text{ext}_c} = -0.5375 \]  
(external magnetic flux density at coercivity point)

\[ BH_{\text{max}_{\text{PtCo_{obs}}}} := 45000 \]  
(Ref. [30], p. 382, but adjusted down)

Starting Values:

\[ B_{\text{ext}_{\text{max}}} := 1 \text{ tesla} \]

\[ \beta := .79 \]

\[ \text{TOL} := .000001 \]

\[ \text{CTOL} := .000001 \]

Given

\[ \mu_{r_{\text{avg}}} = \frac{\text{erf}\left(\frac{-B_{\text{ext}_c}}{B_{\text{ext}_{\text{max}}} \beta}\right) \cdot B_s - 0}{0 - B_{\text{ext}_c}} \]

\[ B_r = \text{erf}\left(\frac{-B_{\text{ext}_c}}{B_{\text{ext}_{\text{max}}} \beta}\right) \cdot B_s \]

\[ \left| \text{erf}\left(-.5 \cdot \frac{B_{\text{ext}_c}}{B_{\text{ext}_{\text{max}}}}\right) \cdot B_s \cdot .5 \cdot \frac{B_{\text{ext}_c}}{\mu_0_{\text{SI}}} \right| = BH_{\text{max}_{\text{PtCo_{obs}}}} \]
\[
\begin{align*}
\left( \frac{\text{B}_{\text{ext\_max}}}{\beta} \right) & := \text{Find} \left( \frac{\text{B}_{\text{ext\_max}}}{\beta} \right) \quad \text{B}_{\text{ext\_max}} = 2.5256 \quad \alpha := \frac{-\text{B}_{\text{ext\_c}}}{\text{B}_{\text{ext\_max}}} \quad \alpha = 0.2128 \\
& \quad \beta = 0.6309
\end{align*}
\]

\[
\mu_{\text{r\_calc}} := \frac{\text{erf} \left( \frac{\alpha}{\beta} \right) \cdot \text{B}_S}{0 - \text{B}_{\text{ext\_c}}} \quad \mu_{\text{r\_calc}} = 1.2 \quad \text{B}_{\text{r\_calc}} := \text{erf} \left( \frac{\alpha}{\beta} \right) \cdot \text{B}_S \quad \text{B}_{\text{r\_calc}} = 0.645
\]

\[
\frac{\mu_{\text{r\_calc}}}{\mu_{\text{r\_avg}}} = 1 \quad \frac{\text{B}_{\text{r\_calc}}}{\text{B}_r} = 1
\]

\[
\text{B}_{\text{H\_max\_PtCo}} := \left| \text{erf} \left( \frac{-0.5 \cdot \text{B}_{\text{ext\_c}}}{\text{B}_{\text{ext\_max}}} \right) \cdot \text{B}_S \cdot \frac{0.5 \cdot \text{B}_{\text{ext\_c}}}{\mu_0_{\text{SI}}} \right| \quad \text{B}_{\text{H\_max\_PtCo}} = 45000 \quad \text{J/m}^3
\]

\[
\text{W}_{\text{H\_PtCo}} := \int_{-1}^{1} \text{erf} \left( \frac{\text{B}_{\text{ext\_ratio}} + \alpha}{\beta} \right) \cdot \text{B}_S \cdot \frac{\text{B}_{\text{ext\_max}}}{\mu_0_{\text{SI}}} - \text{erf} \left( \frac{\text{B}_{\text{ext\_ratio}} - \alpha}{\beta} \right) \cdot \text{B}_S \cdot \frac{\text{B}_{\text{ext\_max}}}{\mu_0_{\text{SI}}} \quad \text{dB}_{\text{ext\_ratio}}
\]

\[
\text{W}_{\text{H\_PtCo}} = 2.9174 \times 10^6 \quad \text{J/m}^3
\]

The references do not provide an experimental value.
Figure 38. Hysteresis Curves for Platinum Cobalt
Platinum cobalt is 77% Pt and 23% Co.

\[ n_{r\_PtCo} := \frac{B_r}{B_{r\_Co}} \quad n_{r\_PtCo} = 1.29 \] (Pt increases the remanence in this case.)

\[ k_{c\_PtCo} := \frac{B_{ext\_c}}{B_{ext\_c\_Co}} \quad k_{c\_PtCo} = 268.75 \]
Nd$_2$-Fe$_{14}$-B

\[ B_S := 1.6 \text{ tesla} \quad \text{(Ref. [30], p. 385)} \]
\[ B_r := 1.3 \text{ tesla} \quad \text{(internal remanent magnetic flux density)} \]
\[ \mu_{r\text{ avg}} := 1.0345 \quad \text{(average permeability, based on but rounded up from } B_{\text{ext c}}, \text{ given in Ref. [30], p. 377)} \]
\[ B_{\text{ext c}} := \frac{-B_r}{\mu_{r\text{ avg}}} \text{ tesla} \quad B_{\text{ext c}} = -1.2566 \quad \text{(external magnetic flux density at coercivity point)} \]
\[ BH_{\text{max NdFeB obs}} := 250000 \quad \text{(some values in the literature go as high as 320000, but that seems improbable)} \]

Starting Values:
\[ B_{\text{ext max}} := 5 \text{ tesla} \quad \beta := .5 \]
\[ TOL := .000001 \quad CTOL := .000001 \]

Given
\[ \mu_{r\text{ avg}} = \frac{\text{erf}\left(\frac{-B_{\text{ext c}}}{B_{\text{ext max}}} \frac{1}{\beta}\right)}{0 - B_{\text{ext c}}} \cdot B_S - 0 \]
\[ B_r = \text{erf}\left(\frac{-B_{\text{ext c}}}{B_{\text{ext max}}} \frac{1}{\beta}\right) \cdot B_S \]
\[ \left| \text{erf}\left(-.5 \cdot \frac{B_{\text{ext c}}}{B_{\text{ext max}}} \frac{1}{\mu_0 SI}\right) \cdot B_S \cdot .5 \cdot \frac{B_{\text{ext c}}}{\mu_0 SI} \right| = BH_{\text{max NdFeB obs}} \]
\[
\begin{align*}
\left( \frac{B_{\text{ext\_max}}}{\beta} \right) & := \text{Find}(B_{\text{ext\_max}}, \beta) \quad B_{\text{ext\_max}} = 2.209 \\
& \quad \alpha := \frac{-B_{\text{ext\_c}}}{B_{\text{ext\_max}}} \\
& \quad \beta = 0.6104
\end{align*}
\]

\[
\begin{align*}
\mu_{\text{r\_calc}} & := \frac{\text{erf}\left(\frac{\alpha}{\beta}\right) \cdot B_s - 0}{0 - B_{\text{ext\_c}}} \\
& \quad \mu_{\text{r\_calc}} = 1.0345 \\
& \quad B_{r\_calc} := \text{erf}\left(\frac{\alpha}{\beta}\right) \cdot B_s \\
& \quad B_{r\_calc} = 1.3
\end{align*}
\]

\[
\begin{align*}
\frac{\mu_{\text{r\_calc}}}{\mu_{\text{r\_avg}}} & = 1 \\
\frac{B_{r\_calc}}{B_r} & = 1
\end{align*}
\]

\[
\begin{align*}
B_{H\text{\_max\_NdFeB}} & := \left| \text{erf}\left(\frac{-0.5 \cdot B_{\text{ext\_c}}}{B_{\text{ext\_max}}} \right) \cdot B_s \cdot \frac{0.5 \cdot B_{\text{ext\_c}}}{\mu_{0\_SI}} \right| \\
& \quad B_{H\text{\_max\_NdFeB}} = 2.5 \times 10^5 \quad \text{J/m}^3
\end{align*}
\]

\[
\begin{align*}
W_{H\text{\_NdFeB}} & := \int_{-1}^{1} \text{erf}\left(\frac{B_{\text{ext\_ratio}} + \alpha}{\beta}\right) \cdot B_s \cdot \frac{B_{\text{ext\_max}}}{\mu_{0\_SI}} - \text{erf}\left(\frac{B_{\text{ext\_ratio}} - \alpha}{\beta}\right) \cdot B_s \cdot \frac{B_{\text{ext\_max}}}{\mu_{0\_SI}} \, dB_{\text{ext\_ratio}}
\end{align*}
\]

\[
W_{H\text{\_NdFeB}} = 5.9948 \times 10^6 \quad \text{J/m}^3
\]

The references do not provide an experimental value.
\[ \text{erf} \left( \frac{B_{\text{ext_ratio}} - \alpha}{\beta} \right) \cdot \frac{B_s}{B_{\text{ext max}}} \]

\[ \text{erf} \left( \frac{B_{\text{ext_ratio}} + \alpha}{\beta} \right) \cdot \frac{B_s}{B_{\text{ext max}}} \]

Figure 39. Hysteresis Curves for \( \text{Nd}_2\text{-Fe}_{14}\text{-B} \)

\[ n_{r_{\text{NdFeB}}} := \frac{B_r}{B_{r_{\text{Fe}}}} \]

\[ n_{r_{\text{NdFeB}}} = 1 \]

\[ k_{c_{\text{NdFeB}}} := \frac{B_{\text{ext c}}}{B_{\text{ext c}_{\text{Fe}}}} \]

\[ k_{c_{\text{NdFeB}}} = 2.1266 \times 10^5 \]

(This is an amazing increase--no wonder this material is so popular.)
**Cu₂MnAl—a Heusler Alloy**

\[ B_S := .6042 \text{ tesla} \]  
(Ref. [28], p. 525—stated to be the same as Ni—but see calc. below)

\[ B_r := .4028 \text{ tesla} \]  
(see calc. below)

\[ \mu_{r\text{ avg}} := .2877 \]  
(average permeability, based on \( B_{ext\_c} \), given in Ref. [32], p. 858 for "Manganese Aluminum")

\[ B_{ext\_c} := \frac{-B_r}{\mu_{r\text{ avg}}} \text{ tesla} \quad B_{ext\_c} = -1.4001 \]  
(external magnetic flux density at coercivity point)

\[ B_{H_{max\_Cu2MnAl\_obs}} := 3.5 \cdot 7957.74715 \quad B_{H_{max\_Cu2MnAl\_obs}} = 27852.115 \]  
(converted from MGO value given in above Ref. to J/m³)

Starting Values:

\[ B_{ext\_max} := 5 \text{ tesla} \quad \beta := .5 \]

\[ TOL := .000001 \quad CTOL := .000001 \]

Given

\[ \mu_{r\text{ avg}} = \frac{\text{erf} \left( \frac{-B_{ext\_c}}{B_{ext\_max}} \right) \cdot B_S}{-0 - B_{ext\_c}} \]

\[ B_r = \text{erf} \left( \frac{-B_{ext\_c}}{B_{ext\_max}} \right) \cdot B_S \]

\[ \text{erf} \left( -0.5 \cdot \frac{B_{ext\_c}}{B_{ext\_max}} \right) \cdot B_S \cdot 0.5 \cdot \frac{B_{ext\_c}}{\mu_0 \text{ SI}} = B_{H_{max\_Cu2MnAl\_obs}} \]
\[ \left( \frac{B_{\text{ext\_max}}}{\beta} \right) := \text{Find}(B_{\text{ext\_max}}, \beta) \quad B_{\text{ext\_max}} = 9.5285 \quad \alpha := \frac{-B_{\text{ext\_c}}}{B_{\text{ext\_max}}} \quad \alpha = 0.1469 \]

\[ \beta = 0.2148 \]

\[ \mu_{\text{r\_calc}} := \frac{\text{erf}(\alpha) \cdot B_s - 0}{0 - B_{\text{ext\_c}}} \quad \mu_{\text{r\_calc}} = 0.2877 \quad B_{\text{r\_calc}} := \text{erf}(\alpha) \cdot B_s \quad B_{\text{r\_calc}} = 0.4028 \]

\[ \frac{\mu_{\text{r\_calc}}}{\mu_{\text{r\_avg}}} = 1 \quad \frac{B_{\text{r\_calc}}}{B_r} = 1 \]

BH_{\text{max\_Cu2MnAl}} := \left| \text{erf}\left(\frac{-0.5 \cdot B_{\text{ext\_c}}}{B_{\text{ext\_max}}} \right) \cdot B_s \cdot \frac{0.5 \cdot B_{\text{ext\_c}}}{\mu_0 \text{SI}} \right| \quad BH_{\text{max\_Cu2MnAl}} = 27852.115 \quad \text{J/m}^3

W_{H\_Cu2MnAl} := \int_{-1}^{1} \left[ \text{erf}\left(\frac{B_{\text{ext\_ratio}} + \alpha}{\beta} \right) \cdot B_s \cdot \frac{B_{\text{ext\_max}}}{\mu_0 \text{SI}} - \text{erf}\left(\frac{B_{\text{ext\_ratio}} - \alpha}{\beta} \right) \cdot B_s \cdot \frac{B_{\text{ext\_max}}}{\mu_0 \text{SI}} \right] dB_{\text{ext\_ratio}}

W_{H\_Cu2MnAl} = 2.6927 \times 10^6 \quad \text{J/m}^3

The references do not provide an experimental value.
Figure 40. Hysteresis Curves for Cu2MnAl

\[ n_{r\_Cu2MnAl} := \frac{B_r}{B_{r\_Ni}} \]

\[ k_{c\_Cu2MnAl} := \frac{B_{ext\_c}}{B_{ext\_c\_Ni}} \]

\[ n_{r\_Cu2MnAl} = 1.007 \]

\[ k_{c\_Cu2MnAl} = 2100.1043 \]
Note: The Heusler alloys are unique in that they are constructed of non-ferromagnetic elements but, which when combined, are ferromagnetic. Ref. [70] states that crystal volume unit cell is FCC, with the edge being 5.95 Å. The Mn atoms are ferromagnetically-charged here, not Cu or Al. Larson specifically says, Ref. [1], p. 216, "Under some special conditions, the [electric] displacements of chromium (6) and manganese (7) are increased to 8 and 9 respectively by reorientation relative to a new zero point, as explained in Chapter 18 of Volume 1 [Ref. 2, 2nd ed.]. These elements are then also able to accept magnetic charges." The Heusler alloys are often compared to nickel.

The two magnetic dipoles are between the two pairs of Mn atoms at the edges of the unit cell.

\[ n_{\text{M}_{\text{Cu2MnAl}}} = 3 \quad Z_{\text{uc}} := 4 \quad n_f := 1 \quad \text{edge}_c := 5.95 \cdot 10^{-10} \quad \text{m} \quad (\text{all edges same}) \]

\[ V_{\text{uc SI}} := \text{edge}_c^3 \quad V_{\text{uc SI}} = 2.1064 \times 10^{-28} \quad \text{m}^3 \quad (\text{assumed to be approx. constant from 0 to 300 K}) \]

\[ B_{\text{s Cu2MnAl}} := \frac{I_{M} \cdot n_f \cdot \text{floor} \left( \frac{Z_{\text{uc}}}{2} \right) \cdot n_{\text{M}_{\text{Cu2MnAl}}} \cdot \mu_{\text{weber}} \cdot \text{edge}_c}{V_{\text{uc SI}}} \]

\[ B_{\text{s Cu2MnAl}} = 1.0049 \quad \text{which compares with} \quad B_{\text{s Ni obs}} = 0.653 \quad \text{tesla} \]

\[ E_{\text{M Cu2MnAl}} := \frac{n_{\text{M}_{\text{Cu2MnAl}}} \cdot \mu_{\text{weber}} \cdot n_{\text{M}_{\text{Cu2MnAl}}} \cdot \mu_{\text{weber}}}{4 \cdot \pi \cdot \mu_0 \cdot \mu_r_0 \cdot \text{edge}_c} \cdot \text{conv joule to ev} \]

\[ E_{\text{M Cu2MnAl}} = 0.1379 \quad \text{eV} \]

\[ T_{\text{c Cu2MnAl}} := \frac{E_{\text{M Cu2MnAl}}}{3 \cdot k_B_{\text{ev}}} \quad T_{\text{c Cu2MnAl}} = 533.5239 \quad \text{K} \quad (\text{which compares with 630.15 K, the supposed observation}) \]
\[ B_{s\_Cu2MnAl\_300} := B_{s\_Cu2MnAl} \left[ 1 - \left( \frac{300}{T_{c\_Cu2MnAl}} \right)^2 \right]^{\frac{1}{4}} \]

\[ B_{s\_Cu2MnAl\_300} = 0.9138 \]

\[ n_{M\_r\_Cu2MnAl} := 2 \]

\[ B_{r\_Cu2MnAl} := \frac{2}{3} B_{s\_Cu2MnAl\_300} \quad B_{r\_Cu2MnAl} = 0.6092 \]

Everything checks out.
Barium Ferrite \( \text{BaO-6Fe}_2\text{O}_3 = \text{BaFe}_{12}\text{O}_{19} \) isotropic form

\[ B_S := .3043 \text{ tesla} \] (see calc. below)

\[ B_r := .2028 \text{ tesla} \] (see calc. below)

\[ \mu_{r_{\text{avg}}} := 1.1958 \] (average permeability, based on \( B_{\text{ext}_c} \), given in Ref. [31], p. 182)

\[ B_{\text{ext}_c} := \frac{-B_r}{\mu_{r_{\text{avg}}}} \text{ tesla} \] \( B_{\text{ext}_c} = -0.1696 \) (external magnetic flux density at coercivity point)

\[ \text{BH}_{\text{max}_\text{BaFe12O19\_obs}} := 5000 \] (adjusted down from Ref. [31], p. 182; Ref. [68] says it's 8350--prob. aniso.)

Starting Values: \[ B_{\text{ext\_max}} := .4 \text{ tesla} \] \( \beta := .5 \)

\[ \text{TOL} := .000001 \] \[ \text{CTOL} := .000001 \]

Given

\[ \mu_{r_{\text{avg}}} = \text{erf} \left( \frac{-B_{\text{ext}_c}}{B_{\text{ext\_max}}} \frac{1}{\beta} \right) \cdot B_S = 0 \]

\[ B_r = \text{erf} \left( \frac{-B_{\text{ext}_c}}{B_{\text{ext\_max}}} \frac{1}{\beta} \right) \cdot B_S \]

\[ \text{erf} \left( -0.5 \cdot \frac{B_{\text{ext}_c}}{B_{\text{ext\_max}}} \cdot B_S \cdot 0.5 \cdot \frac{B_{\text{ext}_c}}{\mu_0 \text{SI}} \right) = \text{BH}_{\text{max}_\text{BaFe12O19\_obs}} \]
\[
\left( \frac{B_{\text{ext\_max}}}{\beta} \right) := \text{Find} \left( B_{\text{ext\_max}} \cdot \beta \right) \quad B_{\text{ext\_max}} = 0.3867 \quad \alpha := -\frac{B_{\text{ext\_c}}}{B_{\text{ext\_max}}} \quad \alpha = 0.4385 \\
\beta = 0.6413
\]

\[
\mu_{r\_calc} := \frac{\text{erf} \left( \frac{\alpha}{\beta} \right) \cdot B_s - 0}{0 - B_{\text{ext\_c}}} \quad \mu_{r\_calc} = 1.1958
\]

\[
B_{r\_calc} := \text{erf} \left( \frac{\alpha}{\beta} \right) \cdot B_s \quad B_{r\_calc} = 0.2028
\]

\[
\frac{\mu_{r\_calc}}{\mu_{r\_avg}} \quad \frac{B_{r\_calc}}{B_r} = 1
\]

\[
B_{H\text{max\_BaFe12O19}} := \left| \text{erf} \left( \frac{-0.5 \cdot B_{\text{ext\_c}}}{B_{\text{ext\_max}}} \right) \cdot B_s \cdot \frac{0.5 \cdot B_{\text{ext\_c}}}{\mu_0 \text{SI}} \right| \quad B_{H\text{max\_BaFe12O19}} = 5000 \quad \text{J/m}^3
\]

\[
W_{H\_BaFe12O19} := \int_{-1}^{1} \left| \text{erf} \left( \frac{B_{\text{ext\_ratio}} + \alpha}{\beta} \right) \cdot B_s \cdot \frac{B_{\text{ext\_max}}}{\mu_0 \text{SI}} - \text{erf} \left( \frac{B_{\text{ext\_ratio}} - \alpha}{\beta} \right) \cdot B_s \cdot \frac{B_{\text{ext\_max}}}{\mu_0 \text{SI}} \right| dB_{\text{ext\_ratio}}
\]

\[
W_{H\_BaFe12O19} = 1.555 \times 10^5 \quad \text{J/m}^3
\]

The references do not provide an experimental value.
Figure 41. Hysteresis Curves for Barium Ferrite

\[
\text{erf}\left( \frac{B_{\text{ext\_ratio}}^\alpha}{\beta} \right) \cdot \frac{B_S}{B_{\text{ext\_max}}}.
\]

\[
\text{erf}\left( \frac{B_{\text{ext\_ratio}}^{-\alpha}}{\beta} \right) \cdot \frac{B_S}{B_{\text{ext\_max}}}
\]

\[
n_{r\_\text{BaFe12O19}} := \frac{B_r}{B_{r\_Fe}}
\]

\[
k_{c\_\text{BaFe12O19}} := \frac{B_{\text{ext\_c}}}{B_{\text{ext\_c\_Fe}}}
\]

\[
n_{r\_\text{BaFe12O19}} = 0.156
\]

\[
k_{c\_\text{BaFe12O19}} = 28700.4516
\]
**Note:** Ref. [71] describes in great detail the crystal structure of barium ferrite, BaFe$_{12}$O$_{19}$. The unit cell is, overall, hexagonal, with $a = 5.893$ Å and $c = 23.194$ Å. There are two formula units per cell, so there are 24 Fe atoms, which means there are 12 magnetic dipoles—8 of these point in the direction of the magnetic field, and 4 point in the direction **opposite** to the direction of the magnetic field—substances like this are called "ferrites" and they exhibit "ferrimagnetism." The iron atoms are in the interstices between the O atoms—not at the corners. The average interatomic distance for the 8 dipoles is 2.910 Å (Fe(4)-Fe(4) in the above Ref.); the average distance for the 4 dipoles is 2.778 Å (Fe(5)-Fe(5) in the above Ref.).

\[
\begin{align*}
n_{M_{\text{BaFe}12\text{O}19}} &:= 3 \quad Z_{uc} := 2 \quad n_f := 1 \quad \text{edge}_a := 5.893 \cdot 10^{-10} \quad \text{edge}_c := 23.194 \cdot 10^{-10} \quad \text{m} \\
 s_{0\_1} := 2.910 \cdot 10^{-10} \quad s_{0\_2} := 2.778 \cdot 10^{-10} \quad \text{m} \\
 V_{uc\_SI} &:= \text{edge}_a^2 \cdot \text{edge}_c \cdot \sin(60 \cdot \text{deg}) \quad V_{uc\_SI} = 6.9756 \times 10^{-28} \quad \text{m}^3 \quad \text{(assumed to be approx. constant from 0 to 300 K)} \\
 B_{s_{\text{BaFe}12\text{O}19}} &:= \frac{l_{M \cdot n_f \cdot 8 \cdot n_{M_{\text{BaFe}12\text{O}19}} \cdot \mu_{\text{weber}} \cdot s_{0\_1}}}{V_{uc\_SI}} - \frac{l_{M \cdot n_f \cdot 4 \cdot n_{M_{\text{BaFe}12\text{O}19}} \cdot \mu_{\text{weber}} \cdot s_{0\_2}}}{V_{uc\_SI}} \\
 B_{s_{\text{BaFe}12\text{O}19}} &= 0.3103 \quad \text{tesla} \\
 E_{M_{\text{BaFe}12\text{O}19}} &:= \frac{n_{M_{\text{BaFe}12\text{O}19}} \cdot \mu_{\text{weber}} \cdot n_{M_{\text{BaFe}12\text{O}19}} \cdot \mu_{\text{weber}} \cdot \text{convjoulestoev}}{4 \cdot \pi \cdot \mu_{0\_SI} \cdot \mu_r \_0 \cdot s_{0\_1}} \\
 E_{M_{\text{BaFe}12\text{O}19}} &= 0.282 \quad \text{eV} \quad \text{(using the interatomic distance of the dominant dipole)}
\end{align*}
\]
\[ T_{c_{BaFe12O19}} := \frac{E_{M_{BaFe12O19}}}{3 \cdot k_{B_{ev}}} \text{ K} \]

\[ T_{c_{BaFe12O19}} = 1090.8822 \text{ K} \quad \text{(which compares with 723.15 K, the supposed observation)} \]

\[ B_{s_{BaFe12O19\_300}} := B_{s_{BaFe12O19}} \left[ 1 - \left( \frac{300}{T_{c_{BaFe12O19}}} \right)^2 \right]^{\frac{1}{4}} \]

\[ B_{s_{BaFe12O19\_300}} = 0.3043 \]

\[ nM_{r_{BaFe12O19l}} := 2 \]

\[ B_{r_{BaFe12O19}} := \frac{2}{3} B_{s_{BaFe12O19\_300}} \quad B_{r_{BaFe12O19}} = 0.2028 \quad \text{obs.} = .20, \text{Ref. [31], p. 182, for "isotropic barium ferrite"} \]

Everything checks out—there is no need for the nonsense of the Quantum Mechanics "superexchange" concept. Anisotropic forms of barium ferrite have higher coercivities and \( BH_{\text{max}} \) values—see Ref. [7], Vol. 4, p. 462.
Magnetite Fe$_3$O$_4$

$B_s := .6247 \quad \text{tesla} \quad \text{(see calc. below)}$

$B_r := .4165 \quad \text{tesla} \quad \text{(see calc. below)}$

$\mu_{r_{\text{avg}}} := 83.3 \quad \text{(average permeability, based on } B_{\text{ext}_c}, \text{ given in Ref. [31], p. 478)}$

$B_{\text{ext}_c} := \frac{-B_r}{\mu_{r_{\text{avg}}}} \quad \text{tesla} \quad B_{\text{ext}_c} = -0.005 \quad \text{(external magnetic flux density at coercivity point)}$

Starting Values: $B_{\text{ext}_{\text{max}}} := 10^{-2} \quad \text{tesla} \quad \beta := .5$

TOL := .000001 \quad CTOL := .000001

Given

$$\mu_{r_{\text{avg}}} = \frac{-B_{\text{ext}_c}}{0 - B_{\text{ext}_c}} \cdot B_s - 0$$

$$B_r = \text{erf} \left( \frac{-B_{\text{ext}_c}}{B_{\text{ext}_{\text{max}}} \beta} \right) \cdot B_s$$
\( \left( \frac{B_{\text{ext\_max}}}{\beta} \right) := \text{Find}(B_{\text{ext\_max}}, \beta) \quad B_{\text{ext\_max}} = 0.0149 \quad \alpha := \frac{-B_{\text{ext\_c}}}{B_{\text{ext\_max}}} \quad \alpha = 0.3355 \)

\( \beta = 0.4903 \)

\[ \mu_{\text{r\_calc}} := \frac{\text{erf} \left( \frac{\alpha}{\beta} \right) \cdot B_s - 0}{0 - B_{\text{ext\_c}}} \quad \mu_{\text{r\_calc}} = 83.3 \]

\[ B_{r\_calc} := \text{erf} \left( \frac{\alpha}{\beta} \right) \cdot B_s \quad B_{r\_calc} = 0.4165 \]

\[ \frac{\mu_{\text{r\_calc}}}{\mu_{\text{r\_avg}}} = 1 \quad \frac{B_{r\_calc}}{B_r} = 1 \]

\[ B_{H_{\text{max\_Fe3O4}}} := \left| \text{erf} \left( \frac{-0.5 \cdot B_{\text{ext\_c}}}{B_{\text{ext\_max}}} \right) \cdot B_s \cdot \frac{0.5 \cdot B_{\text{ext\_c}}}{\mu_0 \text{ SI}} \right| \quad B_{H_{\text{max\_Fe3O4}}} = 233.0233 \quad \text{J/m}^3 \]

The references do not provide an experimental value.

\[ W_{H_{\text{Fe3O4}}} := \int_{-1}^{1} \text{erf} \left( \frac{B_{\text{ext\_ratio}} + \alpha}{\beta} \right) \cdot B_s \cdot \frac{B_{\text{ext\_max}}}{\mu_0 \text{ SI}} - \text{erf} \left( \frac{B_{\text{ext\_ratio}} - \alpha}{\beta} \right) \cdot B_s \cdot \frac{B_{\text{ext\_max}}}{\mu_0 \text{ SI}} \, dB_{\text{ext\_ratio}} \]

\[ W_{H_{\text{Fe3O4}}} = 9833.7969 \quad \text{J/m}^3 \]

The references do not provide an experimental value.
Figure 42. Hysteresis Curves for Magnetite

\[ n_{r\_Fe3O4} := \frac{B_r}{B_{r\_Fe}} \]

\[ k_{c\_Fe3O4} := \frac{B_{ext\_c}}{B_{ext\_c\_Fe}} \]

\[ n_{r\_Fe3O4} = 0.3204 \]

\[ k_{c\_Fe3O4} = 846.1538 \]
Note: Magnetite is "lodestone"—the original natural magnetic material; Lucretius wrote about its wonders prior to 55 B.C.E. Ref. [65], Vol. 2, p.83 describes the crystal structure of magnetite, Fe₃O₄. The unit cell is, overall, FCC, with each edge = 8.3963 Å. There are eight formula units per cell, so there are 24 Fe atoms, which means there are 12 magnetic dipoles—8 of these point in the direction of the magnetic field, and 4 point in the direction opposite to the direction of the magnetic field. The iron atoms are in the interstices between the O atoms—not at the corners. From the Reciprocal System Data Base calculations, the interatomic distance for the 8 dipoles is 5.6519 Å; the interatomic distance for the 4 dipoles is also 5.6519 Å. (Other interatomic distances between the iron atoms are 4.1982, 3.6357, and 3.7842 Å, but these are not used for the dipoles—only the longest distance is used.)

\[
n_{M_{Fe3O4}} := 3 \quad Z_{uc} := 8 \quad n_f := 1 \quad \text{edge}_c := 8.3963 \times 10^{-10} \text{ m (all edges same)}
\]

\[
\text{s}_0_1 := 5.6519 \times 10^{-10} \quad \text{s}_0_2 := 5.6519 \times 10^{-10} \text{ m}
\]

\[
V_{uc_SI} := 591.921 \times 10^{-30} \text{ m}^3 \text{ (assumed to be approx. constant from 0 to 300 K)}
\]

\[
B_{s_{Fe3O4}} := \frac{l_{M} n_f \cdot 8 \cdot n_{M_{Fe3O4}} \cdot M_{u_{weber}} \cdot s_0_1}{V_{uc_SI}} - \frac{l_{M} n_f \cdot 4 \cdot n_{M_{Fe3O4}} \cdot M_{u_{weber}} \cdot s_0_2}{V_{uc_SI}}
\]

\[
B_{s_{Fe3O4}} = 0.6794 \text{ tesla}
\]

\[
E_{M_{Fe3O4}} := \frac{n_{M_{Fe3O4}} \cdot M_{u_{weber}} \cdot n_{M_{Fe3O4}} \cdot M_{u_{weber}}}{4 \cdot \pi \cdot \mu_0_{SI} \cdot H_{r_0} \cdot s_0_1} \cdot \text{conv joulesto ev}
\]

\[
E_{M_{Fe3O4}} = 0.1452 \text{ eV (using the interatomic distance of the dominant dipole)}
\]
\( Tc_{\text{Fe3O4}} := \frac{EM_{\text{Fe3O4}}}{3 \cdot k_{B \text{ ev}}} \) K

\( Tc_{\text{Fe3O4}} = 561.6637 \) K

(which compares with 848.15 K, the supposed observation, Ref. [31], p. 477)

\[ Bs_{\text{Fe3O4}_300} := Bs_{\text{Fe3O4}} \left[ 1 - \left( \frac{300}{Tc_{\text{Fe3O4}}} \right)^2 \right]^{\frac{1}{4}} \]

\( Bs_{\text{Fe3O4}_300} = 0.6247 \) tesla

\( n_{M_r \text{Fe3O4}} := 2 \)

\( Br_{\text{Fe3O4}} := \frac{2}{3} \cdot Bs_{\text{Fe3O4}_300} \)

\( Br_{\text{Fe3O4}} = 0.4165 \) tesla

obs. = .44, Ref. [31], p. 478

Everything checks out—there is no need for the nonsense of the Quantum Mechanics "superexchange" concept.

The sample calculations cover all of the major classes of ferromagnetic materials and verify the conclusions of the Reciprocal System: each ferromagnetic atom has two magnetic charges, one N and one S, orthogonal to one another. In a solid, magnetic dipoles are formed, and all of the observed properties are thereby explained. Therefore, James Alfred Ewing, the great physicist who theorized that atoms were tiny magnets, and who was the first to discover magnetic hysteresis, has been vindicated.
Conclusion

The Reciprocal System represents a new paradigm for theoretical physics. It is a unified, general theory, applicable to all of the categories of physical phenomena, including the electric and magnetic susceptibilities of matter. This paper applies the theory to the calculation of all of the important properties of dielectrics, diamagnets, paramagnets, and ferromagnets; included are 150 equations, 42 figures, and 8 tables covering the calculated and observed values of these properties. The Reciprocal System calculations are usually in agreement with the observations, and where there are non-negligible differences, theoretical explanations are readily available. Unlike conventional physics, the Reciprocal System does not use charged electrons to explain these properties; massless, chargeless electrons are the physical entities involved in ordinary electric circuits, and the additional space and time of the atoms of a dielectric account for the value of the index of refraction and dielectric constant. Magnetic charges do exist, contrary to the assertions of conventional physics, and account for the properties of diamagnets, paramagnets, and ferromagnets.
Acknowledgements

Funding for this work came from my company, Transpower Corporation. Of course, great thanks go to Dewey B. Larson, who served as my theoretical physics mentor from 1965 until his death in 1990. He was, by far, the most intelligent and most logical of any individual I've ever known.
References


Last revised 06/19/2012—corrected dimensions of capacitance and revised RC and RLC circuit calculations
\[ 1 + c_1 \cdot \beta \cdot \sin(t \cdot \beta) \]
\[
\left[ \frac{-V_{\text{max}}}{\sqrt{R^2 + \omega^2 L^2}} \cdot \sin \left( \phi_{ac} - \text{atan} \left( \frac{\omega \cdot L}{R} \right) \right) \right] + \left[ \frac{V_{\text{max}}}{\sqrt{R^2 + \omega^2 L^2}} \cdot \sin \left( \omega \cdot t + \phi_{ac} - \text{atan} \left( \frac{\omega \cdot L}{R} \right) \right) \right] dt
\]
\[
p\left(\frac{-R}{L} \cdot t\right) \cdot \left( \frac{-V_{\text{max}}}{\sqrt{R^2 + \omega^2 \cdot L^2}} \cdot \sin \left( \phi_{\text{ac}} - \tan \left( \frac{\omega \cdot L}{R} \right) \right) \right) + \frac{V_{\text{max}}}{\sqrt{R^2 + \omega^2 \cdot L^2}} \cdot \sin \left( \omega \cdot t + \phi_{\text{ac}} - \tan \left( \frac{\omega \cdot L}{R} \right) \right) \right) \right] \, dt
\]
\[
\frac{\omega L}{R} \left( \frac{V_{\text{max}}}{\sqrt{R^2 + \omega^2 L^2}} \cdot \sin \left( \omega t + \phi_{\text{ac}} - \arctan \left( \frac{\omega L}{R} \right) \right) \right)
\]
\[ p\left(\frac{-R}{L} \cdot t\right) \cdot \left(\frac{-V_{\text{max}}}{\sqrt{R^2 + \omega^2 L^2}} \cdot \sin\left(\phi_{\text{ac}} - \arctan\left(\frac{\omega \cdot L}{R}\right)\right)\right) + \frac{V_{\text{max}}}{\sqrt{R^2 + \omega^2 L^2}} \cdot \sin\left(\omega \cdot t + \phi_{\text{ac}} - \arctan\left(\frac{\omega \cdot L}{R}\right)\right) \right] \text{dt} \]
\[ \Xi_C \omega^2 \cdot R \cdot T - 2 \cdotE_C \cdot \omega^2 \cdot L \left( \frac{1}{2} \right) \]
\[- C \left[ \frac{4 \cdot V_{\text{max}}}{T \cdot \omega} \cdot (1 - \cos(0.25 \cdot T \cdot \omega)) - R \cdot i_{\text{eff}} - L \cdot \frac{2 \cdot i_{\text{eff}}}{T} \right]^2 \cdot \omega^2 \cdot R \cdot T - 2 \cdot C \left[ \frac{4 \cdot V_{\text{max}}}{T \cdot \omega} \cdot (1 - \cos(0.25 \cdot T \cdot \omega)) - R \cdot i_{\text{eff}} - L \cdot \frac{2 \cdot i_{\text{eff}}}{T} \right]^2 \cdot \omega^2 \cdot L \right]^{\frac{1}{2}}\]
\[
- \frac{4 \cdot C \cdot R \cdot V_{\text{max}} \cdot \omega \cdot t \cdot \cos(\omega \cdot t)}{\omega \cdot t} - \frac{V_{\text{max}} \sqrt{t^2 \cdot \sin\left(\frac{\omega \cdot t}{2}\right)^2 - C \cdot R \cdot t \cdot \cos(\omega \cdot t)^2 - C \cdot R \cdot t \cdot \cos(\omega \cdot t) \cdot \sin\left(\frac{\omega \cdot t}{2}\right)^2 + 2 \cdot C \cdot R \cdot t \cdot \cos(\omega \cdot t) + C \cdot R \cdot t \cdot \sin\left(\frac{\omega \cdot t}{2}\right)^2}}{\omega \cdot t} = \frac{4 \cdot C \cdot L^2 \cdot \omega^2 + 4 \cdot C \cdot L \cdot R \cdot \omega^2 \cdot t + C \cdot R^2 \cdot \omega^2 \cdot t^2 + 4 \cdot R \cdot \omega^2 \cdot t^3}{\omega \cdot t}
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