

# Theory of Atomic Spectra and Ionization Energies

by  
Ronald W. Satz, Ph.D.\*  
Transpower Corporation

## Abstract

This paper presents the derivation of the equations for atomic spectra and ionization energies according to the Reciprocal System of physical theory developed by D. B. Larson. The factors include atomic number, ionization level, ion shift, isotope shift, temperature, and integer translational quantum numbers representing atomic energy levels.

**keywords:** atomic spectra, ionization energies, atomic number, atomic energy, ion shift, isotope shift, arc spectra, spark spectra, x-ray spectra, Rydberg series, line spectra, continuous spectra, spectra intensity, spectral line splitting

\*The author is president of Transpower Corporation, a commercial and custom software manufacturing company and engineering/physics consultancy. Mailing address: P. O. Box 7132, Penndel, PA 19047. He is a full member of ASME, SAE, INFORMS, ISUS, and SIAM.

## Introduction and Literature Survey

Spectroscopy began with Newton's work on prisms and refraction.

In the 19th century, spectroscopists discovered various "series" of photon frequencies emitted or absorbed by elements on the earth and from the sun, and there were many attempts to derive equations to represent them. Fowler, Ref. [12], provides an interesting historical account of these attempts, as well as as a brief discussion of the Bohr theory.

By the 1930's, Quantum Mechanics was in ascendancy, and Prof. White, in Ref. [8], explains this theory in considerable detail, along with the experimental data. This textbook is still in use today at our universities and has not been superceded.

A brief treatment of the physical concepts of conventional spectroscopic theory is given by Herzberg, Ref. [9].

More recent treatments of theory and experiment are given by Kuhn, Ref. [10], and Candler, Ref. [11]. Kuhn (p. 1) begins his work with this statement: "The light emitted by atomic gases in a discharge tube is due to the motions of electrons in the atom, and by studying this light we can gain information on the electronic structure which determines the physical and chemical properties of the atom." But this is pure *assumption*--there is no actual physical evidence verifying it. To his credit, though, Kuhn does use "effective quantum numbers" and other *empirical* relations to calculate spectra. Candler's work is on the same level as Kuhn's; he includes numerous Grotrian (energy) diagrams for elements and ions.

Walker and Straw are the authors of two volumes of applied spectroscopy, Ref. [13] and Ref. [14]. These volumes contain numerous diagrams of the instruments used in spectroscopy.

In contrast, Condon and Shortley, Ref. [15], give a purely theoretical account of atomic spectra. Their work is full of the usual impractical, complicated equations of Quantum Mechanics. They talk about the central-field approximation, Russell-Saunders coupling, j-j coupling, intermediate coupling, the perturbation method, etc.

Over at the National Bureau of Standards (now known as NIST), Moore compiled four volumes of atomic energy levels, Ref. [16], [17], [18], and [19]. Much of this material is now online at Ref. [20]. She says, Ref. [16], p. viii, "Each chemical element can emit as many atomic spectra as it has electrons." This statement needs to be revised to say, "Each chemical element can emit as many atomic spectra as its *atomic number*." Also, unfortunately, the work does not contain the Rydberg denominators of the series members.

Moseley, Ref. [21], was the first scientist to grasp the concept of *atomic number*. His famous equations for x-ray spectra and the "screening doublet law" can be generalized to cover *all* spectra, as will be shown later in this paper.

A useful compilation of astrophysical formulae, including those covering continuum radiation and line radiation, is provided by Prof. Lang, Ref. [22]. Another, similar compilation, though focused more on data, is that edited by Cox, Ref. [23].

Several handbooks have tables of spectroscopic data. These include Ref. [24], a Russian compilation; Ref. [25], an American compilation; and Ref. [26], the AIP Handbook.

A fairly extensive search of the physics literature yields just two papers which extend Moseley's work in somewhat the same fashion as what we will attempt, Ref. [27] and Ref. [28]. These papers use the term "screening constant" for what we will call "ion shift." The equations in the papers are still those of Quantum Mechanics and involve the distance between each possible pair of electrons. They are impractical for the computation of *complex* spectra.

Ref. [29] is a clear review of modern physics with many solved problems.

Dewey B. Larson, the developer of the Reciprocal System of theory, thoroughly debunked the nuclear theory of the atom and Quantum Mechanics in Ref. [1]. *This work should be read before continuing with this paper.* His other works, Ref. [2], [3], [4], [5], and [6], describe the Reciprocal System in detail. The fundamentals of spectroscopic theory are given in Ref. [3], and there are minor mentions in the others, such as Ref. [6], p. 216.

The author's *The Unmysterious Universe*, Ref. [7], is a primer on the Reciprocal System. Ref. [30] includes equations for work function and single-level ionization energy; Ref. [31] gives the Reciprocal System equations for subatomic masses and atomic masses; and Ref. [32] provides the Reciprocal System equations for many electrical and magnetic phenomena.

This paper is a runnable *Mathcad* program, so the equations are more detailed than they otherwise would be.

## Nomenclature

A = angstrom unit ( $10^{-8}$  cm or  $10^{-10}$  m)

A = ionization energy, conventional theory, eV

a = conventional theory adjustable constant in spectra equation

a, b = translational quantum numbers used in Reciprocal System for simple spectra

B = magnetic field, tesla (= webers/m<sup>2</sup>)

B = brightness of radiation from a black body, a perfect absorber, in thermal equilibrium, erg/(sec cm<sup>2</sup> Hz steradian)  
 note: subscript ( $\nu, \lambda, \nu'$ ) indicates the abscissa (frequency, wavelength, wave number) used in the intensity plots

$c_{\text{cgs}}$  = speed of light in cgs units

$c_{\text{SI}}$  = speed of light in SI units

ck, cl, cm, cn = *Mathcad* range variables for calculation of ionization energies

$\text{conv}_{\text{evtojoule}}$  = factor to convert eV to joule

D = diameter of atom, m (approx. same as "nucleus" of conventional theory; subscript has element symbol)

$D_{\text{u}}$  = natural Reciprocal System unit of atomic diameter, stated in m

E = total energy of atom, eV (subscript has element symbol), eV

$E_{\text{B}}$  = energy of atom due to magnetic field, eV

$E_{\text{E}}$  = energy of atom due to electric field, eV

$E_i$  or  $E_j$  = individual ionization energy of atom (subscript gives element symbol and ionization stage), eV

$E_{i\_tot}$  = total ionization energy of atom (up to and including current level), eV (subscript has element symbol)

$E_{other}$  = other atomic energy (electric, magnetic, photonic, etc., subscript has element symbol and indication of state), eV

$E_{th}$  = thermal energy of atom, from thermodynamics, eV

$F_V$  = electric field intensity, V/m

$h$  = Planck's constant, eV-sec

$h_{cgs}$  = Planck's constant in cgs units, erg-sec

$h_{SI}$  = Planck's constant in SI units, joule-sec

$I$  = spectral line intensity, eV/sec

$I_R$  = Reciprocal System inter-regional ratio

inf = infinity (the regular infinity symbol does not always work properly in different versions of *Mathcad*)

$k_{B\_ev}$  = Boltzmann's constant in eV/K

$k_{B\_erg}$  = Boltzmann's constant in erg/K

$M$  = atomic or molecular weight, u

$M_{H1\_u}$  = mass of hydrogen 1 isotope, u

$m_e$  = mass of electron, u

$m_l$  = conventional theory magnetic quantum number

$m_1, m_2$  = Reciprocal System time-space region thermal (translational) quantum numbers (dimensionless integers)

$n$  = conventional theory principal quantum number (dimensionless integer)

$n_M$  = number of magnetic charges of atom (dimensionless integer)

$n_p, n_s, n_e$  = principal, subordinate, and electric rotational displacements of atom (dimensionless integers)

$n_1, n_2$  = Reciprocal System time region thermal (translational) quantum numbers (dimensionless integers)

$n_3, n_4$  = Reciprocal System ion motion quantum numbers for  $n_2$  for atom in electric field (dimensionless integers)

$p$  = adjustable constant ("screening constant") in conventional theory (real number)

$Q_{uc}$  = Reciprocal System natural unit of electric charge, stated in coulombs

$R$  = Rydberg fundamental frequency for H1, Hz (if there is a subscript, then the Rydberg frequency for that element)

$R'$  = Rydberg fundamental wave number,  $\text{cm}^{-1}$

$R_v$  = Rydberg fundamental frequency stated in half-cycles/sec

$R_{gas}$  = molecular gas constant, erg/gmole K

$s_u$  = Reciprocal System natural unit of distance, stated in cm

$s_{t_u}$  = Reciprocal System time region natural unit of distance, stated in cm

$T$  = temperature, K

$T_{v\_u}$  = Reciprocal System natural unit of temperature for atoms in vapor state, K

$W$  = number of transitions taking place per second in spectral source

$xx$  = Reciprocal Shift ion shift number (real number, equivalent to screening constant in conventional theory)

$Z$  = atomic number

$\Delta$  = change in (e.g.,  $\Delta v$  = change in wave number,  $\text{cm}^{-1}$ )

$\epsilon_{0\_SI}$  = permittivity of free space in SI units

$\epsilon_r$  = relative permittivity (dimensionless)

$\lambda$  = wavelength of spectral line (usually cm, but could be m or Å depending on context)

$\lambda_B$  = wavelength of spectral line due to magnetic field (cm)

$\lambda_E$  = wavelength of spectral line due to electric field (cm)

$\lambda_m$  = wavelength of spectral line of maximum intensity

$\lambda_u$  = Rydberg fundamental wavelength, cm

$\mu_{0\_SI}$  = magnetic permeability of free space in SI units

$\mu_{admp\_u\_som}$  = Reciprocal System natural unit of atomic magnetic dipole moment stated in Sommerfeld units, joule/tesla

$\mu_{B\_som}$  = Bohr magneton stated in Sommerfeld units, joule/tesla

$\mu_L$  = Larson magneton stated in Sommerfeld units, joule/tesla

$\nu$  = frequency of spectral line, Hz (subscript may have element symbol)

$\nu_B$  = frequency of spectral line due to magnetic field, Hz

$\nu_E$  = frequency of spectral line due to electric field, Hz

$\nu^{\prime}$  = wave number of spectral line,  $\text{cm}^{-1}$

$\nu^{\prime}_0$  = initial value of wave number before application of field

$\nu_m$  = frequency of spectral line of greatest intensity, Hz

$\nu^{\prime}_m$  = wave number of spectral line of greatest intensity,  $\text{cm}^{-1}$

$\Phi_{01}$  = 0-1 step function

Note: A black square in the upper right of an equation means that the equation is disabled from running in *Mathcad*. This is done because not all variables in the equation have, as yet, been given numerical values. Because of the nature of this subject, it is necessary to use a mixture of SI, cgs, and natural units in the paper, but each individual equation utilizes just one set of units.



## Reciprocal System Physical Constants (from Ref. [2]-[6], [31], or derived therefrom)

$$h := 4.14 \cdot 10^{-15} \text{ ev} \cdot \text{sec} \quad h_{\text{SI}} := 6.626 \cdot 10^{-34} \text{ J} \cdot \text{sec} \quad h_{\text{cgs}} := 6.626 \cdot 10^{-27} \text{ erg} \cdot \text{sec}$$

$$c_{\text{cgs}} := 2.997925 \cdot 10^{10} \text{ cm/sec} \quad c_{\text{SI}} := 2.997925 \cdot 10^8 \text{ m/sec} \quad \mu_{0\_SI} := 4 \cdot \pi \cdot 10^{-7} \text{ henry/m}$$

$$I_R := 156.4444 \text{ (inter-regional ratio)} \quad T_{V\_U} := 3.5978 \cdot 10^9 \text{ K} \quad D_U := 3.359 \cdot 10^{-15} \text{ m}$$

$$R_V := 6.576115 \cdot 10^{15} \text{ half-cycles/sec} \quad R := \frac{R_V}{2} \quad R = 3.28806 \times 10^{15} \text{ cycles/sec} \quad s_U := 4.558816 \cdot 10^{-6} \text{ cm}$$

$$\lambda_U := \frac{c_{\text{cgs}}}{R} \quad \lambda_U = 9.11762 \times 10^{-6} \text{ cm} \quad R' := \frac{1}{\lambda_U} \quad R' = 109677.77713 \text{ cm}^{-1} \quad \text{conv}_{\text{evtojoule}} := 1.602062 \cdot 10^{-19}$$

$$m_e := .000548567 \text{ u} \quad k_{B\_ev} := 8.617 \cdot 10^{-5} \text{ ev/K} \quad Q_{UC} := 1.602062 \cdot 10^{-19}$$

$$\Phi_{01}(x) := \begin{cases} 1 & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases} \quad \text{0-1 Step Function} \quad k_{B\_erg} := 1.602062 \cdot 10^{-12} \cdot k_{B\_ev} \quad (\text{Boltzmann constant})$$

$$Z := \sum_{r=1}^{n_p-1} (2 \cdot r^2) + \sum_{r=1}^{n_s} (2 \cdot r^2) + n_e - 2 \quad \epsilon_{0\_SI} := 8.85 \cdot 10^{-12} \text{ "farads/meter"} \quad s_{t\_u} := \frac{s_U}{I_R}$$

atomic number of an element with rotational displacements  $n_p, n_s, n_e$

see Reciprocal System Periodic Table in Ref. [7], pp. 42-43

$$\mu_{B\_som} := 9.2741 \cdot 10^{-24} \text{ joule/tesla} \quad \mu_{\text{admp\_u\_som}} := \frac{9.3175 \cdot 10^{-37}}{\mu_{0\_SI}} \quad \mu_{\text{admp\_u\_som}} = 7.41463 \times 10^{-31} \text{ joule/tesla}$$

## 1. Simple Atomic Spectra

In the Reciprocal System, as expressed by Larson in 1959 (Ref. [3] , p. 122): "The frequency or vibrational velocity of the escaping space unit, the photon, is determined by the characteristics of the atomic vibrational motion from which the motion of the space unit was derived." In other words, it is the *thermal motion* and *ionization level* of the atom that counts here, *not* the motion of some alleged electrons jumping from orbit to orbit around an alleged nucleus.

Continuing with Larson, p. 122: "At zero temperature the thermal vibration period is infinite and the equivalent thermal velocity  $1/t^2$  [(1/t)/t because the atom is inside unit space] is  $1/\infty$  or zero. The addition of thermal energy, which is space displacement, is equivalent to reducing the time displacement. The vibration period and the corresponding equivalent thermal velocity therefore decrease with increasing temperature up to a limit of  $t = 1$ , at which point the molecule has reached the boundary of the time region and is ready to make the transition into the time-space region. Since it is the thermal velocity (with some modifications to be considered later) which is radiated, the distribution of radiated frequencies or *spectrum* emitted by time region structures comprises all values of  $1/t^2$  from  $1/\infty$  to a maximum which depends on the temperature, the absolute maximum being  $1/1$ . Because of the small interval between these values of  $1/t^2$  in the solid and the modifications to which the frequencies are subjected in escaping from the dense solid or liquid structures the actual distribution of the frequencies is essentially continuous and we observe a *continuous spectrum*."

Continuing, p. 122: "At the unit level, the boundary between the time region and the time-space region, there is a directional reversal and further additions of motion of the same nature, motion which is equivalent to thermal energy, go into the reverse velocity component of a compound velocity: velocity of a velocity (mass). The total equivalent thermal energy in the two regions is the sum of the regional components, but since the velocity in the time-space region is inversely directed, the resultant velocity (frequency) of the radiation is the difference between the time region and the time-space region velocity-energy components."

Continuing, p. 123: "As we have found, velocity in the time region is in equilibrium with energy in the time-space region. The latter in turn is proportional to the square of the time-space region velocity. Where the time displacement is  $b$ , the velocity is  $1/b$  and the energy per mass unit is equal to  $1/b^2$ . This is the time-space contribution to the radiation frequency and since it is in the opposite direction from the time region component  $1/a^2$  the resultant is  $1/a^2 - 1/b^2$ . In order to maintain a positive value of the resultant it is necessary that  $b$  exceed  $a$  by at least one unit and the minimum value of  $b$  is therefore 2."

Continuing, p. 123: "It will be noted that the velocity interval for the normal range of temperatures in the time-space (gas) region is relatively large; that is the difference between  $1/2^2$  and  $1/3^2$  involves a reduction of about 55%. Furthermore, there is comparatively little interference on the way out of a gas aggregate. Instead of a continuous spectrum the gas therefore has a *line spectrum*: a regular succession of discrete frequencies resulting from the various possible values of the displacements  $a$  and  $b$ ."

Continuing, p. 123: "In the case of hydrogen there are no modifying factors and the frequencies can be obtained directly from the expression  $1/a^2 - 1/b^2$ , utilizing the value of unit frequency previously employed, which we will designate  $R$ , in accordance with the usual practice.

$$v_H = R(1/a^2 - 1/b^2) \quad (144)$$

Placing  $a$  equal to 1 and assigning the successive values 2, 3, 4, etc. to the displacement  $b$ , we obtain the Lyman series of spectral lines. Although the time region displacement  $a$  can reach unit value before gas motion starts, this is not mandatory and other series based on higher values of  $a$  also occur, becoming less and less probable as  $a$  increases beyond 2. When  $a = 2$  and  $b$  has the values 3, 4, 5, etc., the well-known Balmer series, the first spectral series to be identified, is the result. When  $a = 3$  we obtain the Paschen series,  $a = 4$  gives us the Brackett series, and so on."

Continuing, p. 123: "The frequencies for ionized helium can also be calculated from equation 144 by introducing the factor 4.

$$v_{He} = 4R(1/a^2 - 1/b^2) \quad (145)$$

Continuing, pp. 123-124: In this case the normal one unit of space in the expression  $1/a^2$  and  $1/b^2$  has been increased to two by the addition of one unit of rotational space vibration [positive electric charge]. The velocity  $1/a^2$ , which is actually  $(1/a)^2$ , now becomes  $(2/a)^2$  or  $4(1/a)^2$ . A similar change takes place in the  $b$  component. To generalize we may say that ionization increases the spectral frequencies by the factor  $e^2$ , where  $e$  is the total ionization based on the normal state as unity. The generalized equation for the hydrogen type spectrum is therefore

$$\nu = Re^2(1/a^2 - 1/b^2) \quad (146)$$

Continuing, p. 124: "When we examine the velocity relations of other elements we find that this hydrogen type spectrum is not characteristic of the normal atom but represents a special case in which the effects of all motion other than thermal are eliminated by the absence of any rotational motion with a force integral exceeding unity. In the normal atom effective rotational motions do exist and the radiation frequencies are modified very substantially by these coexisting velocities."

Continuing, p. 125: "...even a preliminary analysis is sufficient to indicate that the numerical values of the spectral terms conform to the general relationship that would be expected on theoretical grounds: that is, each term of the spectral combinations is one of the terms of equation 145 (the thermal motion) plus or minus the effective components of the other motions of the atom, including the rotation, the basic linear vibration, the rotational vibration, the secondary motion of the associated space unit, any electric or magnetic motion that may be present, etc. The splitting of the various terms under certain conditions is obviously due to the fact that the directions of these other motions are not necessarily fixed with reference to the direction of the thermal motion and the corresponding frequency increments may be either plus or minus."

That's about as far as Larson carried the theory of spectra (other than some work on the alkalis), so we're on our own from here on out. Let's first compare the atomic energy terms of conventional spectral theory with those of the Reciprocal System.

## 2. Complex Atomic Spectra

### a. Conventional and Reciprocal System Expressions for Atomic Energies

The conventional expression for the atomic energy level of an atom is given by Ref. [9], p. 197 as

$$E_{el} := A - \frac{(Z - p)^2 \cdot h \cdot R}{(n + a)^2} \quad (A, p, \text{ and } a \text{ are three } \textit{adjustable} \text{ real constants, } n \text{ is an integer, the supposed principal quantum number})$$

From Larson's statements above we can easily write the *generalized* atomic energy level of an atom in the Reciprocal System:

$$E_{el} := E_{el\_I\_tot} + E_{\_el\_I} \cdot \left( \frac{1}{n_1^2} - \frac{1}{m_1^2} \right) + E_{el\_other} \quad (1) \quad \begin{array}{l} \text{(Reciprocal System expression,} \\ \textit{translational} \text{ quantum numbers } n_1, m_1 \\ \text{integers only)} \end{array}$$

$E_{el\_I}$  is the *individual* ionization energy of the *next* level up,  $E_{el\_I\_tot}$  is the *total* ionization energy *up to and including* the *current ionization* level of the atom, and  $n_1$  and  $m_1$  are *integers*, so that  $\left( \frac{1}{n_1^2} - \frac{1}{m_1^2} \right)$  represents a *fraction* of the *next*

higher ionization level,  $E_{el\_I}$ . Of course, the correct Roman numerals should be substituted for I in  $E_{el\_I}$  and  $E_{el\_I\_tot}$  above, with the value of I in  $E_{el\_I\_tot}$  being *one less* than the value of I in  $E_{el\_I}$ .  $E_{el\_other}$  includes *all* other energy, including that of the internal photons, any magnetic charges, etc. (The ground state frequency of both internal photons is 2R; under some circumstances, this can increase to 3R or 4R; but this increase will normally be quickly radiated back out or changed to some other form of energy.)

At the *limit* of  $n$  or  $n_1$  and  $m_1$ :

$$E_{el} := A^n \quad \text{when } n \rightarrow \text{inf} \quad (\text{conventional expression, at } n = \text{inf} \text{ the atom is } \textit{ionized} \text{ or the ion is } \textit{further ionized})$$

$$E_{el\_tot} := E_{el\_I\_tot} + E_{el\_I} + E_{el\_other}^n \quad \text{when } n_1 = 1 \text{ and } m_1 = \text{inf} \quad (\text{Reciprocal System expression})$$

where, again, the value of  $I$  in  $E_{el\_I\_tot}$  is *one less* than the value of  $I$  in  $E_{el\_I}$

**Example:** suppose the atom has two positive electric charges and the temperature is above 0 K. Then, for the Reciprocal System,

$$E_{el\_II\_tot} := E_{el\_I} + E_{el\_II}^n \quad (\text{total ionization energy at level II = sum of } \textit{individual} \text{ ionization levels I and II})$$

$$E_{el\_tot} := E_{el\_II\_tot} + E_{el\_III} \cdot \left( \frac{1}{n_1^2} - \frac{1}{m_1^2} \right) + E_{el\_other}^n \quad n_1 \geq 1 \quad m_1 > n_1 \quad (\text{integers only})$$

This means that the *ground state* for an atom or ion in the Reciprocal System occurs when  $(1/n_1^2 - 1/m_1^2) \rightarrow 1/\text{inf} \rightarrow 0$ , which occurs at  $T = 0$  K. When  $n_1$  is very high, the term values are difficult to distinguish from one another and so we have a "continuous" spectrum at low temperatures. Actually, it's still discrete, but the common spectroscopes cannot distinguish the lines. With greater and greater thermal motion, i.e., *translational space displacement*, eventually a limit is reached and this translational space vibration *converts* to rotational space vibration--a positive electric charge--and the atom is ionized (or raised to the next ionization level). This also explains why there is such a difference in spectra between arc and spark (something that is difficult to explain by conventional theory). In arc spectra the atoms are at approx. 4000 K and usually neutral, whereas in spark spectra the atoms are at approx. 40000 K and usually ionized. X-ray spectra come from atoms at very, very high energy levels and result from *jumps* in ionization levels.

When a photon is emitted or absorbed by an atom, the energy of the atom changes by the energy,  $h\nu$ , of that photon. The frequency of a photon emitted or absorbed by a gaseous atom may thus be represented in the Reciprocal System as

$$\nu := \frac{1}{h} \cdot \left[ E_{\text{el}_l\text{tot}_1} + E_{\text{el}_l_1} \cdot \left( \frac{1}{n_1^2} - \frac{1}{m_1^2} \right) + E_{\text{el\_other}_1} - \left[ E_{\text{el}_l\text{tot}_2} + E_{\text{el}_l_2} \cdot \left( \frac{1}{n_2^2} - \frac{1}{m_2^2} \right) + E_{\text{el\_other}_2} \right] \right] \quad (2)$$

where 1 indicates the first (initial) state of the atom and 2 indicates the second (final) state of the atom. This expression works for *all* spectra, including radio, infrared, optical, ultraviolet, and x-ray, and therefore this equation *completely supercedes any equation from conventional theory*. Again, note that the correct Roman numerals must be substituted for the l's in this equation. The Reciprocal System is a *completely quantized* theory, so  $n_1$ ,  $m_1$ ,  $n_2$ , and  $m_2$  are *integers only*. Note: conventional theory *cannot* use very large values of such integers because it would make the theoretical atom *too large*!

Of course, for optical spectra, there is normally no change in ionization level. If we can neglect the change in  $E_{\text{other}}$ , the expression simplifies to

$$\nu := \frac{E_{\text{el}_l}}{h} \cdot \left[ \left[ \left( \frac{1}{n_1^2} - \frac{1}{m_1^2} \right) - \left( \frac{1}{n_2^2} - \frac{1}{m_2^2} \right) \right] \right] \quad (3)$$

Furthermore, for hydrogen and hydrogen-like atoms, the  $1/m_1$  and  $1/m_2$  can often be neglected, so that we finally have

$$\nu := \frac{E_{\text{el}_l}}{h} \cdot \left[ \left[ \left( \frac{1}{n_1^2} \right) - \left( \frac{1}{n_2^2} \right) \right] \right] \quad (4)$$

With the proper selection of the ionization levels and the  $n$ 's and  $m$ 's, the frequencies of *all* spectra of *all* elements can be found. Other than probability considerations, there is no limitation on the integer values of the  $n$ 's and  $m$ 's; obviously the most probable values of the  $n$ 's and  $m$ 's will give the most intense spectral lines.

## b. Determination of $n$ and $m$ Values

The  $n$ 's generally have a low value (for an atom in the gas or vapor state), whereas the  $m$ 's generally have a high value. Given that the  $n$ 's and  $m$ 's are energy coefficients, we can determine them by equating the atomic thermal energy with the thermodynamic energy expression involving temperature (Ref. [22], p. 21 and see below, Section 4):

$$E_{th} := 3 \cdot k_{B\_ev} \cdot T \quad (5)$$

**Example:** For a very hot arc spectra of H1 (isotope 1 of hydrogen, not ionized),

$$T_1 := 11058 \quad K \quad (\text{from } iteration \text{ with equations below to obtain } integer \text{ values of } n \text{ and } m)$$

From Eq. (5):

$$E_{th} := 3 \cdot k_{B\_ev} \cdot T_1 \quad E_{th} = 2.8586 \quad eV \quad (\text{initial state of atom})$$

$$Z_H := 1 \quad R_{H1} := R$$

$$E_{H1\_I} := Z_H^2 \cdot h \cdot R_{H1} \quad E_{H1\_I} = 13.61256 \quad eV \quad (\text{ionization energy level I})$$

Neglecting  $m_1$ , temporarily,

$$E_{th} = E_{H1\_I} \cdot \left( \frac{1}{n_1^2} \right) \quad n_1 := \sqrt{\frac{E_{H1\_I}}{E_{th}}} \quad n_1 = 2.18219$$



But this is not an integer, so we must "floor" it and then compute  $m_1$  and  $T_1$  by iteration, so that  $m_1$  is also an integer.

$$n_1 := \text{floor}(n_1) \quad n_1 = 2$$

$$E_{\text{th}} = E_{\text{H1}_1} \cdot \left( \frac{1}{n_1^2} - \frac{1}{m_1^2} \right) \quad m_1 := \frac{n_1 \cdot \sqrt{E_{\text{H1}_1}}}{\sqrt{E_{\text{H1}_1} - E_{\text{th}} \cdot n_1^2}} \quad m_1 = 4.99985 \quad m_1 := 5 \quad (\text{close enough})$$

$$E_{\text{H1}_1} := E_{\text{H1}_1} \cdot \left( \frac{1}{n_1^2} - \frac{1}{m_1^2} \right) \quad E_{\text{H1}_1} = 2.85864 \quad \text{eV} \quad (\text{state 1})$$

So we have a Balmer energy hydrogen atom. If it absorbs a photon causing the  $m_1$  coefficient to increase by 1, then:

$$m_2 := m_1 + 1 \quad n_2 := n_1$$

$$\nu := \frac{E_{\text{H1}_1}}{h} \cdot \left| \left( \frac{1}{n_1^2} - \frac{1}{m_1^2} \right) - \left( \frac{1}{n_2^2} - \frac{1}{m_2^2} \right) \right|$$

$$\nu = 4.01874 \times 10^{13} \quad \text{cycle/sec} \quad (\text{absorbed}) \quad \lambda := \frac{c_{\text{cgs}}}{\nu} \cdot 10^8 \quad \lambda = 74598.68714 \quad \text{A} \quad (\text{far infrared})$$

$$E_{\text{H1}_2} := E_{\text{H1}_1} + h \cdot \nu \quad E_{\text{H1}_2} = 3.02501 \quad E_{\text{H1}_2} - E_{\text{H1}_1} = 0.16638 \quad \text{eV} \quad (\text{increase})$$

(state 2)

$$E_{H1\_1} \cdot \left( \frac{1}{n_1^2} - \frac{1}{m_2^2} \right) = 3.02501 \quad (\text{checks})$$

$$T_2 := \frac{E_{H1\_2}}{3 \cdot k_{B\_ev}} \quad T_2 = 11701.72488 \quad \text{K} \quad T_2 - T_1 = 643.72488 \quad \text{K}$$

Any number of such calculations can be done. Therefore, in the Reciprocal System, spectroscopy has nothing to do with the energy states of the alleged electrons jumping around the alleged nucleus of conventional theory. Instead, spectroscopy has everything to do with the *thermal* motions of atoms and ions. The Reciprocal System Data Base will enable a user to enter a range of ionization energy levels and a range of  $n$  and  $m$  values and calculate the corresponding spectra. And, inversely, if a user enters a specific spectral line, the program will determine the most probable values of the  $n$ 's and  $m$ 's.

### c. Doublets, Triplets, Quartets, etc.

In the *fine structure* of atomic spectra,  $1/m_1$  and  $1/m_2$  cannot be neglected. Doublets are observed, and conventional theory explains this as resulting from the supposed two types of electron spin,  $\pm (1/2) h/2\pi$ . But in the Reciprocal System, doublets, triplets, quartets, etc., are all explained as *minor differences between  $m_1$  and  $m_2$* .

**Example:** consider the first Balmer line in H1, neglecting for the moment  $1/m_1$  and  $1/m_2$ :

$$n_1 := 2 \quad n_2 := 3$$

$$\nu_{H1\_1} := \frac{E_{H1\_1}}{h} \cdot \left\| \left[ \left( \frac{1}{n_1^2} \right) - \left[ \frac{1}{(n_2)^2} \right] \right] \right\|$$

$$\nu_{H1\_1} = 4.566747 \times 10^{14} \quad \text{Hz}$$

$$\lambda_{H1\_1} := \frac{c_{cgs}}{\nu_{H1\_1}} \quad \lambda_{H1\_1} = 0.0000656468 \quad \text{cm}$$

But now suppose we don't neglect  $1/m_1$  and  $1/m_2$ . Let

$$m_1 := 88 \quad m_2 := 89$$

values which are probably close to the present resolution limit of spectroscopes.

$$\nu_{H1\_2} := \frac{E_{H1\_1}}{h} \cdot \left[ \left[ \left( \frac{1}{n_1^2} - \frac{1}{m_1^2} \right) - \left( \frac{1}{n_2^2} - \frac{1}{m_2^2} \right) \right] \right] \quad \nu_{H1\_2} = 4.566652 \times 10^{14} \text{ Hz}$$

$$\lambda_{H1\_2} := \frac{c_{\text{cgs}}}{\nu_{H1\_2}} \quad \lambda_{H1\_2} = 0.0000656482 \text{ cm}$$

The difference or "separation" in the usual spectroscopic units of  $\text{cm}^{-1}$  is:

$$\left| \frac{\nu_{H1\_1}}{c_{\text{cgs}}} - \frac{\nu_{H1\_2}}{c_{\text{cgs}}} \right| = 0.31648 \text{ cm}^{-1} \quad \text{which agrees with that observed (Ref. [11], p. 31)}$$

(Balmer doublet separation)

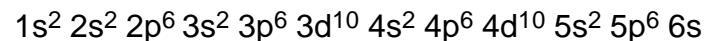
$$\lambda_{H1\_2} - \lambda_{H1\_1} = 1.3639 \times 10^{-9} \text{ cm} \quad (.13639 \text{ \AA})$$

which is probably at the limit of most spectroscopes. Hypothetically,  $m_1$  and  $m_2$  can go even higher and would be observed with spectroscopes of even higher resolving power. *Every* line observed can be explained by choosing the proper values of ionization level and  $n_1$ ,  $m_1$ ,  $n_2$ , and  $m_2$ . Eq. (2) can only be falsified if a line can be found which cannot be accounted for by any integer set of  $n_1$ ,  $n_2$ ,  $m_1$ , and  $m_2$ .

White (Ref. [8], p. 249) and Candler (Ref. [11], p. 113) state the "Alternation Law of Multiplicities": spectral terms arising from successive elements in the periodic table alternate between even and odd multiplicities. If this effect is actually real, and not just due to spectroscopic resolving power, then the Reciprocal System would explain this by means of the electric rotational displacement, which increases across a row of the table. However, ionization energy increases likewise, and so for the same values of the  $n$ 's and  $m$ 's the lines will be spaced *further* apart and thus other lines *intermediate* between these could be found at the same resolution (provided, of course, that they are still integers). So, it's not clear at this time that the "Alternation Law of Multiplicities" is a real effect.

#### d. Spectroscopic Jargon

Conventional spectroscopy and conventional theorists specify "electron configurations" like this, for the ground state of Cs:



Spectra result, supposedly, when these electrons (typically the outer electrons, like the 6 in 5p and the one in 6s here) are raised to a higher level (orbit or shell or "cloud"). Conventional notations for the transitions are given in the References. Of course, the Reciprocal System completely rejects the nuclear theory of the atom and present-day spectroscopic theory and jargon. A much simpler notation for atomic energy transition may be given as follows, for any frequency or wavelength of a photon absorbed or emitted. If  $E_{\text{other}}$  can be neglected, the energy change of an atom can be represented thusly:

$$E_{\text{el\_l\_1}, m_1, n_1} \rightarrow E_{\text{el\_l\_2}, m_2, n_2} \tag{6}$$

The *stage* of ionization (series limit) should be given in Roman numerals, following the convention given previously. So for the example with H above, neutral, we would have

$$E_{H1_I,2,4} \rightarrow E_{H1_I,2,5}$$

Here the energy of ionization, at the series limit, is represented by I, Roman numeral 1, for one charge. Or consider isotope Cs-133, going from 4 charges to 5 charges, at *highest* temperature ( $n_1, n_2 = 1, m_1, m_2 = \text{inf}$ ).

$$E_{Cs133_{IV},1,\text{inf}} \rightarrow E_{Cs133_{V},1,\text{inf}}$$

This is then easily expanded to the totals (but ignoring  $E_{\text{other}}$  for now):

$$E_{Cs133_{tot}_1} := E_{Cs133_{IV}_{tot}} + E_{Cs133_V} \quad \text{or} \quad E_{Cs133_{tot}_1} := E_{Cs133_{V}_{tot}}$$

$$E_{Cs133_{tot}_2} := E_{Cs133_{V}_{tot}} + E_{Cs133_{VI}} \quad \text{or} \quad E_{Cs133_{tot}_2} := E_{Cs133_{VI}_{tot}}$$

Obviously the difference is then

$$E_{Cs133_{tot}_2} - E_{Cs133_{tot}_1} := E_{Cs133_{VI}} \quad (\text{the individual energy required for the 6th charge})$$

A 5-charge atom at maximum temperature is thus equal to a 6-charge atom at minimum temperature.

Because atomic energy depends on ionization state, we now turn to a calculation of the ionization levels of the elements.

### 3. Ionization Energies

#### a. Elements from H to Kr

We will generalize Moseley's x-ray expression to cover *all* spectra, so all energy expressions will involve  $Z^2$ . Moseley's so-called "screening doublets" represent jumps from one ionization level to another adjacent one. Within one ionization level we have the usual optical and surrounding spectra.

Observed values given below for  $E_1$  come from Lang, Ref. [22], pp. 246-247, except where noted from Grigoriev and Meilikhov, Ref. [24], pp. 515-521. Most of Lang's values actually come from Moore's compilation, Ref. [19].

Moore herself says, Ref. [19], p. 1 and p. 5, "The reliability of the data recorded in the literature is often difficult to appraise. In cases where long series are known in the various spectra, the ionization potentials are well determined. With these as key points, good values can be derived by extrapolation or interpolation along isoelectronic sequences, or by comparison along the rows in the Periodic Chart for spectra of similar stages of ionization. Frequently, however, authors give values of ionization potentials without stating the conversion factor [between vacuum value and air value] used and without describing clearly how the quoted value was obtained....Catalan and his associates ...have interpolated values for spectra in which series are not known....In Table 2 all ionization limits were recorded that were derived from observed series, from extrapolation or interpolation as described above."

In other words, the "observed" ionization energies compiled by Moore may really just be *estimates*. The original attempt at calculating the theoretical ionization energies in the Reciprocal System was that given in Ref. [30]. Here, the first ionization energy is shown to be twice that of the electron work function. The problem is that, although the theoretical value of work function in the Reciprocal System is known, the experimental values are uncertain. Even the *CRC Handbook* (Ref. [25], p. 12-118) confesses: "Since [the work function] is dependent on the cleanliness of the surface, measurements reported in the literature often cover a considerable range." Because the first ionization energy = 2 x the work function, this means that the observed ionization energy is also somewhat uncertain. Nonetheless, we will use the observed first ionization levels (which are still quite close to those calculated from the Reciprocal System in Ref. [30]) in the tabulations below..

In all cases, the *final* ionization energy, as indicated earlier by Larson, is simply

$$E_{el\_Z} := h \cdot Z_{el}^2 \cdot R_{el} \quad (7)$$

where  $R_{el}$  is the effective value of R for the isotope:

$$R_{el} := \frac{1 + \frac{m_e}{M_{H1}}}{1 + \frac{m_e}{M_{el}}} \cdot R' \cdot c_{cgs} \quad (8)$$

This *effective* Rydberg frequency depends on the mass M of the isotope, because in creating an ion, it's helpful (though not strictly required) to have a *force couple* so that both a positive charge and a negative charge are created. Therefore the mass values of the atom and an electron come into play, as a ratio in Eq. (8). Incidentally, this is why there is a difference of spectra among the isotopes of a given element--this difference is called the *isotope shift*. It's not due to differences of "nuclear spin."

OK, so where does the electron come from? After all, electrons are not *constituents* of atoms in the Reciprocal System! But: an atom may *harbor* a massless, chargeless electron (or sometimes more than one). This is so because there is an *omnipresent universal flux* of photons, neutrinos, and other massless, chargeless subatomic particles in the universe. Atoms can capture some of the subatomic particles and keep some of them--particularly a massless, chargeless electron. But once an electron becomes charged, as in ionization, it can *escape* the atom. The atom can then accept *another* massless, chargeless electron, and the cycle can then repeat.

It would be simple if all lower ionization levels could be expressed as

$$E_{el\_Z\_xx} := h \cdot (Z_{el} - xx)^2 \cdot R_{el} \quad (9)$$

where  $xx$  is an *integer* in the set  $\{Z_{el}-1, Z_{el}-2, \dots, 1\}$ . But the *rotational displacements* of the atom modify this number into a real number, which we will call the "ion shift" (same as what is called the "screening constant" in conventional theory). For the penultimate (next to last) ionization level

$$xx := \frac{2}{3} \quad (\text{penultimate ionization}) \quad (10)$$

rather than 1. For the remainder of the ionization levels, the ion shift is a *linear function* of the level, with two line segments having different slopes. Normally the first segment has a slope of 1/3, indicating that the *one-dimensional electric rotational displacements* of the atom are the factors affecting  $xx$ , and normally the second segment has a slope of 1/2, indicating that the *two-dimensional magnetic rotational displacements* of the atoms are the factors affecting  $xx$ .

In what follows, for most cases after the first 10 elements, we will compute the *best fit* values for the slopes by manual iteration. You will see that in most cases these best fit values are very close to the *theoretical values*. Then the calculated values of the ionization energies are compared with the observed values. Complete series are known only for the range of elements from H to Kr, so that's what we'll cover. The isotope masses (in u) are those calculated from Reciprocal System theory in Ref. [31]. Subscripts will be used to denote ionization levels here, rather than Roman numerals, for convenience.



$$\mathbf{H^1} \quad Z_H := 1 \quad M_{H1} := 1.0078078 \quad R_{H1} := R \quad E_{H1_1} := Z_H^2 \cdot h \cdot R_{H1} \quad E_{H1_1} = 13.61256 \quad E_{H1\_obs_1} := 13.598$$

(based on Ref. [16], p. 1)

$$\mathbf{H^2} \quad M_{H2} := 2.014169251 \quad R'_{H2} := \frac{1 + \frac{m_e}{M_{H1}}}{1 + \frac{m_e}{M_{H2}}} \cdot R' \quad R_{H2} := c_{cgs} \cdot R'_{H2}$$

$$E_{H2_1} := h \cdot R_{H2} \quad E_{H2_1} = 13.61626 \quad E_{H2\_obs_1} := 13.6012 \quad (\text{based on Ref. [16], p. 3})$$

$$\mathbf{H^3} \quad M_{H3} := 3.0160493 \text{ (obs.)} \quad R'_{H3} := \frac{1 + \frac{m_e}{M_{H1}}}{1 + \frac{m_e}{M_{H3}}} \cdot R' \quad R_{H3} := c_{cgs} \cdot R'_{H3}$$

$$E_{H3_1} := h \cdot R_{H3} \quad E_{H3_1} = 13.61749 \quad E_{H3\_obs_1} := 13.6024 \quad (\text{based on Ref. [16], p. 3})$$

**He**  $Z_{\text{He}} := 2$   $M_{\text{He4}} := 4.002589$

$$R'_{\text{He4}} := \frac{1 + \frac{m_e}{M_{\text{H1}}}}{1 + \frac{m_e}{M_{\text{He4}}}} \cdot R'$$

$$R_{\text{He4}} := c_{\text{cgs}} \cdot R'_{\text{He4}}$$

$$\text{ck} := 1 .. Z_{\text{He}}$$

$$E_{\text{He4\_obs}} := \begin{pmatrix} 24.587 \\ 54.416 \end{pmatrix}$$

$$E_{\text{He4}_1} := \left( Z_{\text{He}} - \frac{2}{3} \right)^2 \cdot h \cdot R_{\text{He4}} \quad E_{\text{He4}_1} = 24.20996$$

$$E_{\text{He4}_2} := Z_{\text{He}}^2 \cdot h \cdot R_{\text{He4}} \quad E_{\text{He4}_2} = 54.4724$$

$$E_{\text{He4}_{\text{ck}}} = \begin{array}{|c|} \hline 24.20996 \\ \hline 54.4724 \\ \hline \end{array}$$

$$E_{\text{He4\_obs}_{\text{ck}}} = \begin{array}{|c|} \hline 24.587 \\ \hline 54.416 \\ \hline \end{array}$$

$$\frac{\sum_{\text{ii}=1}^{Z_{\text{He}}} \frac{E_{\text{He4}_{\text{ii}}}}{E_{\text{He4\_obs}_{\text{ii}}}}}{Z_{\text{He}}} = 0.99285$$

$$\text{Li} \quad Z_{\text{Li}} := 3 \quad M_{\text{Li7}} := 7.042190 \quad R'_{\text{Li7}} := \frac{1 + \frac{m_e}{M_{\text{H1}}}}{1 + \frac{m_e}{M_{\text{Li7}}}} \cdot R' \quad R_{\text{Li7}} := c_{\text{cgs}} \cdot R'_{\text{Li7}}$$

$$E_{\text{Li7\_obs}} := \begin{pmatrix} 5.392 \\ 75.638 \\ 122.451 \end{pmatrix} \quad \text{cm} := 1 .. Z_{\text{Li}} - 2 \quad \text{ck} := 1 .. Z_{\text{Li}} \quad \text{xx1} := Z_{\text{Li}} - \frac{\sqrt{E_{\text{Li7\_obs}_1}}}{\sqrt{R_{\text{Li7}} \cdot h}} \quad \text{xx1} = 2.37078$$

$$\text{cl} := 2 \quad Z_{\text{Li}} - \text{xx1} = 0.62922$$

$$E_{\text{Li7}_1} := (Z_{\text{Li}} - \text{xx1})^2 \cdot h \cdot R_{\text{Li7}} \quad E_{\text{Li7}_{Z_{\text{Li}}-1}} := h \cdot R_{\text{Li7}} \cdot \left( Z_{\text{Li}} - \frac{2}{3} \right)^2 \quad E_{\text{Li7}_{Z_{\text{Li}}}} := h \cdot R_{\text{Li7}} \cdot Z_{\text{Li}}^2$$

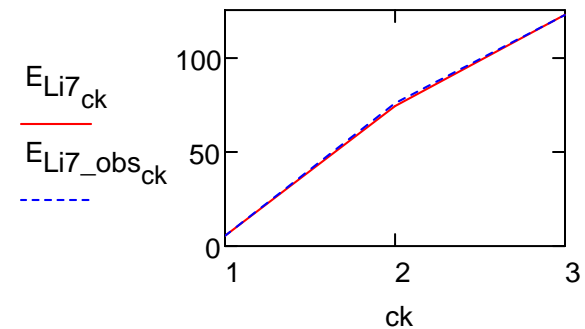
 $E_{\text{Li7}_{\text{ck}}} =$ 

5.392
74.14738
122.57016

 $E_{\text{Li7\_obs}_{\text{ck}}} =$ 

5.392
75.638
122.451

$$\frac{\sum_{\text{ii}=1}^{Z_{\text{Li}}} E_{\text{Li7}_{\text{ii}}}}{Z_{\text{Li}}} = 0.99376$$



$$\text{Be } Z_{\text{Be}} := 4 \quad M_{\text{Be9}} := 9.056359$$

$$R'_{\text{Be9}} := \frac{1 + \frac{m_e}{M_{\text{H1}}}}{1 + \frac{m_e}{M_{\text{Be9}}}} \cdot R' \quad R_{\text{Be9}} := c_{\text{cgs}} \cdot R'_{\text{Be9}}$$

$$E_{\text{Be9\_obs}} := \begin{pmatrix} 9.322 \\ 18.211 \\ 153.893 \\ 217.713 \end{pmatrix} \quad \text{cl} := 2 .. Z_{\text{Be}} - 2 \quad \text{cm} := 1 .. Z_{\text{Be}} - 2 \quad \text{ck} := 1 .. Z_{\text{Be}} \quad \text{xx1} := Z_{\text{Be}} - \frac{\sqrt{E_{\text{Be9\_obs1}}}}{\sqrt{R_{\text{Be9}} \cdot h}}$$

$$\text{xx1} = 3.17267$$

$$Z_{\text{Be}} - \text{xx1} = 0.82733$$

$$\text{xxcl} := \text{xx1} - (\text{cl} - 1) \cdot \frac{1}{3}$$

$$E_{\text{Be9\_cl}} := (Z_{\text{Be}} - \text{xxcl})^2 \cdot h \cdot R_{\text{Be9}}$$

$$E_{\text{Be9}_1} := (Z_{\text{Be}} - \text{xx1})^2 \cdot h \cdot R_{\text{Be9}}$$

$$E_{\text{Be9}_{Z_{\text{Be}}-1}} := h \cdot R_{\text{Be9}} \cdot \left( Z_{\text{Be}} - \frac{2}{3} \right)^2$$

$$E_{\text{Be9}_{Z_{\text{Be}}}} := h \cdot R_{\text{Be9}} \cdot Z_{\text{Be}}^2$$

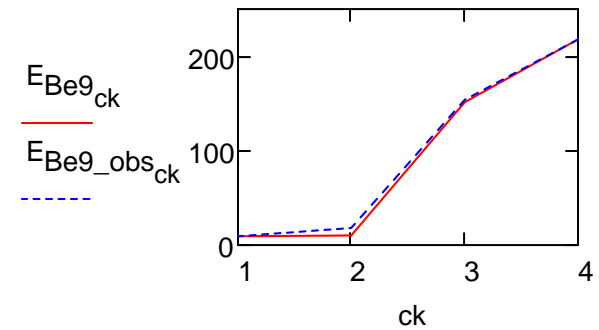
$$E_{\text{Be9}_{\text{ck}}} =$$

9.322
10.0883
151.32381
217.90628

$$E_{\text{Be9\_obs}_{\text{ck}}} =$$

9.322
18.211
153.893
217.713

$$\frac{\sum_{\text{ii}=1}^{Z_{\text{Be}}} E_{\text{Be9}_{\text{ii}}}}{Z_{\text{Be}}} = 0.88454$$



The theoretical slope, 1/3, has been used here; see below.

$$\mathbf{B} \quad Z_B := 5 \quad M_{B11} := 11.012542 \quad R'_{B11} := \frac{1 + \frac{m_e}{M_{H1}}}{1 + \frac{m_e}{M_{B11}}} \cdot R' \quad R_{B11} := c_{cgs} \cdot R'_{B11}$$

$$E_{B11\_obs} := \begin{pmatrix} 8.298 \\ 25.154 \\ 37.930 \\ 259.368 \\ 340.217 \end{pmatrix} \quad cl := 2 .. Z_B - 2 \quad cm := 1 .. Z_B - 2 \quad ck := 1 .. Z_B \quad xx_1 := Z_B - \frac{\sqrt{E_{B11\_obs_1}}}{\sqrt{R_{B11} \cdot h}}$$

$$xx_1 = 4.21943$$

$$Z_B - xx_1 = 0.78057$$

$$xx_{cl} := xx_1 - (cl - 1) \cdot \frac{1}{2} \quad E_{B11_{cl}} := (Z_B - xx_{cl})^2 \cdot h \cdot R_{B11}$$

$$E_{B11_1} := (Z_B - xx_1)^2 \cdot h \cdot R_{B11} \quad E_{B11_{Z_B-1}} := h \cdot R_{B11} \cdot \left(Z_B - \frac{2}{3}\right)^2 \quad E_{B11_{Z_B}} := h \cdot R_{B11} \cdot Z_B^2$$

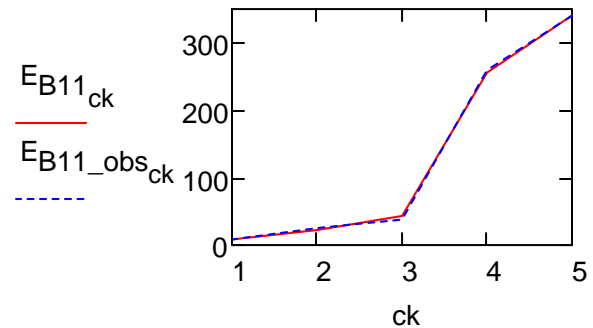
$$E_{B11_{ck}} =$$

8.298
22.33357
43.17879
255.73999
340.48223

$$E_{B11_{obs_{ck}}} =$$

8.298
25.154
37.93
259.368
340.217

$$\frac{\sum_{ii=1}^{Z_B} \frac{E_{B11_{ii}}}{E_{B11_{obs_{ii}}}}}{Z_B} = 1.00261$$



**C**  $Z_C := 6$      $M_{C12} := 12.085016$

$$R'_{C12} := \frac{1 + \frac{m_e}{M_{H1}}}{1 + \frac{m_e}{M_{C12}}} \cdot R'$$

$$R_{C12} := c_{cgs} \cdot R'_{C12}$$

$$E_{C12_{obs}} :=$$

11.26
24.383
47.887
64.492
392.077
489.981

$$cl := 2 .. Z_C - 2$$

$$cm := 1 .. Z_C - 2$$

$$ck := 1 .. Z_C$$

$$xx_1 := Z_C - \frac{\sqrt{E_{C12_{obs_1}}}}{\sqrt{R_{C12} \cdot h}}$$

$$xx_1 = 5.09073$$

$$xx_{cl} := xx_1 - (cl - 1) \cdot \frac{1}{2}$$

$$E_{C12_{cl}} := (Z_C - xx_{cl})^2 \cdot h \cdot R_{C12}$$

$$Z_C - xx_1 = 0.90927$$

$$E_{C12_1} := (Z_C - xx_1)^2 \cdot h \cdot R_{C12}$$

$$E_{C12_{Z_C-1}} := h \cdot R_{C12} \cdot \left( Z_C - \frac{2}{3} \right)^2$$

spectra energy.mcd

$$E_{C12_{Z_C}} := h \cdot R_{C12} \cdot Z_C^2$$

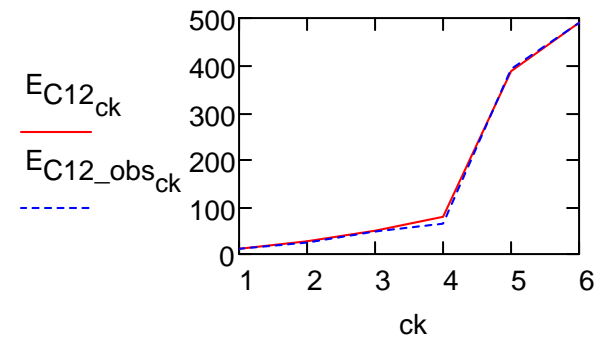
$E_{C12_{ck}} =$

11.26
27.04845
49.64658
79.05438
387.39483
490.29658

$E_{C12_{obs_{ck}}} =$

11.26
24.383
47.887
64.492
392.077
489.981

$$\frac{\sum_{ii=1}^{Z_C} \frac{E_{C12_{ii}}}{E_{C12_{obs_{ii}}}}}{Z_C} = 1.06009$$



$$\mathbf{N} \quad Z_N := 7 \quad M_{N14} := 14.099185 \quad R'_{N14} := \frac{1 + \frac{m_e}{M_{H1}}}{1 + \frac{m_e}{M_{N14}}} \cdot R' \quad R_{N14} := c_{\text{cgs}} \cdot R'_{N14}$$

$$E_{N14\_obs} := \begin{pmatrix} 14.534 \\ 29.601 \\ 47.448 \\ 77.472 \\ 97.888 \\ 552.057 \\ 667.029 \end{pmatrix}$$

$$cl := 2 .. Z_N - 2 \quad cm := 1 .. Z_N - 2 \quad ck := 1 .. Z_N \quad xx_1 := Z_N - \frac{\sqrt{E_{N14\_obs_1}}}{\sqrt{R_{N14} \cdot h}}$$

$$xx_1 = 5.96697$$

$$Z_N - xx_1 = 1.03303$$

$$xx_{cl} := xx_1 - (cl - 1) \cdot \frac{1}{2}$$

$$E_{N14_{cl}} := (Z_N - xx_{cl})^2 \cdot h \cdot R_{N14}$$

$$E_{N14_1} := (Z_N - xx_1)^2 \cdot h \cdot R_{N14}$$

$$E_{N14_{Z_N-1}} := h \cdot R_{N14} \cdot \left(Z_N - \frac{2}{3}\right)^2$$

$$E_{N14_{Z_N}} := h \cdot R_{N14} \cdot Z_N^2$$

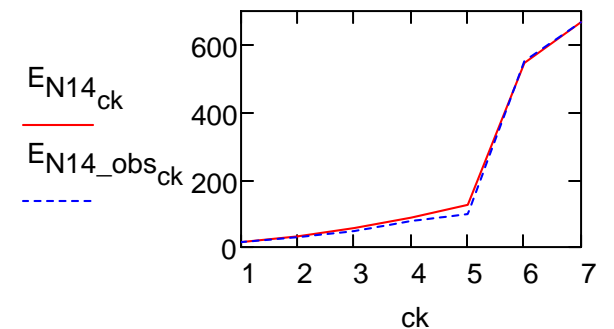
$$E_{N14_{ck}} =$$

14.534
32.00815
56.29202
87.3856
125.28891
546.29078
667.35245

$$E_{N14\_obs_{ck}} =$$

14.534
29.601
47.448
77.472
97.888
552.057
667.029

$$\frac{\sum_{ii=1}^{Z_N} \frac{E_{N14_{ii}}}{E_{N14\_obs_{ii}}}}{Z_N} = 1.09509$$





$Z_O := 8$      $M_{O16} := 16.113354$      $R'_{O16} := \frac{1 + \frac{m_e}{M_{H1}}}{1 + \frac{m_e}{M_{O16}}} \cdot R'$      $R_{O16} := c_{cgs} \cdot R'_{O16}$

$$E_{O16\_obs} := \begin{pmatrix} 13.618 \\ 35.116 \\ 54.934 \\ 77.412 \\ 113.896 \\ 138.116 \\ 739.315 \\ 871.387 \end{pmatrix}$$

$cl := 2 .. Z_O - 2$      $cm := 1 .. Z_O - 2$      $ck := 1 .. Z_O$      $xx1 := Z_O - \frac{\sqrt{E_{O16\_obs_1}}}{\sqrt{R_{O16} \cdot h}}$

$xx1 = 7.00006$

$Z_O - xx1 = 0.99994$

$xx_{cl} := xx1 - (cl - 1) \cdot \frac{1}{2}$      $E_{O16_{cl}} := (Z_O - xx_{cl})^2 \cdot h \cdot R_{O16}$

$E_{O16_1} := (Z_O - xx1)^2 \cdot h \cdot R_{O16}$      $E_{O16_{Z_O-1}} := h \cdot R_{O16} \cdot \left(Z_O - \frac{2}{3}\right)^2$      $E_{O16_{Z_O}} := h \cdot R_{O16} \cdot Z_O^2$

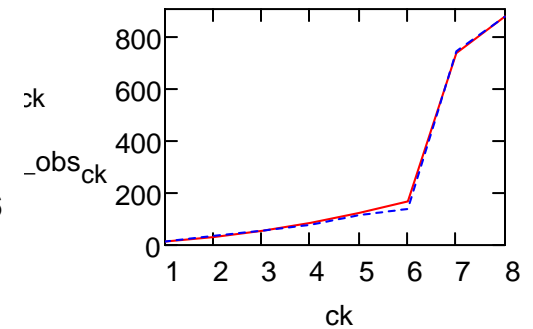
$E_{O16_{ck}} =$

13.618
30.64163
54.47501
85.11814
122.57102
166.83366
732.42666
871.64825

$E_{O16\_obs_{ck}} =$

13.618
35.116
54.934
77.412
113.896
138.116
739.315
871.387

$$\frac{\sum_{ii=1}^{Z_O} \frac{E_{O16_{ii}}}{E_{O16\_obs_{ii}}}}{Z_O} = 1.02986$$



F  $Z_F := 9$      $M_{F19} := 19.127205$      $R'_{F19} := \frac{1 + \frac{m_e}{M_{H1}}}{1 + \frac{m_e}{M_{F19}}} \cdot R'$      $R_{F19} := c_{cgs} \cdot R'_{F19}$

$$E_{F19\_obs} := \begin{pmatrix} 17.422 \\ 34.970 \\ 62.707 \\ 87.138 \\ 114.240 \\ 157.161 \\ 185.182 \\ 953.886 \\ 1103.089 \end{pmatrix}$$

$cl := 2..Z_F - 2$      $cm := 1..Z_F - 2$      $ck := 1..Z_F$      $xx_1 := Z_F - \frac{\sqrt{E_{F19\_obs_1}}}{\sqrt{R_{F19} \cdot h}}$

$xx_1 = 7.86899$   
 $Z_F - xx_1 = 1.13101$

$xx_{cl} := xx_1 - (cl - 1) \cdot \frac{1}{2}$

$E_{F19_{cl}} := (Z_F - xx_{cl})^2 \cdot h \cdot R_{F19}$

$E_{F19_1} := (Z_F - xx_1)^2 \cdot h \cdot R_{F19}$

$E_{F19_{Z_F-1}} := h \cdot R_{F19} \cdot \left(Z_F - \frac{2}{3}\right)^2$

$E_{F19_{Z_F}} := h \cdot R_{F19} \cdot Z_F^2$

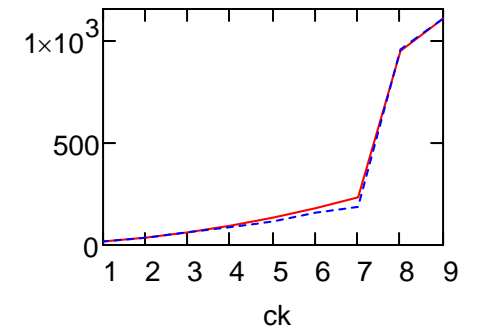
$E_{F19_{ck}} =$

17.422
36.2308
61.84939
94.27776
133.51593
179.56388
232.42162
945.80396
1103.18574

$E_{F19\_obs_{ck}} =$

17.422
34.97
62.707
87.138
114.24
157.161
185.182
953.886
1103.089

$\frac{\sum_{ii=1}^{Z_F} E_{F19_{ii}}}{Z_F} = 1.07359$      $E_{F19_{ck}}$  (red line)  
 $E_{F19\_obs_{ck}}$  (blue dashed line)



$$\text{Ne } Z_{\text{Ne}} := 10 \quad M_{\text{Ne20}} := 20.141693 \quad R'_{\text{Ne20}} := \frac{1 + \frac{m_e}{M_{\text{H1}}}}{1 + \frac{m_e}{M_{\text{Ne20}}}} \cdot R' \quad R_{\text{Ne20}} := c_{\text{cgs}} \cdot R'_{\text{Ne20}}$$

$$E_{\text{Ne20\_obs}} := \begin{pmatrix} 21.564 \\ 40.962 \\ 63.45 \\ 97.11 \\ 126.21 \\ 157.93 \\ 207.27 \\ 239.09 \\ 1195.797 \\ 1362.164 \end{pmatrix}$$

$$\text{cl} := 2 .. Z_{\text{Ne}} - 2 \quad \text{cm} := 1 .. Z_{\text{Ne}} - 2 \quad \text{ck} := 1 .. Z_{\text{Ne}} \quad \text{xx1} := Z_{\text{Ne}} - \frac{\sqrt{E_{\text{Ne20\_obs1}}}}{\sqrt{R_{\text{Ne20}} \cdot h}}$$

$$\text{xx1} = 8.7417$$

$$Z_{\text{Ne}} - \text{xx1} = 1.2583$$

$$E_{\text{Ne20}_1} := (Z_{\text{Ne}} - \text{xx1})^2 \cdot h \cdot R_{\text{Ne20}} \quad E_{\text{Ne20}_{Z_{\text{Ne}}-1}} := h \cdot R_{\text{Ne20}} \cdot \left( Z_{\text{Ne}} - \frac{2}{3} \right)^2 \quad E_{\text{Ne20}_{Z_{\text{Ne}}}} := h \cdot R_{\text{Ne20}} \cdot Z_{\text{Ne}}^2$$

$$\text{xxcl} := \text{xx1} - (\text{cl} - 1) \cdot \frac{1}{2}$$

$$E_{\text{Ne20}_{\text{cl}}} := (Z_{\text{Ne}} - \text{xxcl})^2 \cdot h \cdot R_{\text{Ne20}}$$

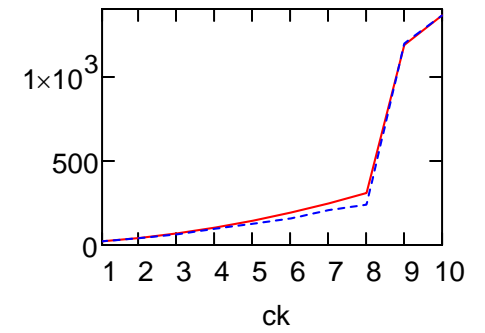
$E_{\text{Ne20}_{\text{ck}}} =$ 

21.564
42.10637
69.45854
103.62051
144.59228
192.37384
246.96521
308.36637
1186.4182
1361.95967

 $E_{\text{Ne20}_{\text{obsck}}} =$ 

21.564
40.962
63.45
97.11
126.21
157.93
207.27
239.09
1195.797
1362.164

$$\frac{\sum_{ii=1}^{Z_{\text{Ne}}} E_{\text{Ne20}_{ii}}}{Z_{\text{Ne}}} = 1.10267$$

 $E_{\text{Ne20}_{\text{ck}}}$  $E_{\text{Ne20}_{\text{obsck}}}$ 

**Na**  $Z_{\text{Na}} := 11$   $M_{\text{Na23}} := 23.155544$

$$R'_{\text{Na23}} := \frac{1 + \frac{m_e}{M_{\text{H1}}}}{\left(1 + \frac{m_e}{M_{\text{Na23}}}\right)} \cdot R'$$

$R_{\text{Na23}} := c_{\text{cgs}} \cdot R'_{\text{Na23}}$

$E_{\text{Na23\_obs}} :=$

5.139
47.286
71.64
98.91
138.39
172.15
208.47
264.18
299.87
1465.091
1648.659

$cl := 3 .. Z_{\text{Na}} - 2$   $cm := 1 .. Z_{\text{Na}} - 2$   $ck := 1 .. Z_{\text{Na}}$   $xx1 := Z_{\text{Na}} - \frac{\sqrt{E_{\text{Na23\_obs}_1}}}{\sqrt{R_{\text{Na23}} \cdot h}}$   $xx1 = 10.38573$

(special)

$Z_{\text{Na}} - xx1 = 0.61427$

$$xx2 := Z_{\text{Na}} - \frac{\sqrt{E_{\text{Na23\_obs}_2}}}{\sqrt{R_{\text{Na23}} \cdot h}}$$

special situation for Na only--have to use observed second ionization

$$E_{\text{Na23}_1} := (Z_{\text{Na}} - \text{xx1})^2 \cdot h \cdot R_{\text{Na23}} \quad E_{\text{Na23}_{Z_{\text{Na}}-1}} := h \cdot R_{\text{Na23}} \cdot \left(Z_{\text{Na}} - \frac{2}{3}\right)^2 \quad E_{\text{Na23}_{Z_{\text{Na}}}} := h \cdot R_{\text{Na23}} \cdot Z_{\text{Na}}^2$$

$$E_{\text{Na23}_2} := (Z_{\text{Na}} - \text{xx2})^2 \cdot h \cdot R_{\text{Na23}}$$

$$\text{xx}_{\text{cl}} := \text{xx2} - \frac{1}{3} \cdot (\text{cl} - 1) \cdot \Phi_{01}(\text{cl} - 2)$$

$$E_{\text{Na23}_{\text{cl}}} := (Z_{\text{Na}} - \text{xx}_{\text{cl}})^2 \cdot h \cdot R_{\text{Na23}}$$

$$E_{\text{Na23}_{\text{ck}}} =$$

5.1390
47.2860
87.1759
111.6607
139.1721
169.7101
203.2747
239.8658
279.4836
1454.2754
1647.9770

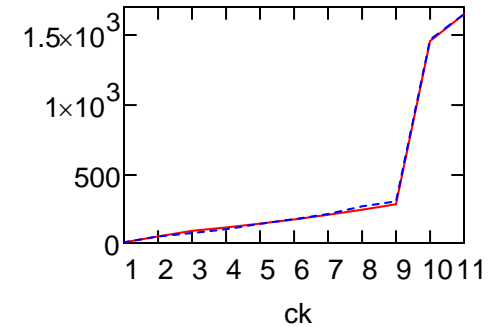
$$E_{\text{Na23}_{\text{obs}_{\text{ck}}}} =$$

5.139
47.286
71.64
98.91
138.39
172.15
208.47
264.18
299.87
1465.091
1648.659

$$\frac{Z_{\text{Na}} \sum_{i=1}^{Z_{\text{Na}}} \frac{E_{\text{Na23}_i}}{E_{\text{Na23}_{\text{obs}_i}}}}{Z_{\text{Na}}} = 1.01314$$

$E_{\text{Na23}_{\text{ck}}}$

$E_{\text{Na23}_{\text{obs}_{\text{ck}}}}$



From here on out we'll determine the "best fit" values for the two slopes, but keep in mind that the theoretical values are 1/3 and 1/2. See above and below for a discussion of this matter.

$$\mathbf{Mg} \quad Z_{\text{Mg}} := 12 \quad M_{\text{Mg24}} := 24.170031 \quad (\text{calc})$$

$$R'_{\text{Mg24}} := \frac{1 + \frac{m_e}{M_{\text{H1}}}}{\left(1 + \frac{m_e}{M_{\text{Mg24}}}\right)} \cdot R'$$

$$R_{\text{Mg24}} := c_{\text{cgs}} \cdot R'_{\text{Mg24}}$$

$$E_{\text{Mg24\_obs}} := \begin{pmatrix} 7.646 \\ 15.035 \\ 80.143 \\ 109.24 \\ 141.26 \\ 186.50 \\ 224.94 \\ 265.90 \\ 327.95 \\ 367.53 \\ 1761.802 \\ 1962.813 \end{pmatrix}$$

$$cl := 2 .. Z_{\text{Mg}} - 2 \quad cm := 1 .. Z_{\text{Mg}} - 2 \quad ck := 1 .. Z_{\text{Mg}} \quad xx1 := Z_{\text{Mg}} - \frac{\sqrt{E_{\text{Mg24\_obs}_1}}}{\sqrt{R_{\text{Mg24}} \cdot h}}$$

$$xx1 = 11.25074$$

$$E_{\text{Mg24}_1} := (Z_{\text{Mg}} - xx1)^2 \cdot h \cdot R_{\text{Mg24}} \quad E_{\text{Mg24}_{Z_{\text{Mg}}-1}} := h \cdot R_{\text{Mg24}} \cdot \left(Z_{\text{Mg}} - \frac{2}{3}\right)^2 \quad E_{\text{Mg24}_{Z_{\text{Mg}}}} := h \cdot R_{\text{Mg24}} \cdot Z_{\text{Mg}}^2$$

$$xx_{cl} := xx_1 - .33 \cdot (cl - 1) \cdot \Phi_{01}(2 - cl) - .575 \cdot (cl - 1) \cdot \Phi_{01}(cl - 3)$$

(segment change after  $cl = 2$ )

$$E_{Mg24_{cl}} := (Z_{Mg} - xx_{cl})^2 \cdot h \cdot R_{Mg24}$$

 $E_{Mg24_{ck}} =$ 

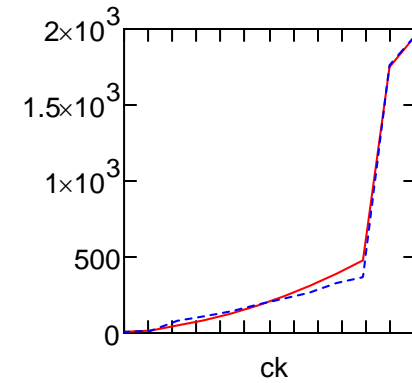
7.646
15.86428
49.12881
83.37921
126.63561
178.89802
240.16642
310.44082
389.72122
478.00761
1749.36947
1961.23082

 $E_{Mg24\_obs_{ck}} =$ 

7.646
15.035
80.143
109.24
141.26
186.5
224.94
265.9
327.95
367.53
1761.802
1962.813

$$\frac{\sum_{ii=1}^{Z_{Mg}} \frac{E_{Mg24_{ii}}}{E_{Mg24\_obs_{ii}}}}{Z_{Mg}} = 1.00029$$

—  $E_{Mg24_{ck}}$   
 - - -  $E_{Mg24\_obs_{ck}}$





$$\text{AI} \quad Z_{\text{AI}} := 13 \quad M_{\text{AI27}} := 27.183882$$

$$R'_{\text{AI27}} := \frac{1 + \frac{m_e}{M_{\text{H1}}}}{\left(1 + \frac{m_e}{M_{\text{AI27}}}\right)} \cdot R'$$

$$R_{\text{AI27}} := c_{\text{CGS}} \cdot R'_{\text{AI27}}$$

$$E_{\text{AI27\_obs}} := \begin{pmatrix} 5.986 \\ 18.828 \\ 28.447 \\ 119.99 \\ 153.71 \\ 190.47 \\ 241.43 \\ 284.59 \\ 330.21 \\ 398.57 \\ 442.07 \\ 2085.983 \\ 2304.080 \end{pmatrix}$$

$$cl := 2 .. Z_{\text{AI}} - 2$$

$$cm := 1 .. Z_{\text{AI}} - 2$$

$$ck := 1 .. Z_{\text{AI}}$$

$$xx1 := Z_{\text{AI}} - \frac{\sqrt{E_{\text{AI27\_obs}_1}}}{\sqrt{R_{\text{AI27}} \cdot h}}$$

$$xx1 = 12.33704$$

$$E_{\text{AI27}_1} := (Z_{\text{AI}} - xx1)^2 \cdot h \cdot R_{\text{AI27}}$$

$$E_{\text{AI27}_{Z_{\text{AI}}-1}} := h \cdot R_{\text{AI27}} \cdot \left(Z_{\text{AI}} - \frac{2}{3}\right)^2$$

$$E_{\text{AI27}_{Z_{\text{AI}}}} := h \cdot R_{\text{AI27}} \cdot Z_{\text{AI}}^2$$

$$xx_{cl} := xx_1 - .33 \cdot (cl - 1) \cdot \Phi_{01}(3 - cl) - .6 \cdot (cl - 1) \cdot \Phi_{01}(cl - 4)$$

(segment change after cl = 3)

$$E_{AI27_{cl}} := (Z_{AI} - xx_{cl})^2 \cdot h \cdot R_{AI27}$$

$E_{AI27_{ck}} =$

5.986
13.42849
23.83736
82.61913
127.77586
182.73878
247.50787
322.08314
406.46459
500.65222
604.64602
2071.7066
2301.72808

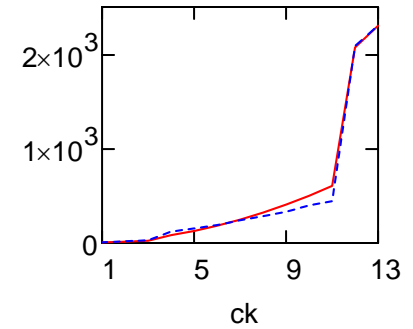
$E_{AI27\_obs_{ck}} =$

5.986
18.828
28.447
119.99
153.71
190.47
241.43
284.59
330.21
398.57
442.07
2085.983
2304.08

$$\frac{\sum_{ii=1}^{Z_{AI}} \frac{E_{AI27_{ii}}}{E_{AI27\_obs_{ii}}}}{Z_{AI}} = 1.00264$$

$E_{AI27_{ck}}$

$E_{AI27\_obs_{ck}}$



$$\text{Si} \quad Z_{\text{Si}} := 14 \quad M_{\text{Si28}} := 28.198370$$

$$R'_{\text{Si28}} := \frac{1 + \frac{m_e}{M_{\text{H1}}}}{\left(1 + \frac{m_e}{M_{\text{Si28}}}\right)} \cdot R'$$

$$R_{\text{Si28}} := c_{\text{CGS}} \cdot R'_{\text{Si28}}$$

$$E_{\text{Si28\_obs}} := \begin{pmatrix} 8.151 \\ 16.345 \\ 33.492 \\ 45.141 \\ 166.77 \\ 205.05 \\ 246.52 \\ 303.17 \\ 351.10 \\ 401.43 \\ 476.06 \\ 523.50 \\ 2437.676 \\ 2673.108 \end{pmatrix}$$

$$cl := 2 .. Z_{\text{Si}} - 2$$

$$cm := 1 .. Z_{\text{Si}} - 2$$

$$ck := 1 .. Z_{\text{Si}}$$

$$xx1 := Z_{\text{Si}} - \frac{\sqrt{E_{\text{Si28\_obs}_1}}}{\sqrt{R_{\text{Si28}} \cdot h}}$$

$$xx1 = 13.22639$$

$$E_{\text{Si28}_1} := (Z_{\text{Si}} - xx1)^2 \cdot h \cdot R_{\text{Si28}}$$

$$E_{\text{Si28}_{Z_{\text{Si}}-1}} := h \cdot R_{\text{Si28}} \cdot \left(Z_{\text{Si}} - \frac{2}{3}\right)^2$$

$$E_{\text{Si28}_{Z_{\text{Si}}}} := h \cdot R_{\text{Si28}} \cdot Z_{\text{Si}}^2$$

$$E_{Si28_{cl}} := (Z_{Si} - xx_{cl})^2 \cdot h \cdot R_{Si28}$$

$E_{Si28_{ck}} =$

8.151
17.81242
31.20291
48.32248
120.4302
169.0998
226.00932
291.15876
364.54812
446.17741
536.04661
634.15573
2421.28047
2669.46172

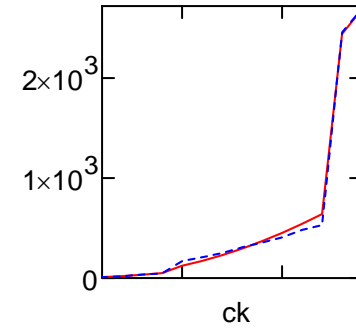
$E_{Si28_{obs_{ck}}} =$

8.151
16.345
33.492
45.141
166.77
205.05
246.52
303.17
351.1
401.43
476.06
523.5
2437.676
2673.108

$$\frac{\sum_{ii=1}^{Z_{Si}} \frac{E_{Si28_{ii}}}{E_{Si28_{obs_{ii}}}}}{Z_{Si}} = 0.99964$$

$E_{Si28_{ck}}$

$E_{Si28_{obs_{ck}}}$



$$\mathbf{P} \quad Z_P := 15 \quad M_{P31} := 31.212221$$

$$R'_{P31} := \frac{1 + \frac{m_e}{M_{H1}}}{\left(1 + \frac{m_e}{M_{P31}}\right)} \cdot R'$$

$$R_{P31} := c_{cgs} \cdot R'_{P31}$$

$$E_{P31\_obs} := \begin{pmatrix} 10.486 \\ 19.725 \\ 30.18 \\ 51.37 \\ 65.023 \\ 220.43 \\ 263.22 \\ 309.41 \\ 371.73 \\ 424.50 \\ 479.57 \\ 560.41 \\ 611.85 \\ 2816.943 \\ 3069.762 \end{pmatrix}$$

$$cl := 2 .. Z_P - 2$$

$$cm := 1 .. Z_P - 2$$

$$ck := 1 .. Z_P$$

$$xx_1 := Z_P - \frac{\sqrt{E_{P31\_obs_1}}}{\sqrt{R_{P31} \cdot h}}$$

$$xx_1 = 14.12255$$

$$E_{P31_1} := (Z_P - xx_1)^2 \cdot h \cdot R_{P31}$$

$$E_{P31_{Z_P-1}} := h \cdot R_{P31} \cdot \left(Z_P - \frac{2}{3}\right)^2$$

$$E_{P31_{Z_P}} := h \cdot R_{P31} \cdot Z_P^2$$

$$xx_{cl} := xx_1 - .32 \cdot (cl - 1) \cdot \Phi_{01}(5 - cl) - .54 \cdot (cl - 1) \cdot \Phi_{01}(cl - 6) \quad (\text{segment change after } cl = 5)$$

$$E_{P31_{cl}} := (Z_P - xx_{cl})^2 \cdot h \cdot R_{P31}$$

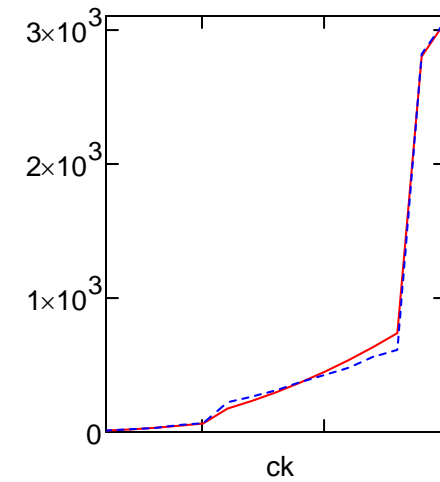
 $E_{P31_{ck}} =$ 

10.486
19.52903
31.36139
45.98306
63.39406
174.30698
230.90025
295.43654
367.91587
448.33821
536.70359
633.01199
737.26341
2798.0975
3064.43885

 $E_{P31_{obs_{ck}}} =$ 

10.486
19.725
30.18
51.37
65.023
220.43
263.22
309.41
371.73
424.5
479.57
560.41
611.85
2816.943
3069.762

$$\frac{\sum_{ii=1}^{Z_P} \frac{E_{P31_{ii}}}{E_{P31_{obs_{ii}}}}}{Z_P} = 1.00088$$

 $E_{P31_{ck}}$ 
 $E_{P31_{obs_{ck}}}$ 


$$\mathbf{S} \quad Z_S := 16 \quad M_{S32} := 32.226708$$

$$R'_{S32} := \frac{1 + \frac{m_e}{M_{H1}}}{\left(1 + \frac{m_e}{M_{S32}}\right)} \cdot R'$$

$$R_{S32} := c_{cgs} \cdot R'_{S32}$$

$$E_{S32\_obs} := \begin{pmatrix} 10.360 \\ 23.33 \\ 34.83 \\ 47.30 \\ 72.68 \\ 88.049 \\ 280.93 \\ 328.23 \\ 379.10 \\ 447.09 \\ 504.78 \\ 564.65 \\ 651.63 \\ 707.14 \\ 3223.836 \\ 3494.099 \end{pmatrix}$$

$$cl := 2 .. Z_S - 2$$

$$cm := 1 .. Z_S - 2$$

$$ck := 1 .. Z_S$$

$$xx_1 := Z_S - \frac{\sqrt{E_{S32\_obs_1}}}{\sqrt{R_{S32} \cdot h}}$$

$$xx_1 = 15.12784$$

$$E_{S32_1} := (Z_S - xx_1)^2 \cdot h \cdot R_{S32}$$

$$E_{S32_{Z_S-1}} := h \cdot R_{S32} \cdot \left(Z_S - \frac{2}{3}\right)^2$$

$$E_{S32_{Z_S}} := h \cdot R_{S32} \cdot Z_S^2$$

$$xx_{cl} := xx_1 - .33 \cdot (cl - 1) \cdot \Phi_{01}(6 - cl) - .545 \cdot (cl - 1) \cdot \Phi_{01}(cl - 7) \quad (\text{segment change after } cl = 6)$$

$$ES32_{cl} := (Z_S - xx_{cl})^2 \cdot h \cdot R_{S32}$$

 $ES32_{ck} =$ 

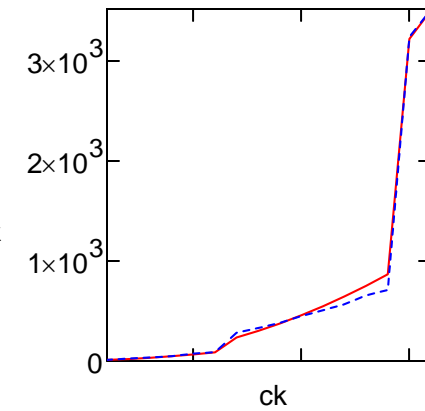
10.36
19.68305
31.97247
47.22827
65.45046
86.63902
233.68033
299.2182
372.84687
454.56635
544.37663
642.27772
748.26961
862.3523
3202.15121
3486.65235

 $ES32_{obs_{ck}} =$ 

10.36
23.33
34.83
47.3
72.68
88.049
280.93
328.23
379.1
447.09
504.78
564.65
651.63
707.14
3223.836
3494.099

$$\frac{\sum_{ii=1}^{Z_S} \frac{ES32_{ii}}{ES32_{obs_{ii}}}}{Z_S} = 0.9977$$

$ES32_{ck}$   
 $ES32_{obs_{ck}}$





$$\text{Cl} \quad Z_{\text{Cl}} := 17 \quad M_{\text{Cl}35} := 35.240559$$

$$R'_{\text{Cl}35} := \frac{1 + \frac{m_e}{M_{\text{H}1}}}{\left(1 + \frac{m_e}{M_{\text{Cl}35}}\right)} \cdot R'$$

$$R_{\text{Cl}35} := c_{\text{cgs}} \cdot R'_{\text{Cl}35}$$

$$E_{\text{Cl}35\_obs} := \begin{pmatrix} 12.967 \\ 23.81 \\ 39.61 \\ 53.46 \\ 67.8 \\ 97.03 \\ 114.193 \\ 348.28 \\ 400.05 \\ 455.62 \\ 529.26 \\ 591.97 \\ 656.69 \\ 749.74 \\ 809.39 \\ 3658.425 \\ 3946.193 \end{pmatrix}$$

$$cl := 2 .. Z_{\text{Cl}} - 2$$

$$cm := 1 .. Z_{\text{Cl}} - 2$$

$$ck := 1 .. Z_{\text{Cl}}$$

$$xx1 := Z_{\text{Cl}} - \frac{\sqrt{E_{\text{Cl}35\_obs1}}}{\sqrt{R_{\text{Cl}35} \cdot h}}$$

$$xx1 = 16.02426$$

$$E_{\text{Cl}35_1} := (Z_{\text{Cl}} - xx1)^2 \cdot h \cdot R_{\text{Cl}35}$$

$$E_{\text{Cl}35_{Z_{\text{Cl}}-1}} := h \cdot R_{\text{Cl}35} \cdot \left(Z_{\text{Cl}} - \frac{2}{3}\right)^2$$

$$E_{\text{Cl}35_{Z_{\text{Cl}}}} := h \cdot R_{\text{Cl}35} \cdot Z_{\text{Cl}}^2$$

$$E_{Cl35_{cl}} := (Z_{Cl} - xx_{cl})^2 \cdot h \cdot R_{Cl35}$$

$E_{Cl35_{ck}} =$

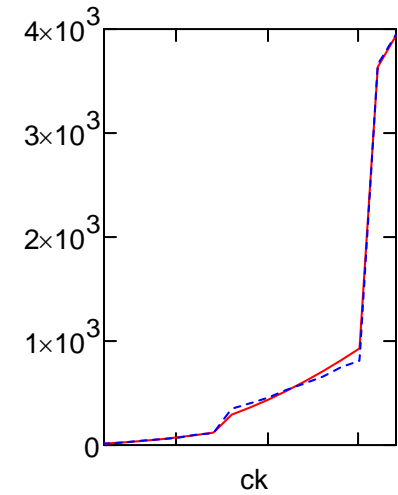
12.967
23.22118
36.44173
52.62868
71.782
93.90171
118.9878
290.16993
359.23261
435.66084
519.45465
610.61401
709.13894
815.02943
928.28549
3633.44813
3936.10936

$E_{Cl35\_obs_{ck}} =$

12.967
23.81
39.61
53.46
67.8
97.03
114.193
348.28
400.05
455.62
529.26
591.97
656.69
749.74
809.39
3658.425
3946.193

$$\frac{\sum_{ii=1}^{Z_{Cl}} \frac{E_{Cl35_{ii}}}{E_{Cl35\_obs_{ii}}}}{Z_{Cl}} = 0.99723$$

$E_{Cl35_{ck}}$  (solid red line)  
 $E_{Cl35\_obs_{ck}}$  (dashed blue line)



$$\text{Ar} \quad Z_{\text{Ar}} := 18 \quad M_{\text{Ar40}} := 40.253775$$

$$R'_{\text{Ar40}} := \frac{1 + \frac{m_e}{M_{\text{H1}}}}{\left(1 + \frac{m_e}{M_{\text{Ar40}}}\right)} \cdot R'$$

$$R_{\text{Ar40}} := c_{\text{cgs}} \cdot R'_{\text{Ar40}}$$

$$E_{\text{Ar40\_obs}} := \begin{pmatrix} 15.759 \\ 27.629 \\ 40.74 \\ 59.81 \\ 75.02 \\ 91.007 \\ 124.319 \\ 143.456 \\ 422.44 \\ 478.68 \\ 538.95 \\ 618.24 \\ 686.09 \\ 755.73 \\ 854.75 \\ 918 \\ 4120.778 \\ 4426.114 \end{pmatrix}$$

$$cl := 2 .. Z_{\text{Ar}} - 2 \quad cm := 1 .. Z_{\text{Ar}} - 2 \quad ck := 1 .. Z_{\text{Ar}}$$

$$xx_1 := Z_{\text{Ar}} - \frac{\sqrt{E_{\text{Ar40\_obs}_1}}}{\sqrt{R_{\text{Ar40}} \cdot h}}$$

$$xx_1 = 16.92433$$

$$xx_{\text{obs}_{ck}} := Z_{\text{Ar}} - \sqrt{\frac{E_{\text{Ar40\_obs}_{ck}}}{R_{\text{Ar40}} \cdot h}}$$

$$E_{\text{Ar40}_1} := (Z_{\text{Ar}} - xx_1)^2 \cdot h \cdot R_{\text{Ar40}}$$

$$E_{\text{Ar40}_{Z_{\text{Ar}}-1}} := h \cdot R_{\text{Ar40}} \cdot \left(Z_{\text{Ar}} - \frac{2}{3}\right)^2$$

$$E_{\text{Ar40}_{Z_{\text{Ar}}}} := h \cdot R_{\text{Ar40}} \cdot Z_{\text{Ar}}^2$$

$$xx_{cl} := xx_1 - .333 \cdot (cl - 1) \cdot \Phi_{01}(8 - cl) - .5 \cdot (cl - 1) \cdot \Phi_{01}(cl - 9)$$

(segment change after cl = 8) spectra\_energy.mcd

$$E_{Ar40_{cl}} := (Z_{Ar} - xx_{cl})^2 \cdot h \cdot R_{Ar40}$$

$E_{Ar40_{ck}} =$

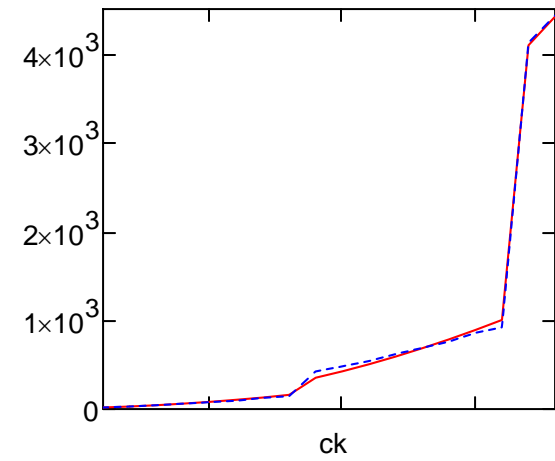
15.759
27.02645
41.31447
58.62305
78.9522
102.30192
128.67221
158.06307
350.87869
423.41316
502.75752
588.91178
681.87592
781.64996
888.23388
1001.6277
4091.98783
4412.80937

$E_{Ar40_{obs_{ck}}} =$

15.759
27.629
40.74
59.81
75.02
91.007
124.319
143.456
422.44
478.68
538.95
618.24
686.09
755.73
854.75
918
4120.778
4426.114

$$\frac{\sum_{ii=1}^{Z_{Ar}} \frac{E_{Ar40_{ii}}}{E_{Ar40_{obs_{ii}}}}}{Z_{Ar}} = 1.00193$$

$E_{Ar40_{ck}}$  ———  
 $E_{Ar40_{obs_{ck}}}$  - - - -



$$\mathbf{K} \quad Z_K := 19 \quad M_{K39} := 39.2688898$$

$$R'_{K39} := \frac{1 + \frac{m_e}{M_{H1}}}{\left(1 + \frac{m_e}{M_{K39}}\right)} \cdot R'$$

$$R_{K39} := c_{cgs} \cdot R'_{K39}$$

$$E_{K39\_obs} := \begin{pmatrix} 4.341 \\ 31.625 \\ 45.72 \\ 60.91 \\ 82.66 \\ 100.0 \\ 117.56 \\ 154.86 \\ 175.814 \\ 503.44 \\ 564.13 \\ 629.09 \\ 714.02 \\ 787.13 \\ 861.77 \\ 968 \\ 1034 \\ 4610.955 \\ 4933.931 \end{pmatrix}$$

$$cl := 2 .. Z_K - 2$$

$$cm := 1 .. Z_K - 2$$

$$ck := 1 .. Z_K$$

$$xx1 := Z_K - \frac{\sqrt{E_{K39\_obs1}}}{\sqrt{R_{K39} \cdot h}}$$

$$xx\_obs_{ck} := Z_K - \sqrt{\frac{E_{K39\_obs_{ck}}}{R_{K39} \cdot h}}$$

$$xx1 = 18.43544$$

$$E_{K39_1} := (Z_K - xx_1)^2 \cdot h \cdot R_{K39} \quad E_{K39_{Z_K-1}} := h \cdot R_{K39} \cdot \left(Z_K - \frac{2}{3}\right)^2 \quad E_{K39_{Z_K}} := h \cdot R_{K39} \cdot Z_K^2$$

$$xx_{cl} := xx_1 - .44 \cdot (cl - 1) \cdot \Phi_{01}(9 - cl) - .57 \cdot (cl - 1) \cdot \Phi_{01}(cl - 10) \quad (\text{segment change after } cl = 9)$$

$$E_{K39_{cl}} := (Z_K - xx_{cl})^2 \cdot h \cdot R_{K39}$$

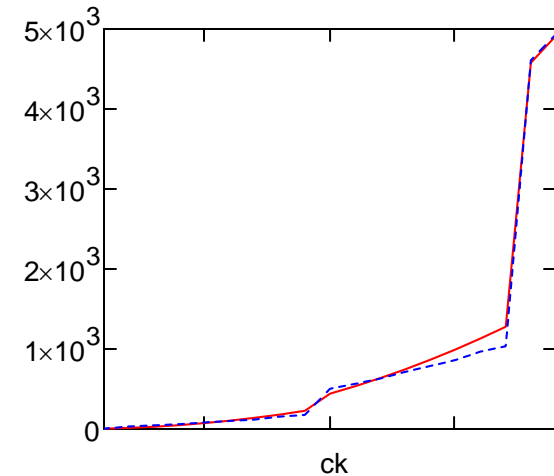
 $E_{K39_{ck}} =$ 

4.341
13.74427
28.42111
48.37153
73.59553
104.0931
139.86426
180.90899
227.2273
441.66228
534.50419
636.19623
746.73841
866.13071
994.37314
1131.46571
1277.40841
4577.75849
4916.73962

 $E_{K39_{obs_{ck}}} =$ 

4.341
31.625
45.72
60.91
82.66
100
117.56
154.86
175.814
503.44
564.13
629.09
714.02
787.13
861.77
968
1034
4610.955
4933.931

$$\frac{\sum_{ii=1}^{Z_K} \frac{E_{K39_{ii}}}{E_{K39_{obs_{ii}}}}}{Z_K} = 0.99799$$

 $E_{K39_{ck}}$   
 $E_{K39_{obs_{ck}}}$ 


$$\text{Ca} \quad Z_{\text{Ca}} := 20 \quad M_{\text{Ca40}} := 40.283385$$

$$R'_{\text{Ca40}} := \frac{1 + \frac{m_e}{M_{\text{H1}}}}{\left(1 + \frac{m_e}{M_{\text{Ca40}}}\right)} \cdot R'$$

$$R_{\text{Ca40}} := c_{\text{cgs}} \cdot R'_{\text{Ca40}}$$

$$E_{\text{Ca40\_obs}} := \left( \begin{array}{l} 6.113 \\ 11.871 \\ 50.908 \\ 67.10 \\ 84.41 \\ 108.78 \\ 127.7 \\ 147.24 \\ 188.54 \\ 211.70 \\ 591.25 \\ 656.39 \\ 726.03 \\ 816.61 \\ 895.12 \\ 974 \\ 1087 \\ 1157 \\ 5129.045 \\ 5469.738 \end{array} \right)$$

$$cl := 2 .. Z_{\text{Ca}} - 2 \quad cm := 1 .. Z_{\text{Ca}} - 2 \quad ck := 1 .. Z_{\text{Ca}}$$

$$xx\_obs_{ck} := Z_{\text{Ca}} - \sqrt{\frac{E_{\text{Ca40\_obs}_{ck}}}{R_{\text{Ca40}} \cdot h}}$$

$$xx1 := Z_{\text{Ca}} - \frac{\sqrt{E_{\text{Ca40\_obs}_1}}}{\sqrt{R_{\text{Ca40}} \cdot h}}$$

$$xx1 = 19.33005$$

$$E_{Ca40_1} := (Z_{Ca} - xx_1)^2 \cdot h \cdot R_{Ca40} \quad E_{Ca40_{Z_{Ca}^{-1}}} := h \cdot R_{Ca40} \cdot \left( Z_{Ca} - \frac{2}{3} \right)^2 \quad E_{Ca40_{Z_{Ca}}} := h \cdot R_{Ca40} \cdot Z_{Ca}^2$$

$$xx_{cl} := xx_1 - .4 \cdot (cl - 1) \cdot \Phi_{01}(10 - cl) - .55 \cdot (cl - 1) \cdot \Phi_{01}(cl - 11) \quad (\text{segment change after } cl = 10)$$

$$E_{Ca40_{cl}} := (Z_{Ca} - xx_{cl})^2 \cdot h \cdot R_{Ca40}$$

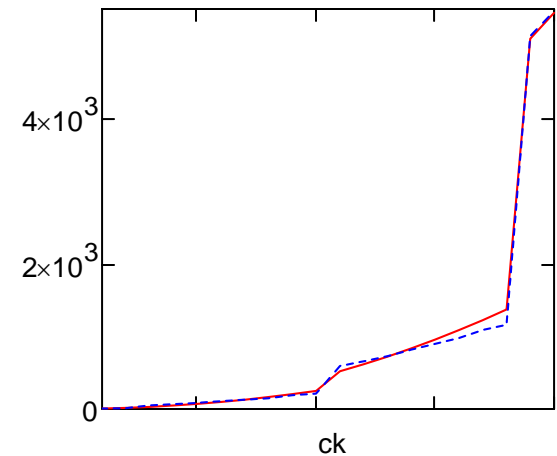
 $E_{Ca40_{ck}} =$ 

6.113
15.59182
29.42897
47.62445
70.17826
97.0904
128.36087
163.98967
203.9768
248.32226
518.48165
615.03834
719.835
832.87162
954.14822
1083.66478
1221.42131
1367.41781
5090.7719
5447.91285

 $E_{Ca40\_obs_{ck}} =$ 

6.113
11.871
50.908
67.1
84.41
108.78
127.7
147.24
188.54
211.7
591.25
656.39
726.03
816.61
895.12
974
1087
1157
5129.045
5469.738

$$\frac{\sum_{ii=1}^{Z_{Ca}} \frac{E_{Ca40_{ii}}}{E_{Ca40\_obs_{ii}}}}{Z_{Ca}} = 0.99985$$

 $\frac{E_{Ca40_{ck}}}{E_{Ca40\_obs_{ck}}}$ 




$$\mathbf{Sc} \quad Z_{\text{Sc}} := 21 \quad M_{\text{Sc45}} := 45.296601$$

$$R'_{\text{Sc45}} := \frac{1 + \frac{m_e}{M_{\text{H1}}}}{\left(1 + \frac{m_e}{M_{\text{Sc45}}}\right)} \cdot R'$$

$$R_{\text{Sc45}} := c_{\text{cgs}} \cdot R'_{\text{Sc45}}$$

$$E_{\text{Sc45\_obs}} := \begin{pmatrix} 6.54 \\ 12.80 \\ 24.76 \\ 73.47 \\ 91.66 \\ 111.1 \\ 138.0 \\ 158.7 \\ 180.02 \\ 225.32 \\ 249.832 \\ 685.89 \\ 755.47 \\ 829.79 \\ 926 \\ 1009 \\ 1094 \\ 1213 \\ 1288 \\ 5674.9 \\ 6033.8 \end{pmatrix}$$

$$cl := 2 .. Z_{\text{Sc}} - 2 \quad cm := 1 .. Z_{\text{Sc}} - 2 \quad ck := 1 .. Z_{\text{Sc}}$$

$$xx_{\text{obs}ck} := Z_{\text{Sc}} - \sqrt{\frac{E_{\text{Sc45\_obs}ck}}{R_{\text{Sc45}} \cdot h}}$$

$$xx_1 := Z_{\text{Sc}} - \frac{\sqrt{E_{\text{Sc45\_obs}_1}}}{\sqrt{R_{\text{Sc45}} \cdot h}}$$

$$xx_1 = 20.30705$$

(from Ref. [24] compilation after XV)

$$E_{\text{Sc45}_1} := (Z_{\text{Ca}} - \text{xx}_1)^2 \cdot h \cdot R_{\text{Sc45}} \quad E_{\text{Sc45}_{Z_{\text{Sc}}-1}} := h \cdot R_{\text{Sc45}} \cdot \left( Z_{\text{Sc}} - \frac{2}{3} \right)^2 \quad E_{\text{Sc45}_{Z_{\text{Sc}}}} := h \cdot R_{\text{Sc45}} \cdot Z_{\text{Sc}}^2$$

$$\text{xx}_{\text{cl}} := \text{xx}_1 - .4 \cdot (\text{cl} - 1) \cdot \Phi_{01}(11 - \text{cl}) - .55 \cdot (\text{cl} - 1) \cdot \Phi_{01}(\text{cl} - 12) \quad (\text{segment change after cl} = 11)$$

$$E_{\text{Sc45}_{\text{cl}}} := (Z_{\text{Sc}} - \text{xx}_{\text{cl}})^2 \cdot h \cdot R_{\text{Sc45}}$$

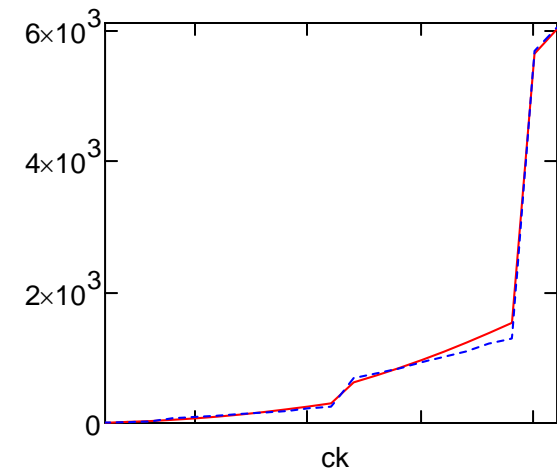
 $E_{\text{Sc45}_{\text{ck}}} =$ 

1.28405
16.26947
30.35728
48.80342
71.6079
98.77072
130.29187
166.17136
206.40919
251.00535
299.95985
619.25713
724.39857
837.77999
959.40139
1089.26278
1227.36414
1373.70548
1528.28681
5631.03174
6006.33297

 $E_{\text{Sc45}_{\text{obs}_{\text{ck}}} =$ 

6.54
12.8
24.76
73.47
91.66
111.1
138
158.7
180.02
225.32
249.832
685.89
755.47
829.79
926
1009
1094
1213
1288
5674.9
6033.8

$$\frac{\sum_{\text{ii}=1}^{Z_{\text{Sc}}} \frac{E_{\text{Sc45}_{\text{ii}}}}{E_{\text{Sc45}_{\text{obs}_{\text{ii}}}}} = 0.99505$$

 $E_{\text{Sc45}_{\text{ck}}}$ 
 $E_{\text{Sc45}_{\text{obs}_{\text{ck}}}}$ 


Ti  $Z_{Ti} := 22$   $M_{Ti48} := 48.310452$

$$R'_{Ti48} := \frac{1 + \frac{m_e}{M_{H1}}}{\left(1 + \frac{m_e}{M_{Ti48}}\right)} \cdot R'$$

$R_{Ti48} := c_{cgs} \cdot R'_{Ti48}$

$E_{Ti48\_obs} :=$  ( 6.82  
13.58  
27.491  
43.266  
99.22  
119.36  
140.8  
168.5  
193.2  
215.91  
265.23  
291.497  
787.33  
861.33  
940.36  
1044  
1131  
1221  
1346  
1426  
6249.1  
6625.8 )

$cl := 2 .. Z_{Ti} - 2$   $cm := 1 .. Z_{Ti} - 2$   $ck := 1 .. Z_{Ti}$

$$xx1 := Z_{Ti} - \frac{\sqrt{E_{Ti48\_obs_1}}}{\sqrt{R_{Ti48} \cdot h}}$$

$$xx\_obs_{ck} := Z_{Ti} - \sqrt{\frac{E_{Ti48\_obs_{ck}}}{R_{Ti48} \cdot h}}$$

$$xx1 = 21.29237$$

(from Ref. [24] compilation after XV)

$$E_{Ti48_1} := (Z_{Ca} - xx_1)^2 \cdot h \cdot R_{Ti48} \quad E_{Ti48_{Z_{Ti}-1}} := h \cdot R_{Ti48} \cdot \left( Z_{Ti} - \frac{2}{3} \right)^2 \quad E_{Ti48_{Z_{Ti}}} := h \cdot R_{Ti48} \cdot Z_{Ti}^2$$

$$xx_{cl} := xx_1 - .32 \cdot (cl - 1) \cdot \Phi_{01}(12 - cl) - .54 \cdot (cl - 1) \cdot \Phi_{01}(cl - 13) \quad (\text{segment change after } cl = 12)$$

$$E_{Ti48_{cl}} := (Z_{Ti} - xx_{cl})^2 \cdot h \cdot R_{Ti48}$$

 $E_{Ti48_{ck}} =$ 

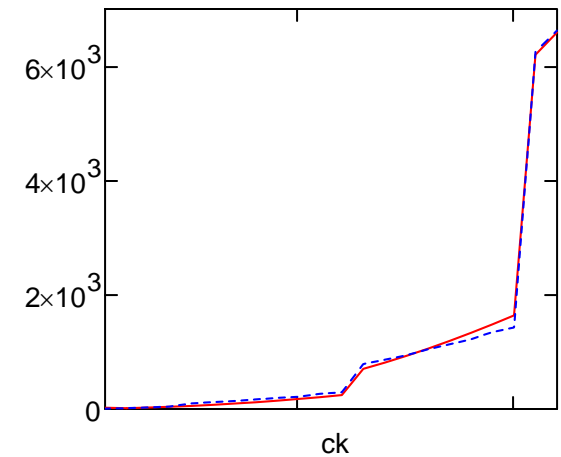
22.74807
14.38286
24.73506
37.87659
53.80746
72.52767
94.03722
118.3361
145.42432
175.30189
207.96878
243.42502
703.62724
813.32449
930.96483
1056.54823
1190.07471
1331.54427
1480.9569
1638.31261
6198.5282
6591.98947

 $E_{Ti48\_obs_{ck}} =$ 

6.82
13.58
27.491
43.266
99.22
119.36
140.8
168.5
193.2
215.91
265.23
291.497
787.33
861.33
940.36
1044
1131
1221
1346
1426
6249.1
6625.8

$$\frac{\sum_{ii=1}^{Z_{Ti}} \frac{E_{Ti48_{ii}}}{E_{Ti48\_obs_{ii}}}}{Z_{Ti}} = 1.0042$$

$E_{Ti48_{ck}}$   
 $E_{Ti48\_obs_{ck}}$



$$\mathbf{V} \quad Z_V := 23 \quad M_{V51} := 51.324304$$

$$R'_{V51} := \frac{1 + \frac{m_e}{M_{H1}}}{\left(1 + \frac{m_e}{M_{V51}}\right)} \cdot R'$$

$$R_{V51} := c_{\text{cgs}} \cdot R'_{V51}$$

$$E_{V51\_obs} := \left( \begin{array}{l} 6.74 \\ 14.65 \\ 29.310 \\ 46.707 \\ 65.23 \\ 128.12 \\ 150.17 \\ 173.7 \\ 205.8 \\ 230.5 \\ 255.04 \\ 308.25 \\ 336.267 \\ 895.58 \\ 974.02 \\ 1060 \\ 1168 \\ 1260 \\ 1355 \\ 1486 \\ 1569.6 \\ 6851.29 \\ 7246.18 \end{array} \right)$$

$$cl := 2 .. Z_V - 2 \quad cm := 1 .. Z_V - 2 \quad ck := 1 .. Z_V$$

$$xx_1 := Z_V - \frac{\sqrt{E_{V51\_obs_1}}}{\sqrt{R_{V51} \cdot h}}$$

$$xx\_obs_{ck} := Z_V - \sqrt{\frac{E_{V51\_obs_{ck}}}{R_{V51} \cdot h}}$$

$$xx_1 = 22.29653$$

(from Ref. [24] compilation after XV)

$$E_{V51_1} := (Z_V - xx_1)^2 \cdot h \cdot R_{V51} \quad E_{V51_{Z_V-1}} := h \cdot R_{V51} \cdot \left(Z_V - \frac{2}{3}\right)^2 \quad E_{V51_{Z_V}} := h \cdot R_{V51} \cdot Z_V^2$$

$$xx_{cl} := xx_1 - .374 \cdot (cl - 1) \cdot \Phi_{01}(13 - cl) - .55 \cdot (cl - 1) \cdot \Phi_{01}(cl - 14) \quad (\text{segment change after } cl = 13)$$

$$E_{V51_{cl}} := (Z_V - xx_{cl})^2 \cdot h \cdot R_{V51}$$

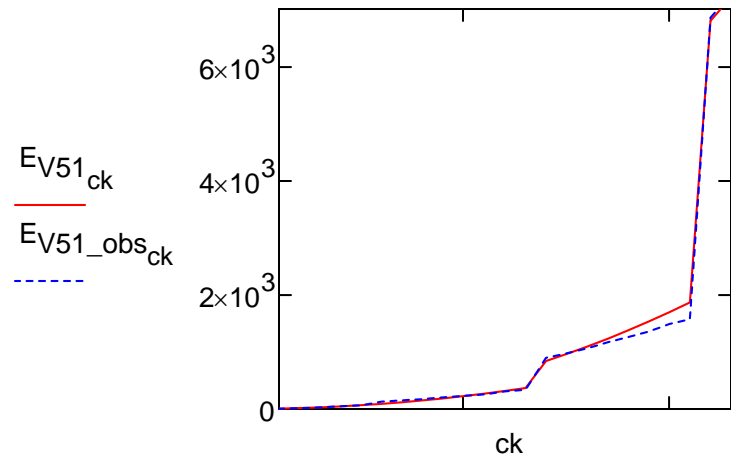
$E_{V51_{ck}} =$

6.74
15.81175
28.69368
45.38578
65.88805
90.20049
118.32311
150.2559
185.99886
225.55199
268.9153
316.08878
367.07243
840.02917
961.80829
1091.82739
1230.08649
1376.58558
1531.32466
1694.30374
1865.52281
6793.26456
7204.88585

$E_{V51\_obs_{ck}} =$

6.74
14.65
29.31
46.707
65.23
128.12
150.17
173.7
205.8
230.5
255.04
308.25
336.267
895.58
974.02
1060
1168
1260
1355
1486
1569.6
6851.29
7246.18

$$\frac{\sum_{ii=1}^{Z_V} \frac{E_{V51_{ii}}}{E_{V51\_obs_{ii}}}}{Z_V} = 0.99985$$



**Cr**     $Z_{Cr} := 24$      $M_{Cr52} := 52.338791$

$$R'_{Cr52} := \frac{1 + \frac{m_e}{M_{H1}}}{\left(1 + \frac{m_e}{M_{Cr52}}\right)} \cdot R'$$

$R_{Cr52} := c_{cgs} \cdot R'_{Cr52}$

$E_{Cr52\_obs} :=$

( 6.766  
16.50  
30.96  
49.1  
69.3  
90.56  
161.1  
184.7  
209.3  
244.4  
270.8  
298.0  
355  
384.30  
1010.64  
1097  
1185  
1299  
1396  
1496  
1634  
1721  
7432  
7894.8 )

$cl := 2 .. Z_{Cr} - 2$      $cm := 1 .. Z_{Cr} - 2$      $ck := 1 .. Z_{Cr}$

$$xx1 := Z_{Cr} - \frac{\sqrt{E_{Cr52\_obs_1}}}{\sqrt{R_{Cr52} \cdot h}}$$

$$xx\_obs_{ck} := Z_{Cr} - \sqrt{\frac{E_{Cr52\_obs_{ck}}}{R_{Cr52} \cdot h}}$$

$xx1 = 23.29518$

(from Russian compilation after XV)

$$E_{Cr52_1} := (Z_{Cr} - xx_1)^2 \cdot h \cdot R_{Cr52} \quad E_{Cr52_{Z_{Cr}-1}} := h \cdot R_{Cr52} \cdot \left(Z_{Cr} - \frac{2}{3}\right)^2 \quad E_{Cr52_{Z_{Cr}}} := h \cdot R_{Cr52} \cdot Z_{Cr}^2$$

$$xx_{cl} := xx_1 - .376 \cdot (cl - 1) \cdot \Phi_{01}(14 - cl) - .55 \cdot (cl - 1) \cdot \Phi_{01}(cl - 15) \quad (\text{segment change after } cl = 14)$$

$$E_{Cr52_{cl}} := (Z_{Cr} - xx_{cl})^2 \cdot h \cdot R_{Cr52}$$

$$E_{Cr52_{ck}} =$$

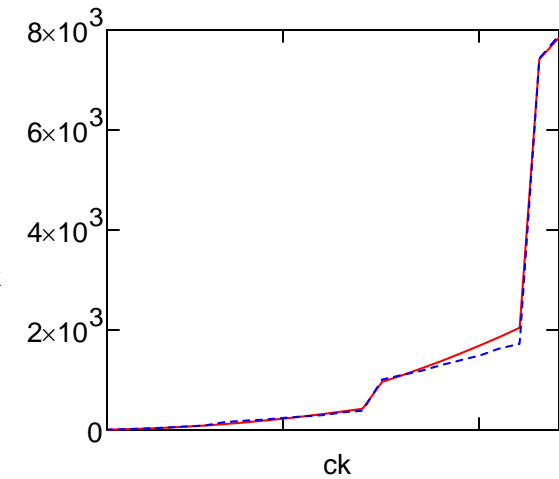
6.766
15.91039
28.90582
45.75228
66.44977
90.99829
119.39785
151.64844
187.75006
227.70271
271.5064
319.16112
370.66687
426.02366
962.11879
1092.15823
1230.43766
1376.95709
1531.71651
1694.71593
1865.95534
2045.43474
7415.23797
7845.01911

$$E_{Cr52_{obs_{ck}}} =$$

6.766
16.5
30.96
49.1
69.3
90.56
161.1
184.7
209.3
244.4
270.8
298
355
384.3
1010.64
1097
1185
1299
1396
1496
1634
1721
7432
7894.8

$$\frac{\sum_{ii=1}^{Z_{Cr}} \frac{E_{Cr52_{ii}}}{E_{Cr52_{obs_{ii}}}}}{Z_{Cr}} = 1.00036$$

$E_{Cr52_{ck}}$   
 $E_{Cr52_{obs_{ck}}}$





\_\_\_\_\_

\_\_\_\_\_

**Mn**  $Z_{\text{Mn}} := 25$   $M_{\text{Mn55}} := 55.352642$

$$R'_{\text{Mn55}} := \frac{1 + \frac{m_e}{M_{\text{H1}}}}{\left(1 + \frac{m_e}{M_{\text{Mn55}}}\right)} \cdot R'$$

$R_{\text{Mn55}} := c_{\text{cgs}} \cdot R'_{\text{Mn55}}$

$E_{\text{Mn55\_obs}} :=$

7.435
15.640
33.667
51.2
72.4
95
119.27
196.46
221.8
243.3
286
314.4
343.6
404
435.3
1136.2
1244
1317
1437
1539
1644
1788
1880
8141

$cl := 2 .. Z_{\text{Mn}} - 2$   $cm := 1 .. Z_{\text{Mn}} - 2$   $ck := 1 .. Z_{\text{Mn}}$

$$xx\_obs_{ck} := Z_{\text{Mn}} - \sqrt{\frac{E_{\text{Mn55\_obs}_{ck}}}{R_{\text{Mn55}} \cdot h}}$$

(from Russian compilation after XVI)

$$xx1 := Z_{\text{Mn}} - \frac{\sqrt{E_{\text{Mn55\_obs}_1}}}{\sqrt{R_{\text{Mn55}} \cdot h}}$$

$xx1 = 24.26115$

(8571.9)

$$E_{\text{Mn55}_1} := (Z_{\text{Mn}} - \text{xx}_1)^2 \cdot h \cdot R_{\text{Mn55}} \quad E_{\text{Mn55}_{Z_{\text{Mn}}-1}} := h \cdot R_{\text{Mn55}} \cdot \left(Z_{\text{Mn}} - \frac{2}{3}\right)^2 \quad E_{\text{Mn55}_{Z_{\text{Mn}}}} := h \cdot R_{\text{Mn55}} \cdot Z_{\text{Mn}}^2$$

$$\text{xx}_{\text{cl}} := \text{xx}_1 - .379 \cdot (\text{cl} - 1) \cdot \Phi_{01}(15 - \text{cl}) - .53 \cdot (\text{cl} - 1) \cdot \Phi_{01}(\text{cl} - 16) \quad (\text{segment change after } \text{cl} = 15) \quad n_{e\_eff} := 15$$

$$E_{\text{Mn55}_{\text{cl}}} := (Z_{\text{Mn}} - \text{xx}_{\text{cl}})^2 \cdot h \cdot R_{\text{Mn55}}$$

 $E_{\text{Mn55}_{\text{ck}}} =$ 

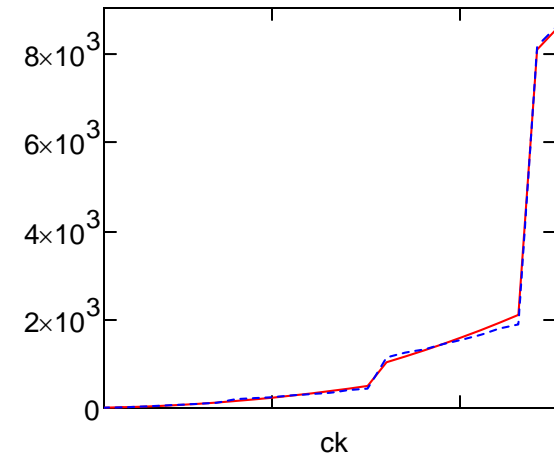
7.435
17.0191
30.51593
47.9255
69.2478
94.48283
123.6306
156.6911
193.66433
234.55029
279.34898
328.06041
380.68457
437.22147
497.6711
1028.24377
1157.51066
1294.42917
1438.99931
1591.22107
1751.09445
1918.61946
2093.79608

 $E_{\text{Mn55\_obs}_{\text{ck}}} =$ 

7.435
15.64
33.667
51.2
72.4
95
119.27
196.46
221.8
243.3
286
314.4
343.6
404
435.3
1136.2
1244
1317
1437
1539
1644
1788
1880

$$\frac{\sum_{ii=1}^{Z_{\text{Mn}}} \frac{E_{\text{Mn55}_{ii}}}{E_{\text{Mn55\_obs}_{ii}}}}{Z_{\text{Mn}}} = 0.99983$$

$E_{\text{Mn55}_{\text{ck}}}$   
 $E_{\text{Mn55\_obs}_{\text{ck}}}$



8064.45423
8512.39539

8141
8571.9

**Fe**     $Z_{\text{Fe}} := 26$      $M_{\text{Fe56}} := 56.367129$

$$R'_{\text{Fe56}} := \frac{1 + \frac{m_e}{M_{\text{H1}}}}{\left(1 + \frac{m_e}{M_{\text{Fe56}}}\right)} \cdot R'$$

$R_{\text{Fe56}} := c_{\text{cgs}} \cdot R'_{\text{Fe56}}$

$E_{\text{Fe56\_obs}} :=$

(  
7.870  
16.18  
30.651  
54.8  
75.0  
99  
125  
151.06  
235.04  
262.1  
290.4  
330.8  
361.0  
392.2  
457  
489.5  
1266.1  
1360  
1470  
1582  
1690  
1800  
1960  
2046  
)

$cl := 2 .. Z_{\text{Fe}} - 2$      $cm := 1 .. Z_{\text{Fe}} - 2$      $ck := 1 .. Z_{\text{Fe}}$

$$xx\_obs_{ck} := Z_{\text{Fe}} - \sqrt{\frac{E_{\text{Fe56\_obs}_{ck}}}{R_{\text{Fe56}} \cdot h}}$$

(from Ref. [24] compilation after XVII)

$$xx_1 := Z_{\text{Fe}} - \frac{\sqrt{E_{\text{Fe56\_obs}_1}}}{\sqrt{R_{\text{Fe56}} \cdot h}}$$

$xx_1 = 25.23985$

$$\begin{pmatrix} 8828 \\ 9277.7 \end{pmatrix}$$

$$E_{\text{Fe56}_1} := (Z_{\text{Fe}} - \text{xx}_1)^2 \cdot h \cdot R_{\text{Fe56}} \quad E_{\text{Fe56}_{Z_{\text{Fe}}-1}} := h \cdot R_{\text{Fe56}} \cdot \left( Z_{\text{Fe}} - \frac{2}{3} \right)^2 \quad E_{\text{Fe56}_{Z_{\text{Fe}}}} := h \cdot R_{\text{Fe56}} \cdot Z_{\text{Fe}}^2$$

$$\text{xx}_{\text{cl}} := \text{xx}_1 - .379 \cdot (\text{cl} - 1) \cdot \Phi_{01}(16 - \text{cl}) - .52 \cdot (\text{cl} - 1) \cdot \Phi_{01}(\text{cl} - 17) \quad (\text{segment change after } \text{cl} = 16) \quad n_{e\_eff} := 16$$

$$E_{\text{Fe56}_{\text{cl}}} := (Z_{\text{Fe}} - \text{xx}_{\text{cl}})^2 \cdot h \cdot R_{\text{Fe56}}$$

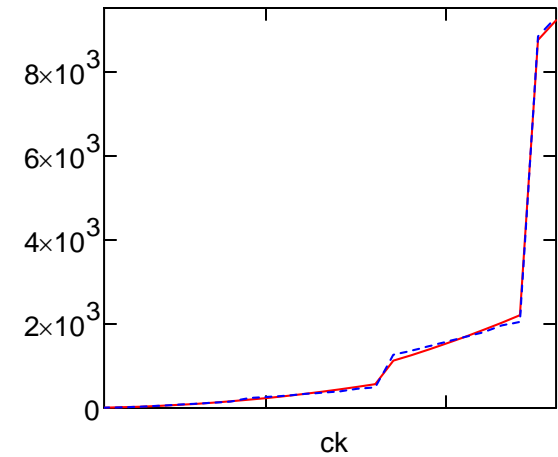
 $E_{\text{Fe56}_{\text{ck}}} =$ 

7.87
17.67407
31.39087
49.02041
70.56268
96.01768
125.38541
158.66588
195.85909
236.96502
281.98369
330.91509
383.75923
440.5161
501.1857
565.76804
1122.9444
1255.2442
1394.90962
1541.94064
1696.33727
1858.0995
2027.22734

 $E_{\text{Fe56}_{\text{obs}_{\text{ck}}} =$ 

7.87
16.18
30.651
54.8
75
99
125
151.06
235.04
262.1
290.4
330.8
361
392.2
457
489.5
1266.1
1360
1470
1582
1690
1800
1960

$$\frac{\sum_{\text{ii}=1}^{Z_{\text{Fe}}} \frac{E_{\text{Fe56}_{\text{ii}}}}{E_{\text{Fe56}_{\text{obs}_{\text{ii}}}}} = 0.99946}{Z_{\text{Fe}}}$$

 $E_{\text{Fe56}_{\text{ck}}}$   
 $E_{\text{Fe56}_{\text{obs}_{\text{ck}}}}$ 


2203.72079
8740.90747
9207.00849

2046
8828
9277.7

**Co**  $Z_{\text{Co}} := 27$   $M_{\text{Co59}} := 59.380981$

$$R'_{\text{Co59}} := \frac{1 + \frac{m_e}{M_{\text{H1}}}}{\left(1 + \frac{m_e}{M_{\text{Co59}}}\right)} \cdot R'$$

$R_{\text{Co59}} := c_{\text{CGS}} \cdot R'_{\text{Co59}}$

$E_{\text{Co59\_obs}} :=$

7.86  
17.06  
33.50  
51.3  
79.5  
102  
129  
157  
186.13  
276  
305  
336  
379  
411  
444  
512  
546.8  
1403.0  
1505  
1603  
1735  
1846  
1962  
2119

$cl := 2 .. Z_{\text{Co}} - 2$      $cm := 1 .. Z_{\text{Co}} - 2$      $ck := 1 .. Z_{\text{Co}}$

$$xx\_obs_{ck} := Z_{\text{Co}} - \sqrt{\frac{E_{\text{Co59\_obs}_{ck}}}{R_{\text{Co59}} \cdot h}}$$

(from Ref. [24] compilation after XVIII)

$$xx_1 := Z_{\text{Co}} - \frac{\sqrt{E_{\text{Co59\_obs}_1}}}{\sqrt{R_{\text{Co59}} \cdot h}}$$

$xx_1 = 26.24033$



$\begin{pmatrix} 2219 \\ 9544 \\ 10012.1 \end{pmatrix}$

$$E_{Co59_1} := (Z_{Co} - xx_1)^2 \cdot h \cdot R_{Co59} \quad E_{Co59_{Z_{Co}-1}} := h \cdot R_{Co59} \cdot \left( Z_{Co} - \frac{2}{3} \right)^2 \quad E_{Co59_{Z_{Co}}} := h \cdot R_{Co59} \cdot Z_{Co}^2$$

$$xx_{cl} := xx_1 - .38 \cdot (cl - 1) \cdot \Phi_{01}(17 - cl) - .52 \cdot (cl - 1) \cdot \Phi_{01}(cl - 18) \quad (\text{segment change after } cl = 17) \quad n_{e\_eff} := 17$$

$$E_{Co59_{cl}} := (Z_{Co} - xx_{cl})^2 \cdot h \cdot R_{Co59}$$

$E_{Co59_{ck}} =$

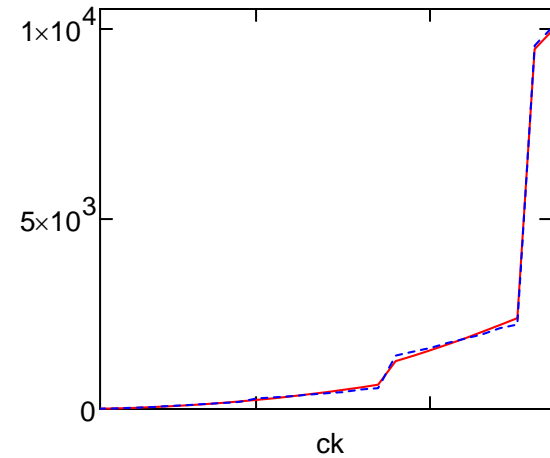
7.86
17.69011
31.45364
49.15058
70.78092
96.34468
125.84184
159.27242
196.63641
237.9338
283.16461
332.32883
385.42646
442.4575
503.42194
568.3198
637.15107
1255.11845
1394.77708
1541.80133

$E_{Co59\_obs_{ck}} =$

7.86
17.06
33.5
51.3
79.5
102
129
157
186.13
276
305
336
379
411
444
512
546.8
1403
1505
1603

$$\frac{\sum_{ii=1}^{Z_{Co}} \frac{E_{Co59_{ii}}}{E_{Co59\_obs_{ii}}}}{Z_{Co}} = 0.99981$$

$E_{Co59_{ck}}$  (red solid line)  
 $E_{Co59\_obs_{ck}}$  (blue dashed line)



1696.19119
1696.19119
1857.94666
2027.06774
2203.55443
2387.40673
9444.60361
9928.86466

1735
1735
1846
1962
2119
2219
9544
10012.1

**Ni**     $Z_{\text{Ni}} := 28$      $M_{\text{Ni58}} := 58.396103$

$$R'_{\text{Ni58}} := \frac{1 + \frac{m_e}{M_{\text{H1}}}}{\left(1 + \frac{m_e}{M_{\text{Ni58}}}\right)} \cdot R'$$

$$R_{\text{Ni58}} := c_{\text{cgs}} \cdot R'_{\text{Ni58}}$$

$E_{\text{Ni58\_obs}} :=$

7.635
18.168
35.17
54.9
75.5
108
133
162
193
224.5
321.2
352
384
430
464
499
571
607.2
1547
1648
1756
1894
2011
2121

$cl := 2 .. Z_{\text{Ni}} - 2$      $cm := 1 .. Z_{\text{Ni}} - 2$      $ck := 1 .. Z_{\text{Ni}}$

$$xx\_obs_{ck} := Z_{\text{Ni}} - \sqrt{\frac{E_{\text{Ni58\_obs}_{ck}}}{R_{\text{Ni58}} \cdot h}}$$

(from Ref. [24] compilation after XIX)

$$xx_1 := Z_{\text{Ni}} - \frac{\sqrt{E_{\text{Ni58\_obs}_1}}}{\sqrt{R_{\text{Ni58}} \cdot h}}$$

$$xx_1 = 27.25128$$

2131
2295
2399
10290
10775

$$E_{Ni58_1} := (Z_{Ni} - xx_1)^2 \cdot h \cdot R_{Ni58}$$

$$E_{Ni58_{Z_{Ni}-1}} := h \cdot R_{Ni58} \cdot \left( Z_{Ni} - \frac{2}{3} \right)^2$$

$$E_{Ni58_{Z_{Ni}}} := h \cdot R_{Ni58} \cdot Z_{Ni}^2$$

$$xx_{cl} := xx_1 - .38 \cdot (cl - 1) \cdot \Phi_{01}(18 - cl) - .528 \cdot (cl - 1) \cdot \Phi_{01}(cl - 19)$$

(segment change after cl = 18)  $n_{e\_eff} := 18$

$$E_{Ni58_{cl}} := (Z_{Ni} - xx_{cl})^2 \cdot h \cdot R_{Ni58}$$

$E_{Ni58_{ck}} =$

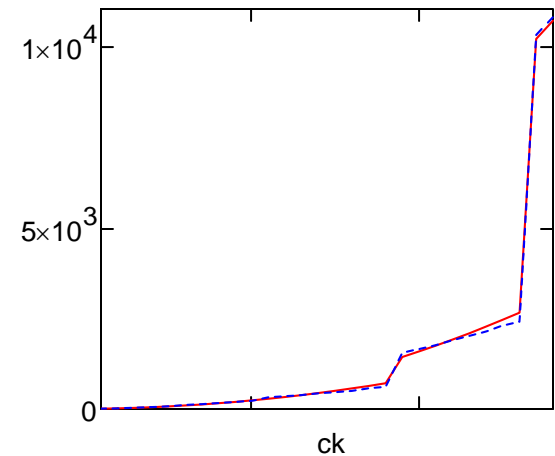
7.635
17.35175
31.00191
48.58547
70.10245
95.55284
124.93663
158.25384
195.50445
236.68848
281.80591
330.85676
383.84101
440.75969

$E_{Ni58\_obs_{ck}} =$

7.635
18.168
35.17
54.9
75.5
108
133
162
193
224.5
321.2
352
384
430

$$\frac{\sum_{ii=1}^{Z_{Ni}} \frac{E_{Ni58_{ii}}}{E_{Ni58\_obs_{ii}}}}{Z_{Ni}} = 0.99967$$

$E_{Ni58_{ck}}$  (red solid line)  
 $E_{Ni58\_obs_{ck}}$  (blue dashed line)



440.75888
501.60975
566.39423
635.11213
707.76343
1431.69351
1582.95074
1741.80197
1908.24718
2082.28637
2263.91955
2453.14672
2649.96788
10175.53354
10677.95429

464
499
571
607.2
1547
1648
1756
1894
2011
2131
2295
2399
10290
10775

**Cu**  $Z_{\text{Cu}} := 29$   $M_{\text{Cu63}} := 63.409319$

$$R'_{\text{Cu63}} := \frac{1 + \frac{m_e}{M_{\text{H1}}}}{\left(1 + \frac{m_e}{M_{\text{Cu63}}}\right)} \cdot R'$$

$$R_{\text{Cu63}} := c_{\text{CGS}} \cdot R'_{\text{Cu63}}$$

$E_{\text{Cu63\_obs}} :=$

7.726
20.292
36.83
55.2
79.9
103
139
166
199
232
266
368.8
401
435
484
520
557
633
671
1698
1793
1905
2045
2173

$$cl := 2 .. Z_{\text{Cu}} - 2 \quad cm := 1 .. Z_{\text{Cu}} - 2 \quad ck := 1 .. Z_{\text{Cu}}$$

$$xx\_obs_{ck} := Z_{\text{Cu}} - \sqrt{\frac{E_{\text{Cu63\_obs}_{ck}}}{R_{\text{Cu63}} \cdot h}}$$

(from Ref. [24] compilation after XX)

$$xx_1 := Z_{\text{Cu}} - \frac{\sqrt{E_{\text{Cu63\_obs}_1}}}{\sqrt{R_{\text{Cu63}} \cdot h}}$$

$$xx_1 = 28.24683$$

2298
2460
2585
11062
11568

$$E_{Cu63_1} := (Z_{Cu} - xx_1)^2 \cdot h \cdot R_{Cu63} \quad E_{Cu63_{Z_{Cu}-1}} := h \cdot R_{Cu63} \cdot \left(Z_{Cu} - \frac{2}{3}\right)^2 \quad E_{Cu63_{Z_{Cu}}} := h \cdot R_{Cu63} \cdot Z_{Cu}^2$$

$$xx_{cl} := xx_1 - .378 \cdot (cl - 1) \cdot \Phi_{01}(19 - cl) - .534 \cdot (cl - 1) \cdot \Phi_{01}(cl - 20) \quad (\text{segment change after } cl = 19) \quad n_{e\_eff} := 19$$

$$E_{Cu63_{cl}} := (Z_{Cu} - xx_{cl})^2 \cdot h \cdot R_{Cu63}$$

$E_{Cu63_{ck}} =$

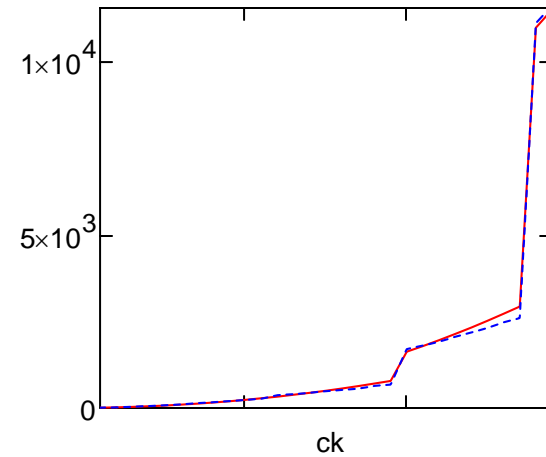
7.726
17.42712
31.02036
48.50572
69.88319
95.15278
124.31449
157.36831
194.31425
235.15231
279.88249
328.50478
381.01919
437.42572
497.72436
561.91513
629.99801
701.972

$E_{Cu63\_obs_{ck}} =$

7.726
20.292
36.83
55.2
79.9
103
139
166
199
232
266
368.8
401
435
484
520
557
622

$$\frac{\sum_{ii=1}^{Z_{Cu}} \frac{E_{Cu63_{ii}}}{E_{Cu63\_obs_{ii}}}}{Z_{Cu}} = 0.99976$$

$E_{Cu63_{ck}}$   
 $E_{Cu63\_obs_{ck}}$



701.775
777.84012
1617.92699
1780.35004
1950.54067
2128.49885
2314.2246
2507.71791
2708.97879
2918.00723
10933.71273
11454.29366

555
671
1698
1793
1905
2045
2173
2298
2460
2585
11062
11568



**Zn**     $Z_{\text{Zn}} := 30$      $M_{\text{Zn64}} := 63.409319$

$$R'_{\text{Zn64}} := \frac{1 + \frac{m_e}{M_{\text{H1}}}}{\left(1 + \frac{m_e}{M_{\text{Zn64}}}\right)} \cdot R'$$

$$R_{\text{Zn64}} := c_{\text{cgs}} \cdot R'_{\text{Zn64}}$$

$E_{\text{Zn64\_obs}} :=$

9.394
17.964
39.722
59.4
82.6
108
134
174
203
238
274
310.8
419.7
454
490
542
579
619
698
738
1856
1953
2070
2216

$$cl := 2 .. Z_{\text{Zn}} - 2 \quad cm := 1 .. Z_{\text{Zn}} - 2 \quad ck := 1 .. Z_{\text{Zn}}$$

$$xx_{\text{obs}ck} := Z_{\text{Zn}} - \sqrt{\frac{E_{\text{Zn64\_obs}ck}}{R_{\text{Zn64}} \cdot h}}$$

(from Ref. [24] compilation after XXI)

$$xx_1 := Z_{\text{Zn}} - \frac{\sqrt{E_{\text{Zn64\_obs}_1}}}{\sqrt{R_{\text{Zn64}} \cdot h}}$$

$$xx_1 = 29.1695$$

$$\begin{pmatrix} 2350 \\ 2479 \\ 2647 \\ 2780 \\ 11865 \\ 12389 \end{pmatrix}$$

$$E_{Zn64_1} := (Z_{Zn} - xx_1)^2 \cdot h \cdot R_{Zn64} \quad E_{Zn64_{Z_{Zn}-1}} := h \cdot R_{Zn64} \cdot \left( Z_{Zn} - \frac{2}{3} \right)^2 \quad E_{Zn64_{Z_{Zn}}} := h \cdot R_{Zn64} \cdot Z_{Zn}^2$$

$$xx_{cl} := xx_1 - .379 \cdot (cl - 1) \cdot \Phi_{01}(20 - cl) - .5 \cdot (cl - 1) \cdot \Phi_{01}(cl - 21) \quad (\text{segment change after } cl = 20) \quad n_{e\_eff} := 20$$

$$E_{Zn64_{cl}} := (Z_{Zn} - xx_{cl})^2 \cdot h \cdot R_{Zn64}$$

$$E_{Zn64_{cl}} =$$

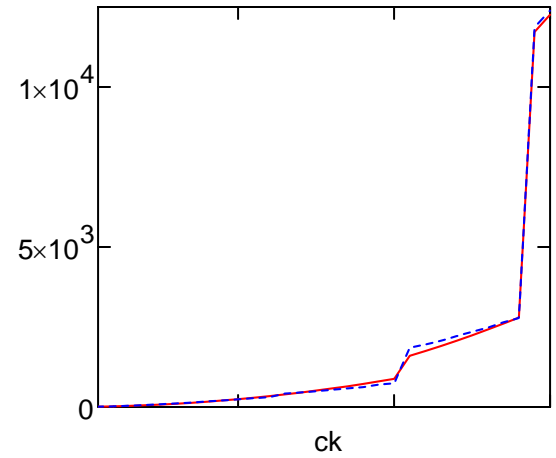
$$E_{Zn64 \text{ obs}_{cl}} =$$

ck
9.394
19.92431
34.36736
52.72315
74.99168
101.17294
131.26694
165.27368
203.19316
245.02537
290.77032
340.42801
393.99844
451.48161
512.87751
578.18615
647.40753
720.54164
797.5885
878.54809
1597.60441
1748.51914
1906.2438
2070.77838
2242.12289
2420.27732
2605.24168
2797.01596
11719.12407
12257.86479

ck
9.394
17.964
39.722
59.4
82.6
108
134
174
203
238
274
310.8
419.7
454
490
542
579
619
698
738
1856
1953
2070
2216
2350
2479
2647
2780
11865
12389

$$\frac{\sum_{ii=1}^{Z_{Zn}} \frac{E_{Zn64_{ii}}}{E_{Zn64\_obs_{ii}}}}{Z_{Zn}} = 0.99981$$

$E_{Zn64_{ck}}$   
 ———  
 $E_{Zn64\_obs_{ck}}$   
 - - - -



**Ga**  $Z_{\text{Ga}} := 31$   $M_{\text{Ga69}} := 69.437011$

$$R'_{\text{Ga69}} := \frac{1 + \frac{m_e}{M_{\text{H1}}}}{\left(1 + \frac{m_e}{M_{\text{Ga69}}}\right)} \cdot R'$$

$$R_{\text{Ga69}} := c_{\text{cgs}} \cdot R'_{\text{Ga69}}$$

$E_{\text{Ga69\_obs}} :=$

5.99
20.51
30.71
64
90
117
147
179
214
248
284
321
358
475
510
546
596
637
677
765
807
2010
2120
2242

2393  
2533  
2668  
2840  
2982  
12696  
13239

$$cl := 2 .. Z_{Ga} - 2 \quad cm := 1 .. Z_{Ga} - 2 \quad ck := 1 .. Z_{Ga}$$

$$xx_{obs_{ck}} := Z_{Ga} - \sqrt{\frac{E_{Ga69\_obs_{ck}}}{R_{Ga69} \cdot h}}$$

$$xx_1 := Z_{Ga} - \frac{\sqrt{E_{Ga69\_obs_1}}}{\sqrt{R_{Ga69} \cdot h}}$$

(from Ref. [24] compilation after IV)

$$xx_1 = 30.33683$$

$$E_{Ga69_1} := (Z_{Ga} - xx_1)^2 \cdot h \cdot R_{Ga69} \quad E_{Ga69_{Z_{Ga}-1}} := h \cdot R_{Ga69} \cdot \left(Z_{Ga} - \frac{2}{3}\right)^2 \quad E_{Ga69_{Z_{Ga}}} := h \cdot R_{Ga69} \cdot Z_{Ga}^2$$

$$xx_{cl} := xx_1 - .4 \cdot (cl - 1) \cdot \Phi_{01}(21 - cl) - .515 \cdot (cl - 1) \cdot \Phi_{01}(cl - 22) \quad (\text{segment change after } cl = 21) \quad n_{e\_eff} := 21$$

$$E_{Ga69_{cl}} := (Z_{Ga} - xx_{cl})^2 \cdot h \cdot R_{Ga69}$$

$E_{Ga69_{ck}} =$

5.99
15.39504
29.15844
47.28019
69.76029
96.59875
127.79557
163.35074
203.26427
247.53615
296.16639
349.15498
406.50193
468.20723
534.07000

$E_{Ga69\_obs_{ck}} =$

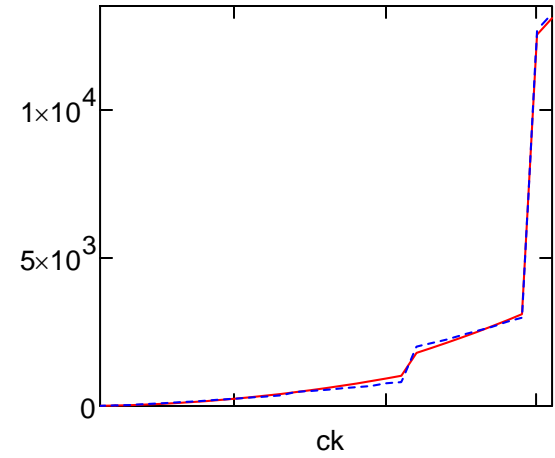
5.99
20.51
30.71
64
90
117
147
179
214
248
284
321
358
475
510

534.27089
604.6929
679.47327
758.612
842.10908
929.96451
1022.1783
1794.39564
1959.02901
2130.88704
2309.96973
2496.27707
2689.80906
2890.56571
3098.54702
12531.78452
13088.68546

510
546
596
637
677
765
807
2010
2120
2242
2393
2533
2668
2840
2982
12696
13239

$$\frac{\sum_{ii=1}^{Z_{Ga}} \frac{E_{Ga69_{ii}}}{E_{Ga69_{obs_{ii}}}}}{Z_{Ga}} = 0.99975$$

$E_{Ga69_{ck}}$   
 —  
 $E_{Ga69_{obs_{ck}}}$   
 - - -



**Ge**  $Z_{\text{Ge}} := 32$   $M_{\text{Ge74}} := 74.450238$

$$R'_{\text{Ge74}} := \frac{1 + \frac{m_e}{M_{\text{H1}}}}{\left(1 + \frac{m_e}{M_{\text{Ge74}}}\right)} \cdot R'$$

$$R_{\text{Ge74}} := c_{\text{cgs}} \cdot R'_{\text{Ge74}}$$

$E_{\text{Ge74\_obs}} :=$

7.899
15.934
34.22
45.71
93.5
120
151
183
217
255
291
330
369
409
533
568
607
658
701
744
836
880
2180
2294

- 2421
- 2577
- 2723
- 2863
- 3041
- 3192
- 13550
- 14119

$$cl := 2 .. Z_{Ge} - 2 \quad cm := 1 .. Z_{Ge} - 2 \quad ck := 1 .. Z_{Ge}$$

$$xx_{obsck} := Z_{Ge} - \sqrt{\frac{E_{Ge74\_obsck}}{R_{Ge74} \cdot h}}$$

$$xx_1 := Z_{Ge} - \frac{\sqrt{E_{Ge74\_obs_1}}}{\sqrt{R_{Ge74} \cdot h}}$$

(from Ref. [24] compilation after V)

$$xx_1 = 31.23845$$

$$E_{Ge74_1} := (Z_{Ge} - xx_1)^2 \cdot h \cdot R_{Ge74} \quad E_{Ge74_{Z_{Ge}-1}} := h \cdot R_{Ge74} \cdot \left(Z_{Ge} - \frac{2}{3}\right)^2 \quad E_{Ge74_{Z_{Ge}}} := h \cdot R_{Ge74} \cdot Z_{Ge}^2$$

$$xx_{cl} := xx_1 - .386 \cdot (cl - 1) \cdot \Phi_{01}(22 - cl) - .5 \cdot (cl - 1) \cdot \Phi_{01}(cl - 23) \quad (\text{segment change after } cl = 22) \quad n_{e\_eff} := 22$$

$$E_{Ge74_{cl}} := (Z_{Ge} - xx_{cl})^2 \cdot h \cdot R_{Ge74}$$

$E_{Ge74_{ck}} =$

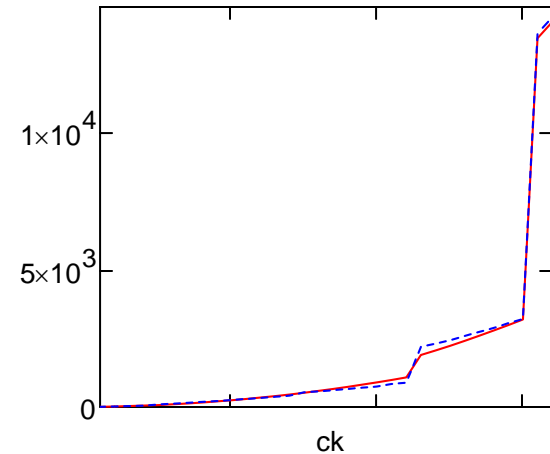
7.899
17.93567
32.03096
50.18486
72.39736
98.66848
128.99822
163.38656
201.83351
244.33908
290.90326
341.52604
396.20744
454.94746

$E_{Ge74\_obsck} =$

7.899
15.934
34.22
45.71
93.5
120
151
183
217
255
291
330
369
409

$$\frac{\sum_{ii=1}^{Z_{Ge}} \frac{E_{Ge74_{ii}}}{E_{Ge74\_obs_{ii}}}}{Z_{Ge}} = 1.00026$$

$E_{Ge74_{ck}}$   
 $E_{Ge74\_obsck}$





517.74608
584.60331
655.51916
730.49362
809.52669
892.61837
979.76866
1070.97756
1884.09218
2047.68793
2218.09361
2395.30922
2579.33476
2770.17024
2967.81566
3172.271
13371.683
13946.74406

533
568
607
658
701
744
836
880
2180
2294
2421
2577
2723
2863
3041
3192
13550
14119

**As**     $Z_{\text{As}} := 33$      $M_{\text{As75}} := 75.464725$

$$R'_{\text{As75}} := \frac{1 + \frac{m_e}{M_{\text{H1}}}}{\left(1 + \frac{m_e}{M_{\text{As75}}}\right)} \cdot R'$$

$$R_{\text{As75}} := c_{\text{cgs}} \cdot R'_{\text{As75}}$$

$E_{\text{As75\_obs}} :=$

9.81
18.633
28.351
50.13
62.63
127.6
154
187
222
259
300
338
379
421
462
594
630
670
724
769
813
910
957
2357

2474  
2606  
2768  
2920  
3065  
3248  
3409  
14440  
15029

$$cl := 2 .. Z_{As} - 2 \quad cm := 1 .. Z_{As} - 2 \quad ck := 1 .. Z_{As}$$

$$xx\_obs_{ck} := Z_{As} - \sqrt{\frac{E_{As75\_obs_{ck}}}{R_{As75} \cdot h}}$$

(from Ref. [24] compilation after VI)

$$xx_1 := Z_{As} - \frac{\sqrt{E_{As75\_obs_1}}}{\sqrt{R_{As75} \cdot h}}$$

$$xx_1 = 32.15131$$

$$E_{As75_1} := (Z_{As} - xx_1)^2 \cdot h \cdot R_{As75} \quad E_{As75_{Z_{As}-1}} := h \cdot R_{As75} \cdot \left(Z_{As} - \frac{2}{3}\right)^2 \quad E_{As75_{Z_{As}}} := h \cdot R_{As75} \cdot Z_{As}^2$$

$$xx_{cl} := xx_1 - .371 \cdot (cl - 1) \cdot \Phi_{01}(23 - cl) - .5 \cdot (cl - 1) \cdot \Phi_{01}(cl - 24) \quad (\text{segment change after } cl = 22) \quad n_{e\_eff} := 23$$

$$E_{As75_{cl}} := (Z_{As} - xx_{cl})^2 \cdot h \cdot R_{As75}$$

$E_{As75_{ck}} =$ 

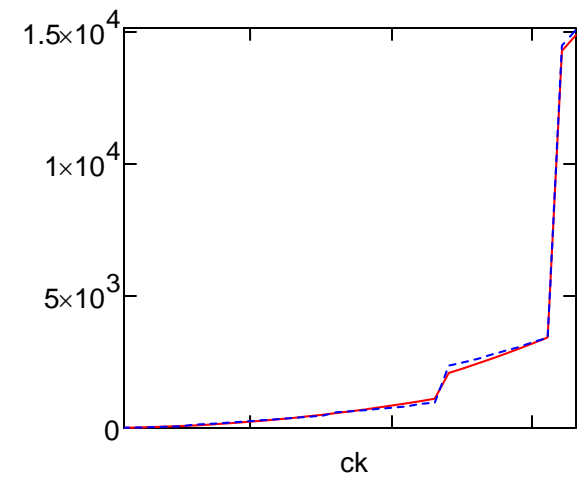
9.81
20.26144
34.46219
52.41225
74.11116
99.56027
128.75823
161.70551
198.40208
238.84797
283.04315
330.98764
382.68144
438.12454
497.31694
560.25865
626.94967
697.38998
771.57961
849.51854
931.20677
1016.64431
1105.83115
2076.89505
2248.48752
2426.88993
2612.10228
2804.12455
3002.95676
3208.59891
3421.05099
14238.81595

 $E_{As75\_obs_{ck}} =$ 

9.81
18.633
28.351
50.13
62.63
127.6
154
187
222
259
300
338
379
421
462
594
630
670
724
769
813
910
957
2357
2474
2606
2768
2920
3065
3248
3409
14440

$$\frac{\sum_{ii=1}^{Z_{As}} \frac{E_{As75_{ii}}}{E_{As75\_obs_{ii}}}}{Z_{As}} = 1.00047$$

$E_{As75_{ck}}$   
 $E_{As75\_obs_{ck}}$



14832.0369

15029

**Se**     $Z_{\text{Se}} := 34$      $M_{\text{Se80}} := 80.477941$

$$R'_{\text{Se80}} := \frac{1 + \frac{m_e}{M_{\text{H1}}}}{\left(1 + \frac{m_e}{M_{\text{Se80}}}\right)} \cdot R'$$

$$R_{\text{Se80}} := c_{\text{cgs}} \cdot R'_{\text{Se80}}$$

$E_{\text{Se80\_obs}} :=$

( 9.752 )  
 21.19  
 30.820  
 42.944  
 68.3  
 81.70  
 155.4  
 191  
 227  
 264  
 304  
 347  
 388  
 431  
 475  
 519  
 657  
 695  
 736  
 793  
 839  
 886  
 987  
 1036.2

1030.3  
 2542  
 2661  
 2798  
 2965  
 3123  
 3274  
 3463  
 3633  
 15370  
 15968

$$cl := 2 .. Z_{Se} - 2 \quad cm := 1 .. Z_{Se} - 2 \quad ck := 1 .. Z_{Se}$$

$$xx_{obsck} := Z_{Se} - \sqrt{\frac{E_{Se80\_obsck}}{R_{Se80} \cdot h}}$$

(from Ref. [24] compilation after VII)

$$xx_1 := Z_{Se} - \frac{\sqrt{E_{Se80\_obs_1}}}{\sqrt{R_{Se80} \cdot h}}$$

$$xx_1 = 33.15382$$

$$E_{Se80_1} := (Z_{Se} - xx_1)^2 \cdot h \cdot R_{Se80}$$

$$E_{Se80_{Z_{Se}-1}} := h \cdot R_{Se80} \cdot \left(Z_{Se} - \frac{2}{3}\right)^2$$

$$E_{Se80_{Z_{Se}}} := h \cdot R_{Se80} \cdot Z_{Se}^2$$

$$xx_{cl} := xx_1 - .369 \cdot (cl - 1) \cdot \Phi_{01}(24 - cl) - .5 \cdot (cl - 1) \cdot \Phi_{01}(cl - 25) \quad (\text{segment change after } cl = 24) \quad n_{e\_eff} := 24$$

$$E_{Se80_{cl}} := (Z_{Se} - xx_{cl})^2 \cdot h \cdot R_{Se80}$$

$E_{Se80_{ck}} =$

9.752
20.1118
34.18059
51.95837
73.44514
98.64091
127.54566
160.15941

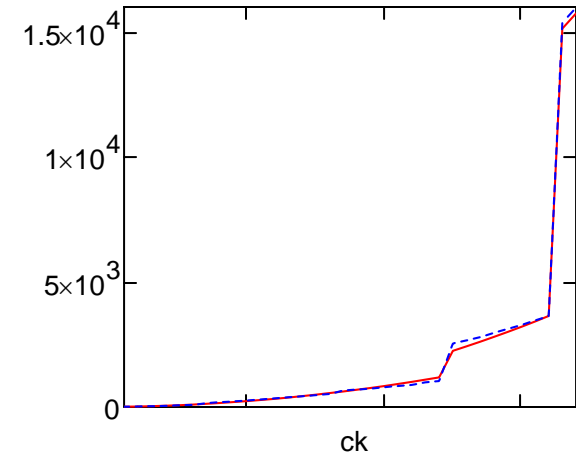
$E_{Se80\_obsck} =$

9.752
21.19
30.82
42.944
68.3
81.7
155.4
191

196.48215	227
236.51388	264
280.2546	304
327.70432	347
378.86302	388
433.73072	431
492.30741	475
554.59309	519
620.58776	657
690.29142	695
763.70407	736
840.82572	793
921.65636	839
1006.19599	886
1094.44461	987
1186.40222	1036.3
2247.60917	2542
2425.97744	2661
2611.15564	2798
2803.14378	2965
3001.94186	3123
3207.54987	3274
3419.96783	3463
3639.19572	3633
15133.19418	15370
15744.57522	15968

$$\frac{\sum_{ii=1}^{Z_{Se}} \frac{E_{Se80_{ii}}}{E_{Se80\_obs_{ii}}}}{Z_{Se}} = 1.00072$$

$E_{Se80_{ck}}$   
 ———  
 $E_{Se80\_obs_{ck}}$   
 - - - -





**Br**     $Z_{\text{Br}} := 35$      $M_{\text{Br79}} := 79.493064$

$$R'_{\text{Br79}} := \frac{1 + \frac{m_e}{M_{\text{H1}}}}{\left(1 + \frac{m_e}{M_{\text{Br79}}}\right)} \cdot R'$$

$R_{\text{Br79}} := c_{\text{cgs}} \cdot R'_{\text{Br79}}$

$E_{\text{Br79\_obs}} :=$

11.814
21.8
36
47.3
59.7
88.6
103.0
192.8
232
270
310
352
398
441
486
533
579
724
762
806
864
913
961
1000

1000  
1119  
2733  
2855  
2997  
3170  
3334  
3490  
3684  
3865  
16315  
16937

$$cl := 2 .. Z_{Br} - 2 \quad cm := 1 .. Z_{Br} - 2 \quad ck := 1 .. Z_{Br}$$

$$xx_{obsck} := Z_{Br} - \sqrt{\frac{E_{Br79\_obsck}}{R_{Br79} \cdot h}}$$

(from Ref. [24] compilation after VIII)

$$xx_1 := Z_{Br} - \frac{\sqrt{E_{Br79\_obs_1}}}{\sqrt{R_{Br79} \cdot h}}$$

$$xx_1 = 34.06865$$

$$E_{Br79_1} := (Z_{Br} - xx_1)^2 \cdot h \cdot R_{Br79} \quad E_{Br79_{Z_{Br}-1}} := h \cdot R_{Br79} \cdot \left(Z_{Br} - \frac{2}{3}\right)^2 \quad E_{Br79_{Z_{Br}}} := h \cdot R_{Br79} \cdot Z_{Br}^2$$

$$xx_{cl} := xx_1 - .356 \cdot (cl - 1) \cdot \Phi_{01}(25 - cl) - .5 \cdot (cl - 1) \cdot \Phi_{01}(cl - 26) \quad (\text{segment change after } cl = 25) \quad n_{e\_eff} := 25$$

$$E_{Br79_{cl}} := (Z_{Br} - xx_{cl})^2 \cdot h \cdot R_{Br79}$$

$E_{Br79_{ck}} =$

11.814
22.57173
36.78173
54.44397
75.55848
100.12524
128.14426
159.61553

$E_{Br79\_obsck} =$

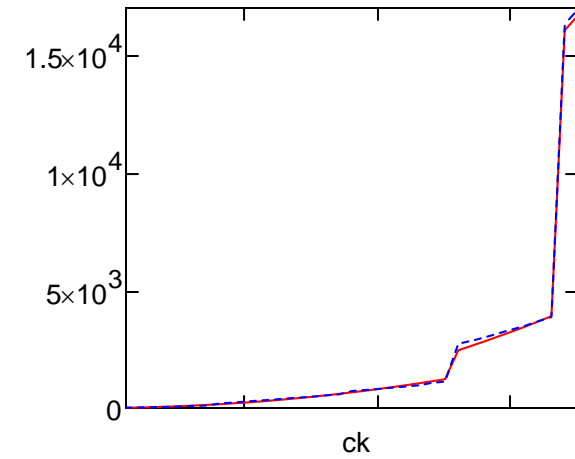
11.814
21.8
36
47.3
59.7
88.6
103
192.8

194.53906
232.91485
274.7429
320.0232
368.75576
420.94057
476.57764
535.66697
598.20855
664.20239
733.64849
806.54684
882.89745
962.70032
1045.95544
1132.66283
1222.82246
2457.04027
2643.3785
2836.52667
3036.48477
3243.25281
3456.83079
3677.2187
3904.41655
16054.80435
16684.34517

232
270
310
352
398
441
486
533
579
724
762
806
864
913
961
1068
1119
2733
2855
2997
3170
3334
3490
3684
3865
16315
16937

$$\frac{\sum_{ii=1}^{Z_{\text{Br}}} \frac{E_{\text{Br79}_{ii}}}{E_{\text{Br79\_obs}_{ii}}}}{Z_{\text{Br}}} = 0.99832$$

$E_{\text{Br79}_{ck}}$   
 $E_{\text{Br79\_obs}_{ck}}$





**Kr**     $Z_{\text{Kr}} := 36$      $M_{\text{Kr84}} := 84.506279$

$$R'_{\text{Kr84}} := \frac{1 + \frac{m_e}{M_{\text{H1}}}}{\left(1 + \frac{m_e}{M_{\text{Kr84}}}\right)} \cdot R'$$

$$R_{\text{Kr84}} := c_{\text{cgs}} \cdot R'_{\text{Kr84}}$$

$E_{\text{Kr84\_obs}} :=$

13.999
24.359
36.95
52.5
64.7
78.5
111.0
126
230.9
275
316
358
403
451
497
545
593
642
794
833
878
939
989
1039

1151

1205

2931

3056

3203

3381

3551

3712

3912

4105

17290

17936

$$cl := 2 .. Z_{Kr} - 2 \quad cm := 1 .. Z_{Kr} - 2 \quad ck := 1 .. Z_{Kr}$$

$$xx_{obsck} := Z_{Kr} - \sqrt{\frac{E_{Kr84\_obsck}}{R_{Kr84} \cdot h}}$$

(from Ref. [24] compilation after IX)

$$xx_1 := Z_{Kr} - \frac{\sqrt{E_{Kr84\_obs_1}}}{\sqrt{R_{Kr84} \cdot h}}$$

$$xx_1 = 34.98618$$

$$E_{Kr84_1} := (Z_{Kr} - xx_1)^2 \cdot h \cdot R_{Kr84} \quad E_{Kr84_{Z_{Kr}-1}} := h \cdot R_{Kr84} \cdot \left(Z_{Kr} - \frac{2}{3}\right)^2 \quad E_{Kr84_{Z_{Kr}}} := h \cdot R_{Kr84} \cdot Z_{Kr}^2$$

$$xx_{cl} := xx_1 - .345 \cdot (cl - 1) \cdot \Phi_{01}(26 - cl) - .5 \cdot (cl - 1) \cdot \Phi_{01}(cl - 27) \quad (\text{segment change after } cl = 26) \quad n_{e\_eff} := 26$$

$$E_{Kr84_{cl}} := (Z_{Kr} - xx_{cl})^2 \cdot h \cdot R_{Kr84}$$

$E_{\text{Kr84}_{\text{ck}}} =$ 

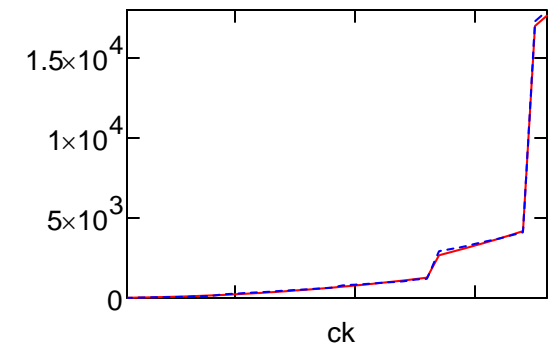
13.999
25.14772
39.53866
57.1718
78.04716
102.16473
129.52451
160.12651
193.97072
231.05714
271.38577
314.95661
361.76967
411.82494
465.12242
521.66211
581.44402
644.46813
710.73446
780.24301
852.99376
928.98673
1008.2219
1090.6993
1176.4189
1265.38071
2674.77018
2869.04172
3070.1232
3278.01462
3492.71597
3714.22727

 $E_{\text{Kr84}_{\text{obs}_{\text{ck}}} =$ 

13.999
24.359
36.95
52.5
64.7
78.5
111
126
230.9
275
316
358
403
451
497
545
593
642
794
833
878
939
989
1039
1151
1205
2931
3056
3203
3381
3551
3712

$$\frac{Z_{\text{Kr}} \sum_{ii=1} E_{\text{Kr84}_{ii}}}{Z_{\text{Kr}} E_{\text{Kr84}_{\text{obs}_{ii}}}} = 0.99819$$

$E_{\text{Kr84}_{\text{ck}}}$   
 $E_{\text{Kr84}_{\text{obs}_{\text{ck}}}}$



3942.5485
4177.67968
17003.6625
17651.36342

3912
4105
17290
17936

Unfortunately there are no complete series of observed ionization energy values for elements beyond Kr. Based on the previous equations, the Reciprocal System equations may be generalized to apply to any isotope, as follows.

### b. Generalized Expressions for Ionization Energies

The results above indicate quite clearly that the *true* slopes are 1/3 and 1/2--the differences with that observed are due to experimental errors or faulty extrapolation (see quotation from Moore above). Therefore, generalizing:

$$cl := 2 \dots Z_{el} - 2^{\blacksquare} \quad cm := 1 \dots Z_{el} - 2^{\blacksquare} \quad ck := 1 \dots Z_{el}^{\blacksquare}$$

$$xx_{cl} := xx_1 - \frac{1}{3} \cdot (cl - 1) \cdot \Phi_{01} \left[ (Z_{el} - 10) - cl \right] - \frac{1}{2} \cdot (cl - 1) \cdot \Phi_{01} \left[ cl - (Z_{el} - 9) \right] \quad Z_{el} > 10$$

(segment change after  $cl = Z_{el} - 10$ )

$$E_{el_1} := (Z_{el} - xx_1)^2 \cdot h \cdot R_{el}^{\blacksquare} \quad E_{el_{Z_{el}-1}} := h \cdot R_{el_1} \cdot \left( Z_{el} - \frac{2}{3} \right)^2^{\blacksquare} \quad E_{el_{Z_{el}}} := h \cdot R_{el} \cdot Z_{el}^2^{\blacksquare}$$

$$E_{el_{cl}} := (Z_{el} - xx_{cl})^2 \cdot h \cdot R_{el}^{\blacksquare}$$



### Example: extrapolation of ionization energy equation to uranium

We can extrapolate the above ionization energy equations to the natural isotope with the highest Z-value, U-238

$$Z_U := 92 \quad M_{U238} := 239.286410 \quad R'_{U238} := \frac{1 + \frac{m_e}{M_{H1}}}{\left(1 + \frac{m_e}{M_{U238}}\right)} \cdot R' \quad R_{U238} := c_{\text{CGS}} \cdot R'_{U238}$$

$$cl := 2 .. Z_U - 2 \quad cm := 1 .. Z_U - 2 \quad ck := 1 .. Z_U$$

$$E_{U238\_obs} := \begin{pmatrix} 6.194 \\ 11.9 \\ 20 \\ 37 \end{pmatrix} \quad \text{From Ref. [24] compilation--only known values} \quad cn := 1 .. 4$$

$$xx_1 := Z_U - \frac{\sqrt{E_{U238\_obs_1}}}{\sqrt{R_{U238} \cdot h}}$$

$$xx_{cl} := xx_1 - \frac{1}{3} \cdot (cl - 1) \cdot \Phi_{01} \left[ (Z_U - 10) - cl \right] - \frac{1}{2} \cdot (cl - 1) \cdot \Phi_{01} \left[ cl - (Z_U - 9) \right] \quad (\text{segment change after } cl = Z_U - 10)$$

$$E_{U238_1} := (Z_U - xx_1)^2 \cdot h \cdot R_{U238} \quad E_{U238_{Z_U-1}} := h \cdot R_{U238} \cdot \left( Z_U - \frac{2}{3} \right)^2 \quad E_{U238_{Z_U}} := h \cdot R_{U238} \cdot Z_U^2$$

$$E_{U238_{cl}} := (Z_U - xx_{cl})^2 \cdot h \cdot R_{U238}$$

$E_{U238_{ck}} =$ 

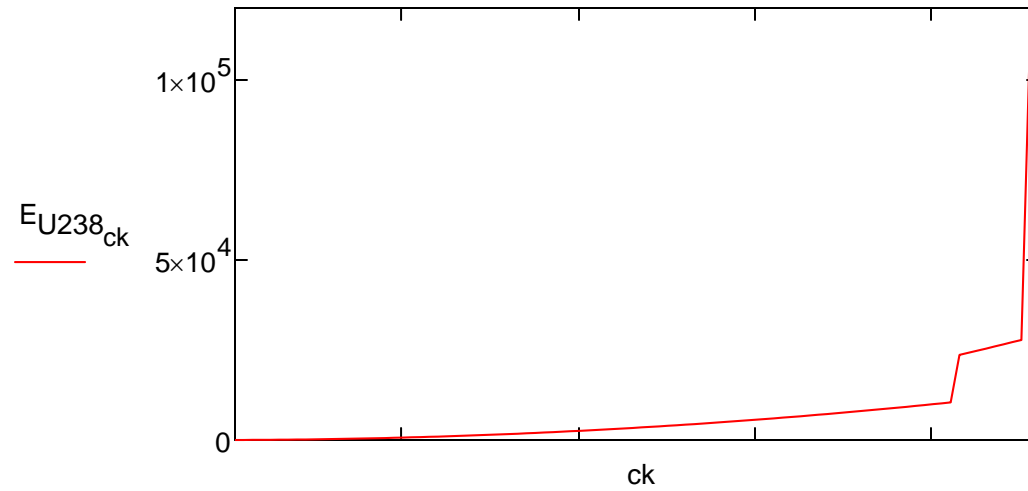
6.194
13.83057
24.4938
38.18368
54.90021
74.6434
97.41324
123.20973
152.03287
183.88266
218.75911
256.66221
297.59196
341.54837
388.53143
438.54114
491.5775
547.64051
606.73018
668.8465
733.98947
802.1591
873.35538
947.57831
1024.82789
1105.10413
1188.40701
1274.73655
1364.09275
1456.47559
1551.88509
1650.32124

 $E_{U238_{obs_{cn}}} =$ 

6.194
11.9
20
37

1751.78404
1856.2735
1963.78961
2074.33237
2187.90178
2304.49785
2424.12057
2546.76994
2672.44596
2801.14864
2932.87796
3067.63394
3205.41658
3346.22586
3490.0618
3636.92439
3786.81364
3939.72953
4095.67208
4254.64128
4416.63714
4581.65964
4749.7088
4920.78461
5094.88708
5272.01619
5452.17196
5635.35438
5821.56346
6010.79918
6203.06156
6398.35059

6596.66628
6798.00861
7002.3776
7209.77324
7420.19554
7633.64448
7850.12008
8069.62234
8292.15124
8517.7068
8746.28901
8977.89787
9212.53338
9450.19555
9690.88437
9934.59984
10181.34196
10431.11074
23654.4666
24225.47385
24803.29107
25387.91825
25979.3554
26577.60253
27182.65961
27794.52667
113614.48258
115279.14147



This is, of course, a prediction. And, of course, it's possible that there may be additional line segments, in which case the extrapolation would be wrong. But there is, presently, no theoretical indication that this would be the case, so we'll go with this extrapolation.

#### 4. Intensity of Spectral Lines

Based on the definition given by Herzberg, p. 152, the intensity of an emission spectrum line corresponding to the transition from  $n_1, m_1$  to  $n_2, m_2$  can be given by

$$I(n_1, m_1, n_2, m_2) := W(n_1, m_1, n_2, m_2) \cdot h \cdot \nu \quad \frac{\text{eV}}{\text{sec}} \quad (11)$$

where  $W(n_1, m_1, n_2, m_2)$  represents the *number* of transitions taking place per second in the light source, and  $h\nu$  is the energy of the radiated quantum. However,  $W$  is very difficult to calculate.

Kuhn, Ref. [10], p. 171, states: "The absolute strength of spectral lines is difficult to measure and difficult to establish. The same applies to the relative strength of lines of appreciably differing frequencies such as the different lines of a series. In consequence of this, accurate comparisons between experiment and theory are not very numerous." Relative intensities are tabulated in the *CRC Handbook* (Ref. [25], p. 10-1, ff.), but the Introduction says: "Because different observers frequently use different scales for their intensity estimates, these intensities are useful only as a rough indication of the appearance of a spectrum."

However, now that we know that emission and absorption lines are the result of changes in the *thermal* motions of the atoms, not electrons jumping from orbit to orbit around nuclei, we can, as an approximation, use the well-known regular "black body" relations derived by Wien.

The brightest lines observed should indicate the *most likely* temperature. Wien's Law is (Lang, Ref. [22], p. 21):

$$B_\nu(T, \nu) := \frac{2 \cdot h_{\text{cgs}} \cdot \nu^3}{c_{\text{cgs}}^2} \cdot \exp\left(\frac{-h_{\text{cgs}} \cdot \nu}{k_{\text{B\_erg}} \cdot T}\right) \quad \frac{\text{erg}}{\text{sec} \cdot \text{cm}^2 \cdot \text{Hz} \cdot \text{rad}^2} \quad h_{\text{cgs}} \nu \gg k_{\text{B\_erg}} T \quad (12)$$

The frequency at which  $B_\nu$  is maximized is then computed from:

$$\frac{d}{d\nu} \left( \frac{2 \cdot h_{\text{cgs}} \cdot \nu^3}{c_{\text{cgs}}^2} \cdot \exp\left(\frac{-h_{\text{cgs}} \cdot \nu}{k_{\text{B\_erg}} \cdot T}\right) \right) = 0 \quad (13)$$

$$\nu_m := \frac{3 \cdot k_{\text{B\_erg}} \cdot T}{h_{\text{cgs}}} \quad \text{or} \quad \nu_m := 6.2442 \cdot 10^{10} \cdot T \quad \text{Hz} \quad \lambda_m := \frac{c_{\text{cgs}}}{\nu_m} \quad \lambda_m := \frac{.4801}{T} \quad \text{cm} \quad \text{spectra\_energy.mcd} \quad (14)$$

$$\nu_m(T) := \frac{3 \cdot k_{\text{B\_erg}} \cdot T}{h_{\text{cgs}}} \quad (\text{functional representation}) \quad (\text{all temperatures are in K})$$

### Example calculations for arc spectra:

For a wide range of arc spectra, we'll assume 5 temperatures for 5 bright lines (interspersed with dark lines).

$$T_1 := 2000 \quad \nu_{m1} := \nu_m(T_1) \quad \nu_{m1} = 1.25007 \times 10^{14}$$

$$B_{\nu m1} := B_{\nu}(T_1, \nu_{m1}) \quad B_{\nu m1} = 1.43404 \times 10^{-6}$$

$$T_2 := 4000 \quad \nu_{m2} := \nu_m(T_2) \quad \nu_{m2} = 2.50015 \times 10^{14}$$

$$B_{\nu m2} := B_{\nu}(T_2, \nu_{m2}) \quad B_{\nu m2} = 0.00001$$

$$T_3 := 6000 \quad \nu_{m3} := \nu_m(T_3) \quad \nu_{m3} = 3.75022 \times 10^{14}$$

$$B_{\nu m3} := B_{\nu}(T_3, \nu_{m3}) \quad B_{\nu m3} = 0.00004$$

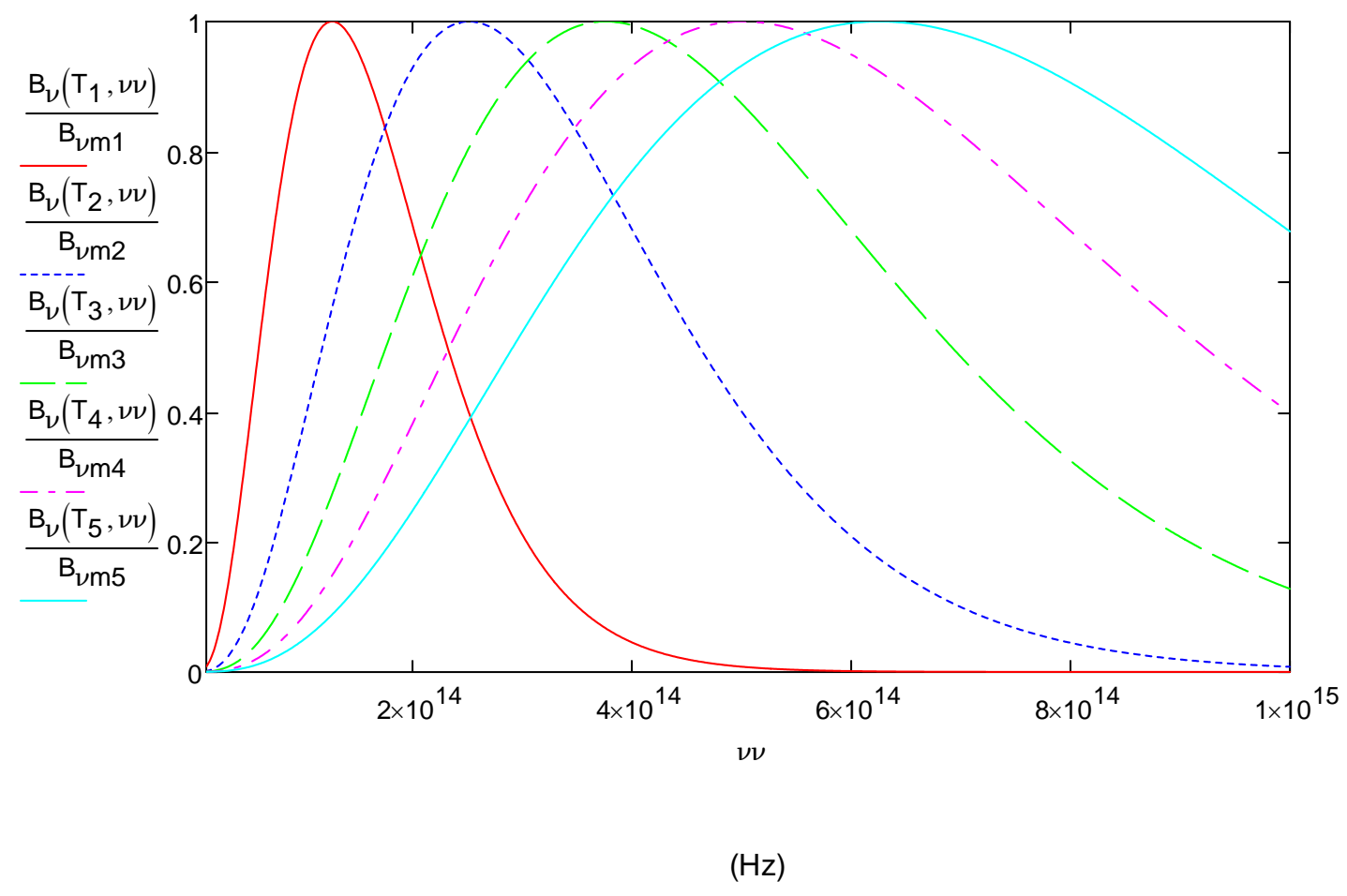
$$T_4 := 8000 \quad \nu_{m4} := \nu_m(T_4) \quad \nu_{m4} = 5.00029 \times 10^{14}$$

$$B_{\nu m4} := B_\nu(T_4, \nu_{m4}) \quad B_{\nu m4} = 0.00009$$

$$T_5 := 10000 \quad \nu_{m5} := \nu_m(T_5) \quad \nu_{m5} = 6.25036 \times 10^{14}$$

$$B_{\nu m5} := B_\nu(T_5, \nu_{m5}) \quad B_{\nu m5} = 0.00018$$

Now we can plot the *ratio* of brightness of any frequency to the maximum brightness for that temperature:





It's also helpful to plot the corresponding wavelengths and wave numbers:

$$B_{\lambda}(T, \lambda) := 2 \cdot h_{\text{cgs}} \cdot \frac{c_{\text{cgs}}}{\lambda^3} \cdot \exp\left(\frac{-h_{\text{cgs}} \cdot \frac{c_{\text{cgs}}}{\lambda}}{k_{\text{B\_erg}} \cdot T}\right) \frac{\text{erg}}{\text{sec} \cdot \text{cm}^2 \cdot \text{Hz} \cdot \text{rad}^2} \quad (\text{functional form for wavelength})$$

$$B_{\nu'}(T, \nu') := 2 \cdot h_{\text{cgs}} \cdot c_{\text{cgs}} \cdot \nu'^3 \cdot \exp\left(\frac{-h_{\text{cgs}} \cdot c_{\text{cgs}} \cdot \nu'}{k_{\text{B\_erg}} \cdot T}\right) \frac{\text{erg}}{\text{sec} \cdot \text{cm}^2 \cdot \text{Hz} \cdot \text{rad}^2} \quad (\text{functional form for wavenumber})$$

$$\lambda_{\text{m1}} := \frac{c_{\text{cgs}}}{\nu_{\text{m1}}} \quad \lambda_{\text{m1}} = 0.00024 \text{ cm} \quad B_{\lambda_{\text{m1}}} := B_{\lambda}(T_1, \lambda_{\text{m1}})$$

$$\lambda_{\text{m2}} := \frac{c_{\text{cgs}}}{\nu_{\text{m2}}} \quad \lambda_{\text{m2}} = 0.00012 \text{ cm} \quad B_{\lambda_{\text{m2}}} := B_{\lambda}(T_2, \lambda_{\text{m2}})$$

$$\lambda_{\text{m3}} := \frac{c_{\text{cgs}}}{\nu_{\text{m3}}} \quad \lambda_{\text{m3}} = 0.00008 \text{ cm} \quad B_{\lambda_{\text{m3}}} := B_{\lambda}(T_3, \lambda_{\text{m3}})$$

$$\lambda_{\text{m4}} := \frac{c_{\text{cgs}}}{\nu_{\text{m4}}} \quad \lambda_{\text{m4}} = 0.00006 \text{ cm} \quad B_{\lambda_{\text{m4}}} := B_{\lambda}(T_4, \lambda_{\text{m4}})$$

$$\lambda_{\text{m5}} := \frac{c_{\text{cgs}}}{\nu_{\text{m5}}} \quad \lambda_{\text{m5}} = 0.000048 \text{ cm} \quad B_{\lambda_{\text{m5}}} := B_{\lambda}(T_5, \lambda_{\text{m5}})$$

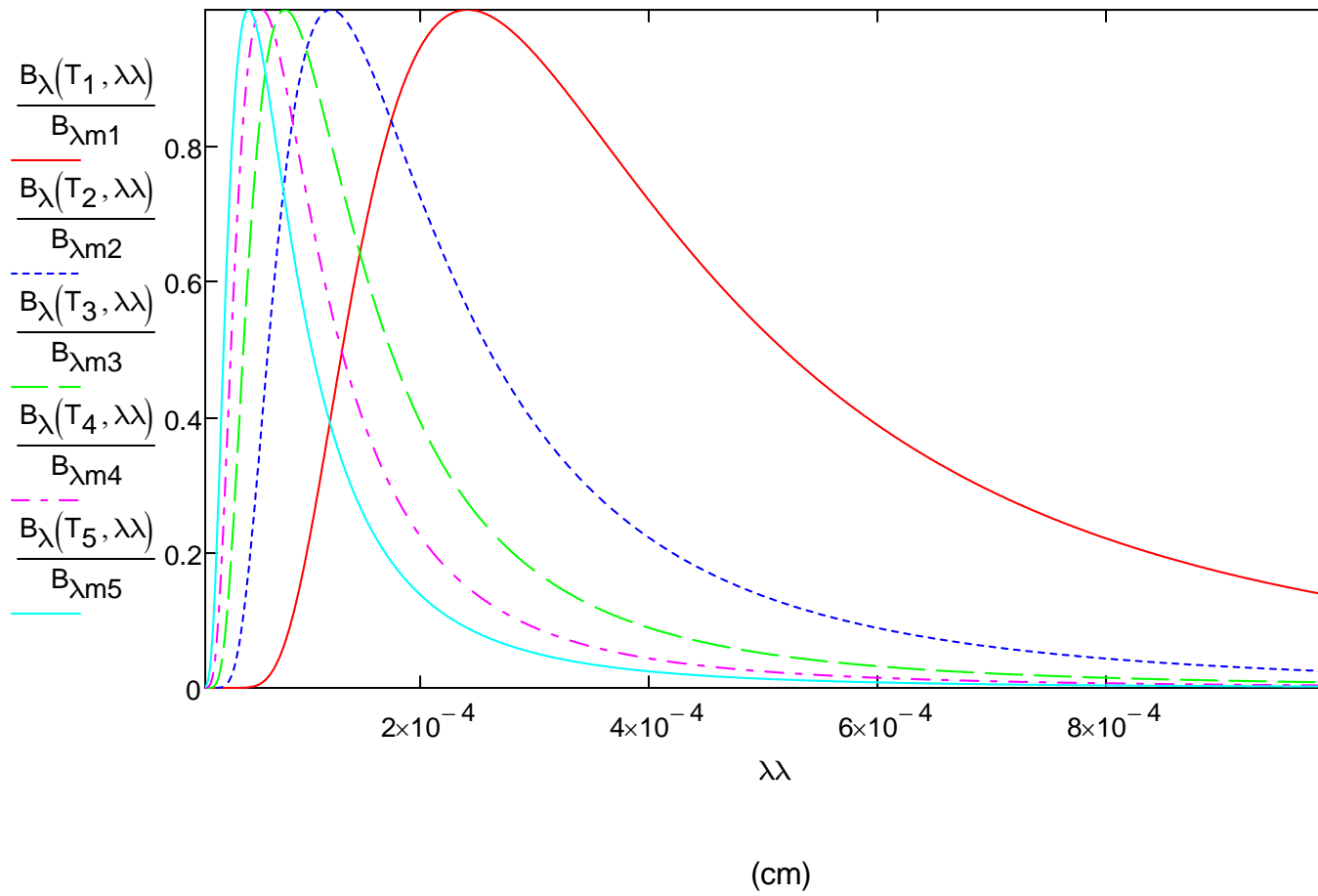
$$\nu'_{m1} := \frac{1}{\lambda_{m1}} \quad \nu'_{m1} = 4169.79273 \quad \text{cm}^{-1} \quad B_{\nu'_{m1}} := B_{\nu'}(T_1, \nu'_{m1})$$

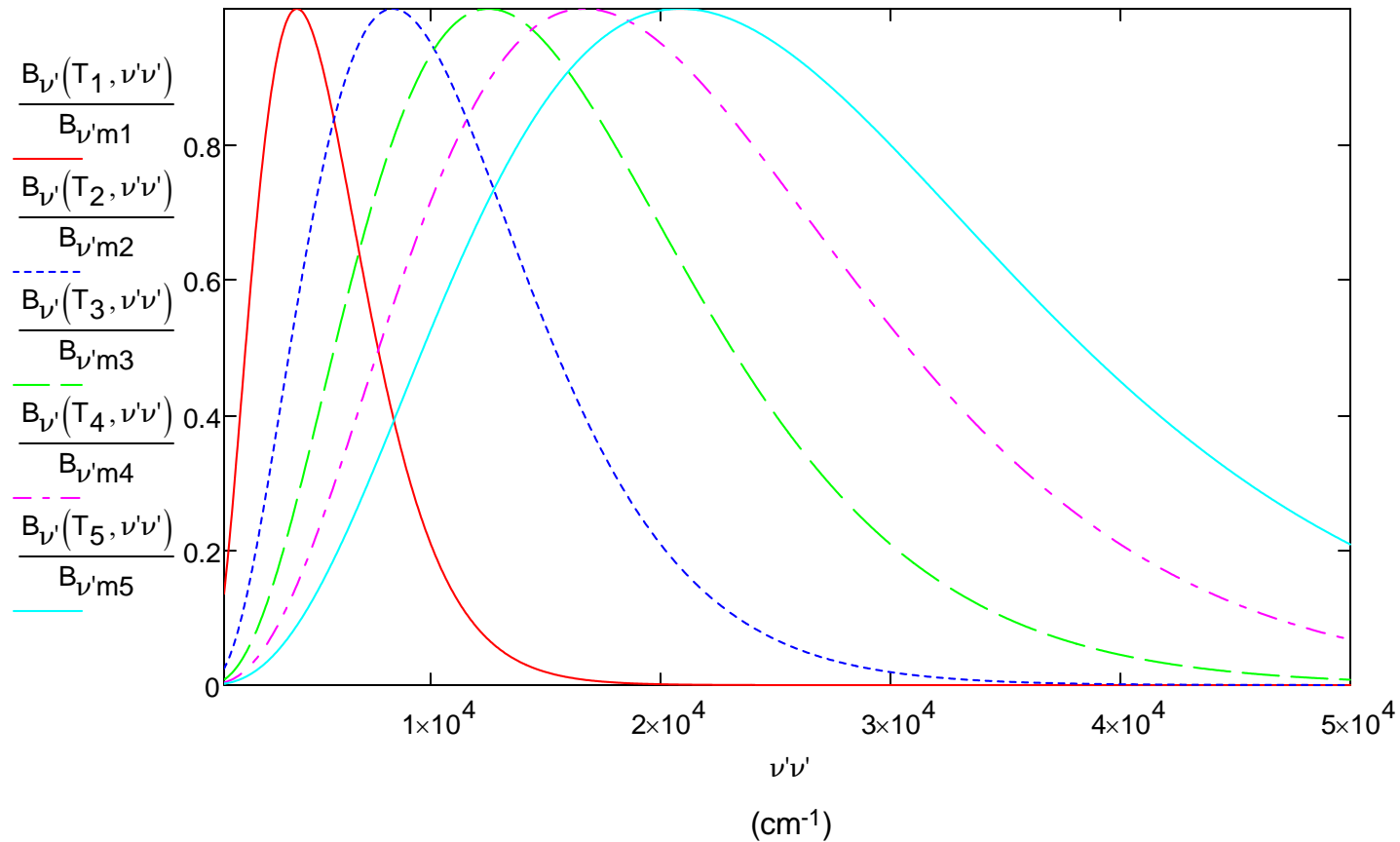
$$\nu'_{m2} := \frac{1}{\lambda_{m2}} \quad \nu'_{m2} = 8339.58545 \quad \text{cm}^{-1} \quad B_{\nu'_{m2}} := B_{\nu'}(T_2, \nu'_{m2})$$

$$\nu'_{m3} := \frac{1}{\lambda_{m3}} \quad \nu'_{m3} = 12509.37818 \quad \text{cm}^{-1} \quad B_{\nu'_{m3}} := B_{\nu'}(T_3, \nu'_{m3})$$

$$\nu'_{m4} := \frac{1}{\lambda_{m4}} \quad \nu'_{m4} = 16679.1709 \quad \text{cm}^{-1} \quad B_{\nu'_{m4}} := B_{\nu'}(T_4, \nu'_{m4})$$

$$\nu'_{m5} := \frac{1}{\lambda_{m5}} \quad \nu'_{m5} = 20848.96363 \quad \text{cm}^{-1} \quad B_{\nu'_{m5}} := B_{\nu'}(T_5, \nu'_{m5})$$





If one examines the numerous spectra illustrated in the References, one usually sees widely separated bright lines, with relatively less bright lines in between. Our interpretation is that this means that these bright lines are at considerably *different temperatures*. The nearby darker lines surrounding each bright line are then at approximately the same temperature as this bright line. Therefore we conclude that the plots above are, at least approximately, correct.

**Example calculations for spark spectra:**

For a wide range of spark spectra, we'll assume 5 temperatures for 5 bright lines.

$$T_1 := 30000 \quad \nu_{m1} := \nu_m(T_1) \quad \nu_{m1} = 1.87511 \times 10^{15}$$

$$B_{\nu m1} := B_{\nu}(T_1, \nu_{m1}) \quad B_{\nu m1} = 0.00484$$

$$T_2 := 35000 \quad \nu_{m2} := \nu_m(T_2) \quad \nu_{m2} = 2.18763 \times 10^{15}$$

$$B_{\nu m2} := B_{\nu}(T_2, \nu_{m2}) \quad B_{\nu m2} = 0.00769$$

$$T_3 := 40000 \quad \nu_{m3} := \nu_m(T_3) \quad \nu_{m3} = 2.50015 \times 10^{15}$$

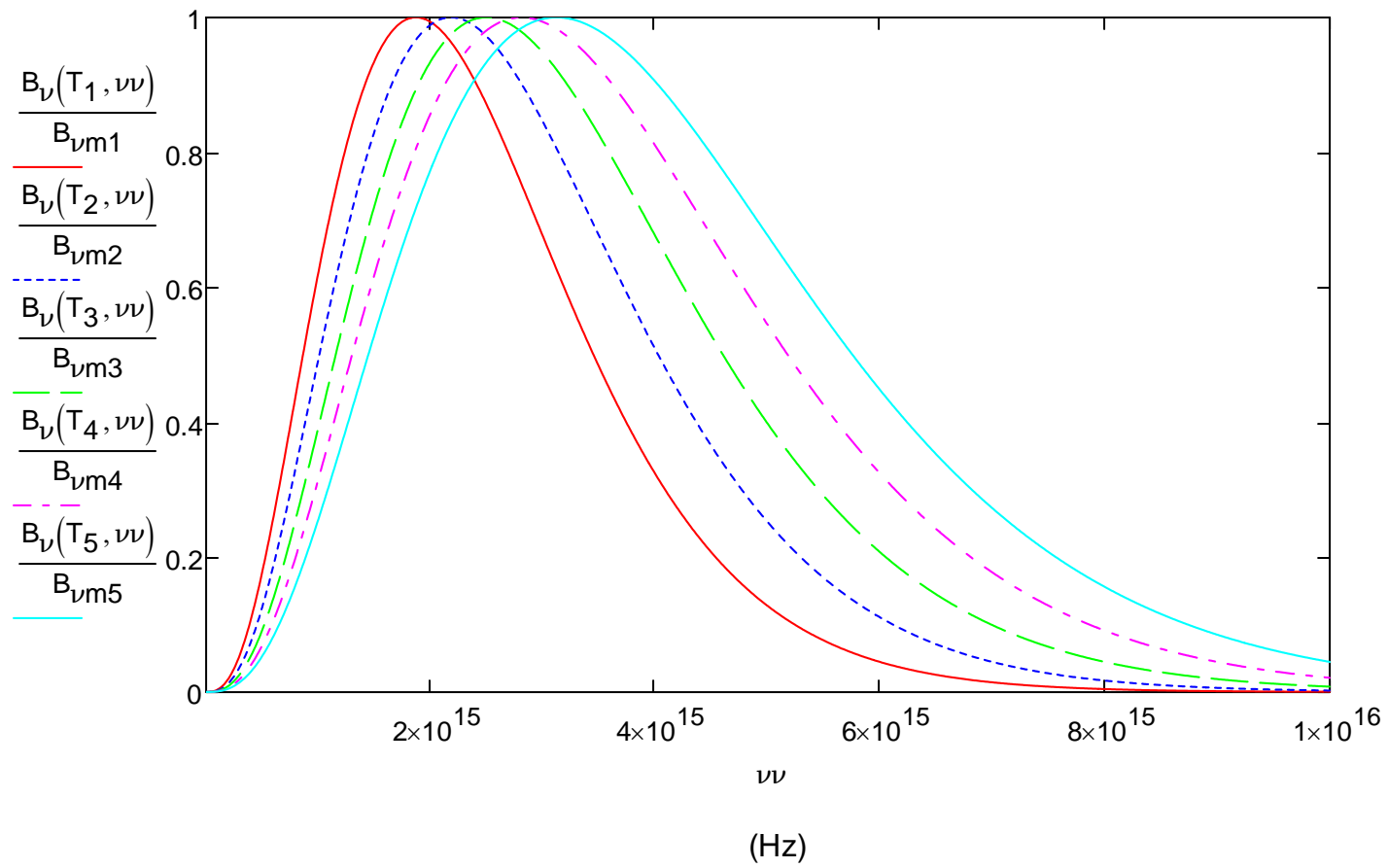
$$B_{\nu m3} := B_{\nu}(T_3, \nu_{m3}) \quad B_{\nu m3} = 0.01147$$

$$T_4 := 45000 \quad \nu_{m4} := \nu_m(T_4) \quad \nu_{m4} = 2.81266 \times 10^{15}$$

$$B_{\nu m4} := B_{\nu}(T_4, \nu_{m4}) \quad B_{\nu m4} = 0.01633$$

$$T_5 := 50000 \quad \nu_{m5} := \nu_m(T_5) \quad \nu_{m5} = 3.12518 \times 10^{15}$$

$$B_{\nu m5} := B_{\nu}(T_5, \nu_{m5}) \quad B_{\nu m5} = 0.02241$$



As with the arc spectra, it's also helpful to plot the corresponding wavelengths and wave numbers:

$$\lambda_{m1} := \frac{c_{\text{cgs}}}{\nu_{m1}} \quad \lambda_{m1} = 0.000016 \quad \text{cm} \quad B_{\lambda m1} := B_{\lambda}(T_1, \lambda_{m1})$$

$$\lambda_{m2} := \frac{c_{\text{cgs}}}{\nu_{m2}} \quad \lambda_{m2} = 0.000014 \quad \text{cm} \quad B_{\lambda m2} := B_{\lambda}(T_2, \lambda_{m2})$$

$$\lambda_{m3} := \frac{c_{\text{cgs}}}{\nu_{m3}} \quad \lambda_{m3} = 0.000012 \quad \text{cm} \quad B_{\lambda m3} := B_{\lambda}(T_3, \lambda_{m3})$$

$$\lambda_{m4} := \frac{c_{\text{cgs}}}{\nu_{m4}} \quad \lambda_{m4} = 0.000011 \quad \text{cm} \quad B_{\lambda m4} := B_{\lambda}(T_4, \lambda_{m4})$$

$$\lambda_{m5} := \frac{c_{\text{cgs}}}{\nu_{m5}} \quad \lambda_{m5} = 9.592803 \times 10^{-6} \quad \text{cm} \quad B_{\lambda m5} := B_{\lambda}(T_5, \lambda_{m5})$$

$$\nu'_{m1} := \frac{1}{\lambda_{m1}} \quad \nu'_{m1} = 62546.89088 \quad \text{cm}^{-1} \quad B_{\nu'_{m1}} := B_{\nu'}(T_1, \nu'_{m1})$$

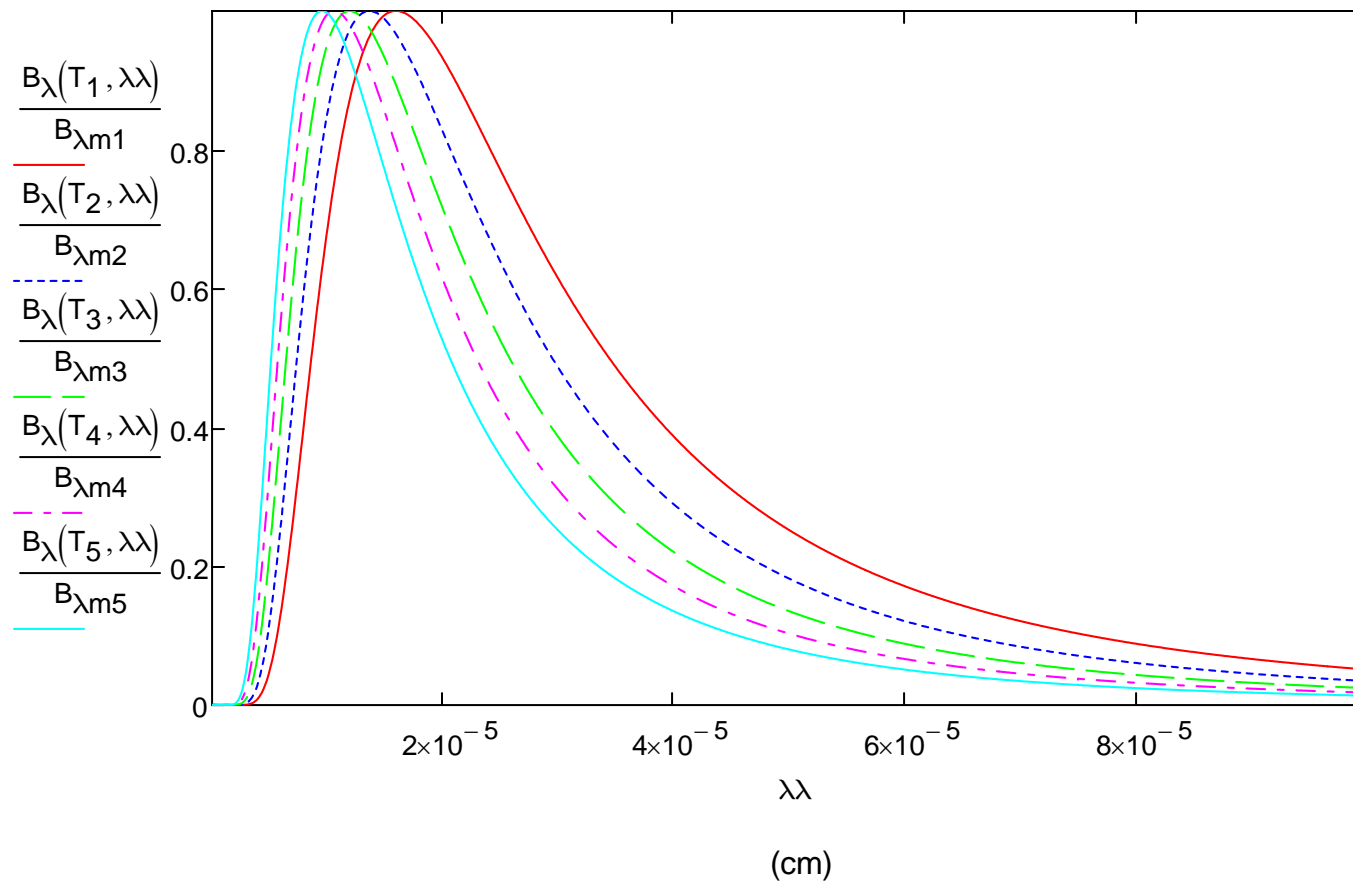
$$\nu'_{m2} := \frac{1}{\lambda_{m2}} \quad \nu'_{m2} = 72971.37269 \quad \text{cm}^{-1} \quad B_{\nu'_{m2}} := B_{\nu'}(T_2, \nu'_{m2})$$

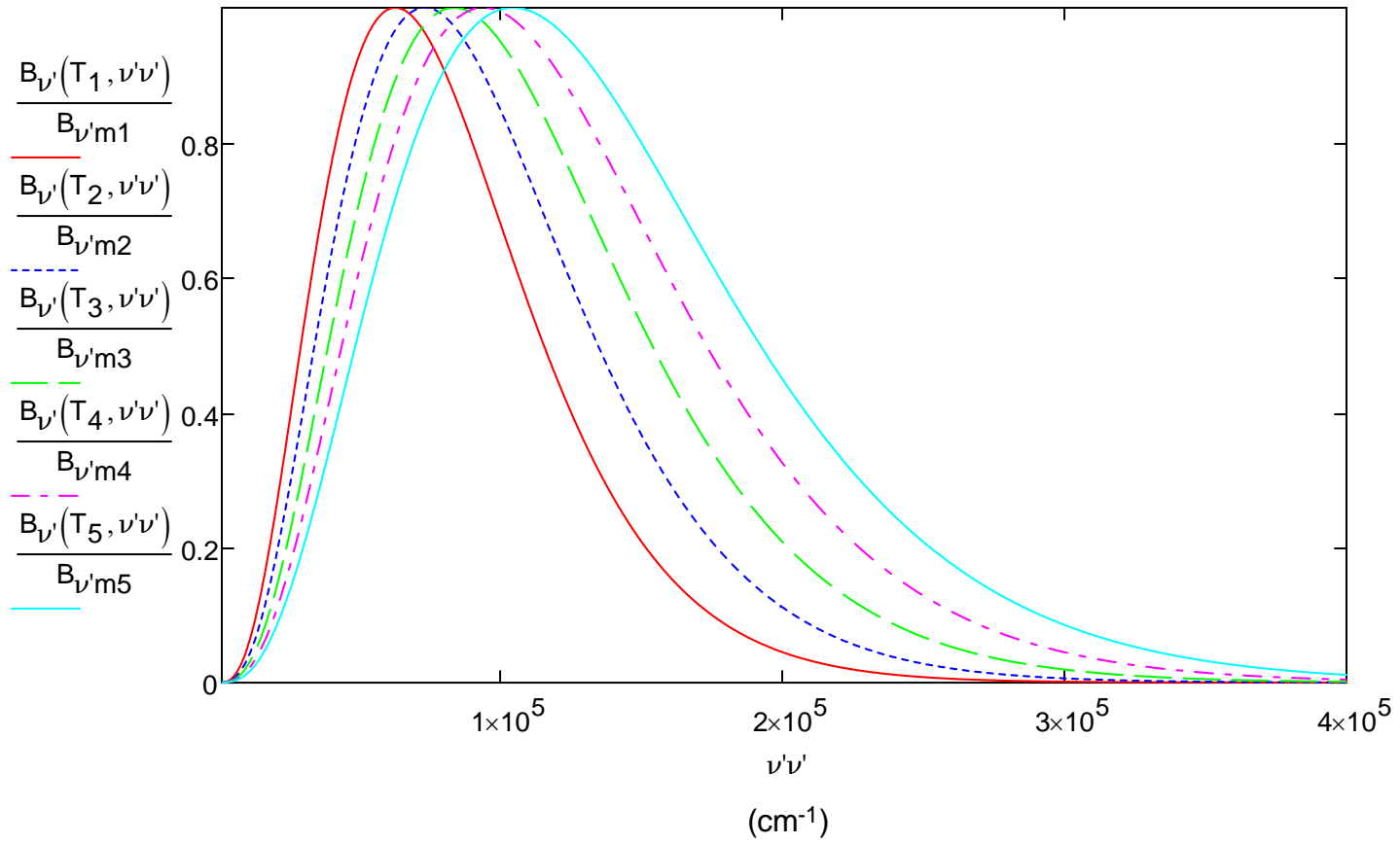
$$\nu'_{m3} := \frac{1}{\lambda_{m3}} \quad \nu'_{m3} = 83395.85451 \quad \text{cm}^{-1} \quad B_{\nu'_{m3}} := B_{\nu'}(T_3, \nu'_{m3})$$

$$\nu'_{m4} := \frac{1}{\lambda_{m4}} \quad \nu'_{m4} = 93820.33632 \quad \text{cm}^{-1} \quad B_{\nu'_{m4}} := B_{\nu'}(T_4, \nu'_{m4})$$

$$\nu'_{m5} := \frac{1}{\lambda_{m5}} \quad \nu'_{m5} = 104244.81814 \quad \text{cm}^{-1} \quad B_{\nu'_{m5}} := B_{\nu'}(T_5, \nu'_{m5})$$







For a wide range of x-ray spectra, we'll assume 5 temperatures for 5 bright lines. Note: the temperatures may be thought of here as *energy equivalents*, because x-ray spectra are usually observed after bombarding *solids* by *energetic charged electrons*.

$$T_1 := 30000000 \quad \nu_{m1} := \nu_m(T_1) \quad \nu_{m1} = 1.87511 \times 10^{18}$$

$$B_{\nu m1} := B_\nu(T_1, \nu_{m1}) \quad B_{\nu m1} = 4.83989 \times 10^6$$

$$T_2 := 35000000 \quad \nu_{m2} := \nu_m(T_2) \quad \nu_{m2} = 2.18763 \times 10^{18}$$

$$B_{\nu m2} := B_\nu(T_2, \nu_{m2}) \quad B_{\nu m2} = 7.68557 \times 10^6$$

$$T_3 := 40000000 \quad \nu_{m3} := \nu_m(T_3) \quad \nu_{m3} = 2.50015 \times 10^{18}$$

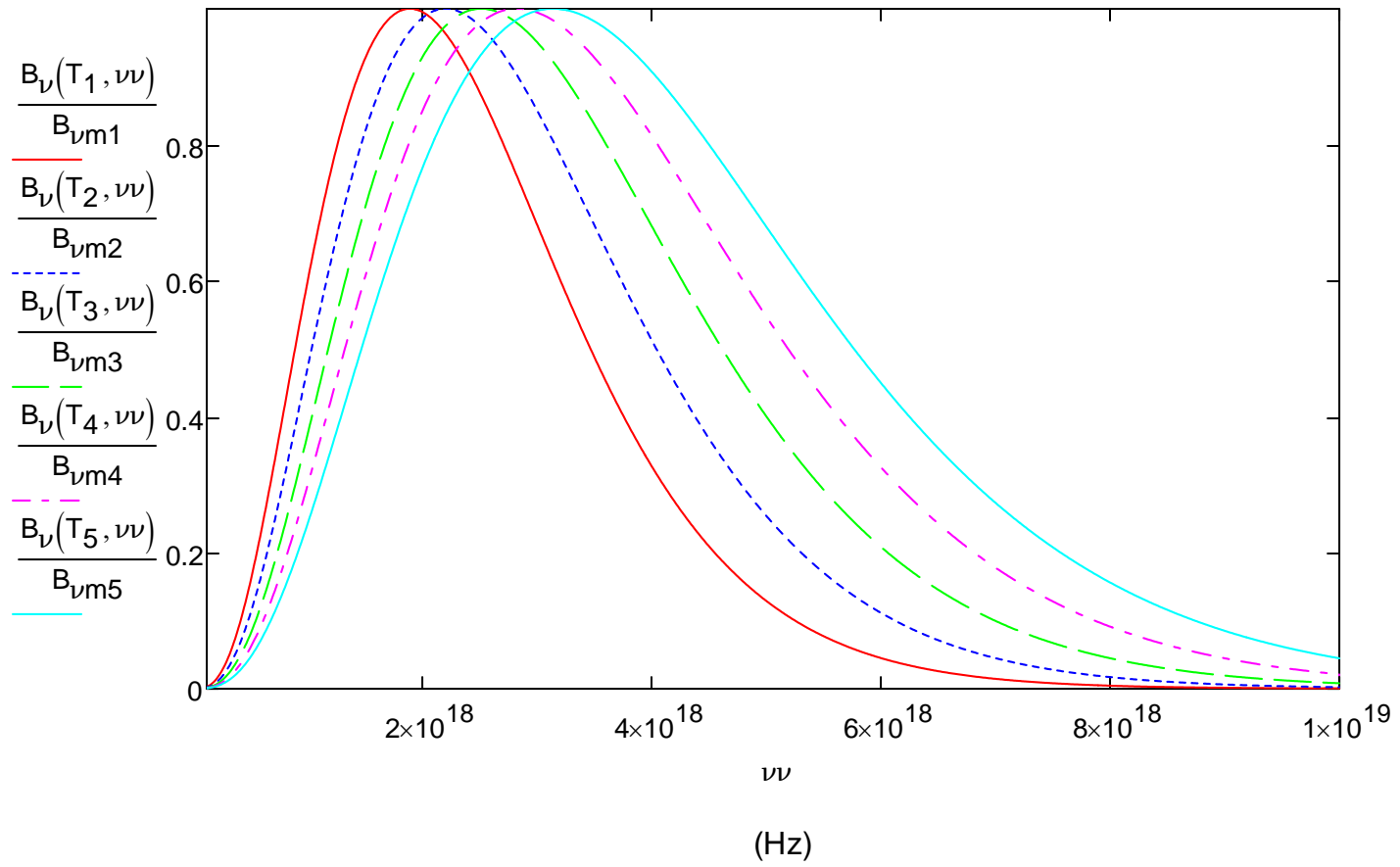
$$B_{\nu m3} := B_\nu(T_3, \nu_{m3}) \quad B_{\nu m3} = 1.14723 \times 10^7$$

$$T_4 := 45000000 \quad \nu_{m4} := \nu_m(T_4) \quad \nu_{m4} = 2.81266 \times 10^{18}$$

$$B_{\nu m4} := B_\nu(T_4, \nu_{m4}) \quad B_{\nu m4} = 1.63346 \times 10^7$$

$$T_5 := 50000000 \quad \nu_{m5} := \nu_m(T_5) \quad \nu_{m5} = 3.12518 \times 10^{18}$$

$$B_{\nu m5} := B_\nu(T_5, \nu_{m5}) \quad B_{\nu m5} = 2.24069 \times 10^7$$



As with the arc and spark spectra, it's also helpful to plot the corresponding wavelengths and wave numbers:

$$\lambda_{m1} := \frac{c_{\text{cgs}}}{\nu_{m1}} \quad \lambda_{m1} = 1.5988 \times 10^{-8} \quad \text{cm} \quad B_{\lambda m1} := B_{\lambda}(T_1, \lambda_{m1})$$

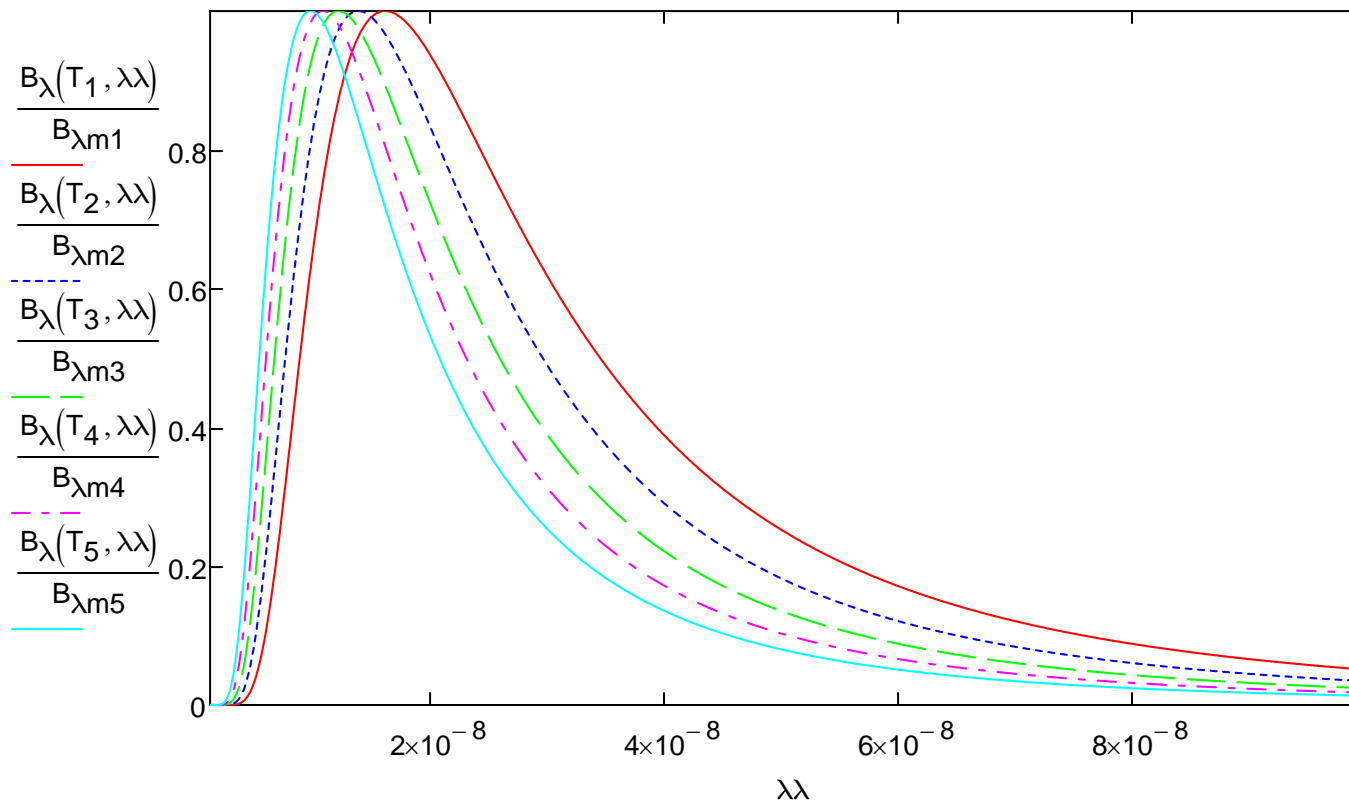
$$\lambda_{m2} := \frac{c_{\text{cgs}}}{\nu_{m2}} \quad \lambda_{m2} = 1.3704 \times 10^{-8} \quad \text{cm} \quad B_{\lambda m2} := B_{\lambda}(T_2, \lambda_{m2})$$

$$\lambda_{m3} := \frac{c_{\text{cgs}}}{\nu_{m3}} \quad \lambda_{m3} = 1.1991 \times 10^{-8} \quad \text{cm} \quad B_{\lambda m3} := B_{\lambda}(T_3, \lambda_{m3})$$

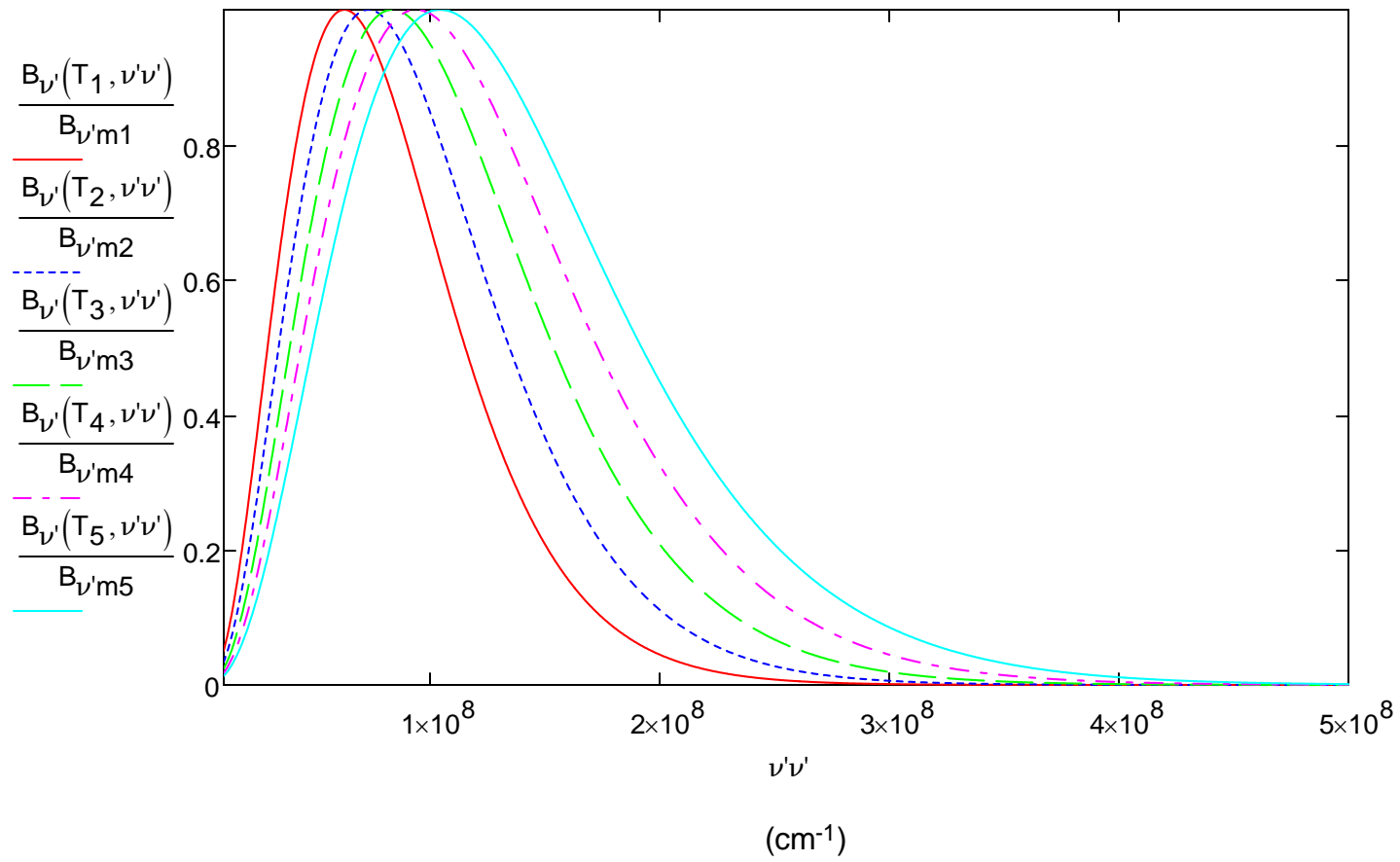
$$\lambda_{m4} := \frac{c_{\text{cgs}}}{\nu_{m4}} \quad \lambda_{m4} = 1.065867 \times 10^{-8} \quad \text{cm} \quad B_{\lambda m4} := B_{\lambda}(T_4, \lambda_{m4})$$

$$\lambda_{m5} := \frac{c_{\text{cgs}}}{\nu_{m5}} \quad \lambda_{m5} = 9.592803 \times 10^{-9} \quad \text{cm} \quad B_{\lambda m5} := B_{\lambda}(T_5, \lambda_{m5})$$

$$\begin{aligned}\nu'_{m1} &:= \frac{1}{\lambda_{m1}} & \nu'_{m1} &= 6.25469 \times 10^7 \text{ cm}^{-1} & B_{\nu'_{m1}} &:= B_{\nu'}(T_1, \nu'_{m1}) \\ \nu'_{m2} &:= \frac{1}{\lambda_{m2}} & \nu'_{m2} &= 7.29714 \times 10^7 \text{ cm}^{-1} & B_{\nu'_{m2}} &:= B_{\nu'}(T_2, \nu'_{m2}) \\ \nu'_{m3} &:= \frac{1}{\lambda_{m3}} & \nu'_{m3} &= 8.33959 \times 10^7 \text{ cm}^{-1} & B_{\nu'_{m3}} &:= B_{\nu'}(T_3, \nu'_{m3}) \\ \nu'_{m4} &:= \frac{1}{\lambda_{m4}} & \nu'_{m4} &= 9.38203 \times 10^7 \text{ cm}^{-1} & B_{\nu'_{m4}} &:= B_{\nu'}(T_4, \nu'_{m4}) \\ \nu'_{m5} &:= \frac{1}{\lambda_{m5}} & \nu'_{m5} &= 1.04245 \times 10^8 \text{ cm}^{-1} & B_{\nu'_{m5}} &:= B_{\nu'}(T_5, \nu'_{m5})\end{aligned}$$



(cm) (note:  $1 \text{ \AA} = 10^{-8} \text{ cm}$ , so x-ray wavelengths are commonly expressed in  $\text{\AA}$ )





## 5. Spectral Line Width

Quoting Kuhn, Ref. [10], pp. 385-386: "If a monochromatic light source is moving with a velocity whose component in the line of sight is  $v_x$ , the frequency appears shifted by an amount  $v_0 v_x / c$  compared with the frequency  $\nu_0$  of the source at rest. Owing to this *Doppler Effect*, the random motion of the atoms of a gas causes the spectral lines emitted or absorbed by these atoms to be broadened. If the lines emitted by atoms at rest are considered as infinitely narrow, i.e., if natural line width and broadening by external influences are neglected, the intensity distribution follows immediately from Maxwell's distribution law....The half-value width in wave-numbers is found to be

$$\Delta\nu' := \frac{2 \cdot \nu'_0}{c} \cdot \sqrt{\frac{2 \cdot R_{\text{gas}} \cdot T \cdot \ln(2)}{M}}$$

$$\Delta\nu' := 7.16 \cdot 10^{-7} \cdot \nu'_0 \cdot \sqrt{\frac{T}{M}}$$

Here, for once, the Reciprocal System is in agreement with conventional theory. But: isn't it odd that conventional theory uses thermal theory to explain line width but *not* the frequency itself, as does the Reciprocal System?

### Example:

$$M_{\text{U238}} = 239.28641 \text{ u} \quad T := 23900 \text{ K} \quad \nu'_0 := \frac{1}{5000 \cdot 10^{-8}} \text{ cm}^{-1}$$

$$\Delta\nu' := 7.16 \cdot 10^{-7} \cdot \nu'_0 \cdot \sqrt{\frac{T}{M_{\text{U238}}}} \quad \Delta\nu' = 0.14311 \text{ cm}^{-1}$$

note: this is of the same order of magnitude as present-day spectroscopy resolution

## 6. Spectra in Magnetic Fields

As explained in Ref. [29], pp. 113-115, and many other texts, when an atom is placed in a magnetic field its spectral lines are split into a number of discrete lines. The observed change in frequency of the lines is directly proportional to the magnitude of the applied field,  $B$ . The conventional expression (in SI units, for convenience) for this additional energy is

$$E_B := m_l \cdot \mu_{B\_som} \cdot B$$

where  $m_l$  is the magnetic quantum number and  $\mu_B$  is the Bohr magneton (in the Sommerfeld system). From Ref. [32], it's easy to derive the corresponding equation from the Reciprocal System (also in SI units, for convenience):

$$E_B := n_M \cdot \mu_L \cdot B \tag{15}$$

where  $n_M$  is the number of magnetic charges *induced* per atom and  $\mu_L$  is the Larson magneton. This latter magneton can be defined as

$$\mu_L := \mu_{admp\_u\_som} \cdot \frac{T_{v\_u}}{T} \cdot \frac{D}{D_u} \quad \text{joule/tesla} \tag{16}$$

in Sommerfeld units, rather than Kennelly units. Note that the Larson magneton varies with atomic diameter and temperature. For either theory, the change in wavelength can be found by taking the differential of  $E = hc/\lambda$ :

$$dE := -h \cdot c \cdot \frac{d\lambda}{\lambda^2} \quad \text{or} \quad |d\lambda| := \frac{\lambda^2 \cdot |dE|}{h \cdot c}$$

where  $dE = E_B$ .

**Example:** Cd red line in magnetic field of 1 tesla

$$B := 1 \text{ tesla} \quad \lambda := 6438 \cdot 10^{-10} \text{ m} \quad \mu_{B\_som} = 9.2741 \times 10^{-24} \text{ joule/tesla}$$

conventional theory:

$$dE := \mu_{B\_som} \cdot B \quad dE = 0 \quad \text{joules}$$

$$d\lambda := \frac{\lambda^2 \cdot |dE|}{h_{SI} \cdot c_{SI}} \quad d\lambda = 1.93509 \times 10^{-11} \text{ m} \quad (\text{plus or minus from } \lambda)$$

here  $m_l = +/- 1$  ("regular" Zeeman effect)

Reciprocal System:

Cd has rotational displacements 4-3-(6)  $n_p := 4$   $n_s := 3$   $n_e := -6$

$D_{Cd} := 14.922 \cdot 10^{-15} \text{ m}$  (from Reciprocal System data base)

$T := 3850 \text{ K}$  (for arc spectra) (either from experiment or from iteration with  $E_B$ )

$n_{M\_Cd} := 3$  (could be 1, 2, 3, or 4, i.e.  $\leq n_p$ ; 3 is for a strong field for "regular" Zeeman effect)

$$\mu_{L\_Cd} := \mu_{admp\_u\_som} \cdot \frac{T_{v\_u}}{T} \cdot \frac{D_{Cd}}{D_u} \quad \mu_{L\_Cd} = 3.0781 \times 10^{-24} \text{ joule/tesla}$$

$$E_B := n_{M\_Cd} \cdot \mu_{L\_Cd} \cdot B \quad E_B = 9.2343 \times 10^{-24} \text{ joule}$$

$$d\lambda := \frac{\lambda^2 \cdot E_B}{h_{SI} \cdot c_{SI}} \quad d\lambda = 1.92679 \times 10^{-11} \text{ m} \quad (\text{plus or minus from } \lambda)$$

$$\lambda_{Cd} := \begin{pmatrix} \lambda - d\lambda \\ \lambda \\ \lambda + d\lambda \end{pmatrix} \cdot 10^{10} \quad \lambda_{Cd} = \begin{pmatrix} 6437.81 \\ 6438 \\ 6438.19 \end{pmatrix} \text{ A}$$

For use with Eq. (2), for an atom initially in zero field and then in a field of magnitude B:

$$E_{el\_other\_1} := 0 \quad E_{el\_other\_2} := E_B \quad (17)$$

As stated at the beginning of this paper, the frequency or wavelength of a spectral line emitted or absorbed in the Reciprocal System is ordinarily due to a change in the thermal motion of the atom, not electrons jumping around. The *vectorial* direction of the atomic thermal motion (scalarly outward) is *random* and thus there is a net component either in the direction of an imposed magnetic field or opposite to it at any moment. Hence, the calculation above.

For the "regular" Zeeman effect, there are only two displacements from the no-field spectral line. This means, for the Reciprocal System, that all atoms have the *same* number of magnetic charges (and very nearly the same temperature). For the "anomalous" Zeeman effect, there would be a probability distribution of the number of magnetic charges per atom, and thus there would have to be displacements for *each* set. So, for a situation in a *relatively weak* field, where some atoms have zero magnetic charge, some have one magnetic charge, some have two, and some have three, there would be six lines, plus the undisplaced center line. The strength of the lines would be dependent on the relative number of atoms with each possible number of charges.

The "anomalous" Zeeman effect is actually more common than the "regular" Zeeman effect. The "anomalous" Zeeman effect reduces to the "regular" Zeeman effect for *sufficiently strong* magnetic fields. This transition is termed the "Paschen-Back effect."

Both conventional theory and the Reciprocal System are consistent with the experimental observations of the Zeeman effect. But: the conventional theory equation is *not* dependent on temperature, whereas the Reciprocal System equation is. Therefore, it would be interesting to compare arc and spark spectra in a magnetic field--the problem, of course, is that normal electromagnets cannot be used at very high temperatures!

The surface temperature of many main sequence stars (excepting their coronas) is at approximately the same temperature as the atoms in arc spectra, which explains, perhaps, why conventional theory does not include a temperature variation for the Bohr magneton. The magnetic field of most stars is only a fraction of a tesla.

Just as a magnetic field splits atomic spectra, so does an electric field. This phenomenon is termed the Stark effect, after its discoverer. The electric field intensity is commonly expressed in units of volts/m--but electric flux density units are more appropriate here. Analogous to webers/m<sup>2</sup> (=tesla) for the magnetic field, we have coulombs/m<sup>2</sup> for the electric field. To convert the electric field intensity,  $F_V$ , to the electric flux density, we must multiply  $F_V$  by the permittivity, of the medium,  $\epsilon_r \times \epsilon_{0\_SI}$ , in SI units. This quantity must then be divided by the Reciprocal System natural unit of electric flux density stated in SI units. This ratio then multiplies the *total* ionization energy level of the atoms. The translational motion quantum number for state 2,  $n_2$ , is modified by the motion caused by the electric field,  $n_4 - n_3$ , with  $n_3$  and  $n_4$  being two additional quantum numbers. Conventional theory adds two quantum numbers to explain the "parabolic" motion of the electrons of an atom in an electric field. This is nonsense, of course; the two new numbers are simply due to the motion of the ions in the electric field (either in the same or opposite direction of the thermal motion). Putting all this together, we have

$$E_E := E_{el\_I\_tot} \cdot \frac{\epsilon_r \cdot \epsilon_{0\_SI} \cdot F_V}{Q_{uc}} \cdot n_2 \cdot (n_4 - n_3) \quad \blacksquare \quad \text{(for hydrogen and hydrogen-like ions)} \quad (18)$$

$$\frac{1}{(s_{t\_u} \cdot 10^{-2})^2}$$

$$n_3 \leq 0, 1, \dots, n_2 - 1 \quad n_4 \leq 0, 1, \dots, n_2 - 1 \quad \text{(integers only)}$$

for  $2n_2 - 1$  equidistant levels. The experimental data for the Stark effect is somewhat uncertain for elements higher than H; the splitting factor appears to be different (and may include modification of the time-space thermal translational quantum number,  $m_2$ , as well as  $n_2$ ). Application of the theory to these elements will have to wait for later work, but the terms in front of the splitting factor should be the same.

For use with Eq. (2), for an atom initially in zero field and then in a field of magnitude  $F_V$ :

$$E_{el\_other\_1} := 0 \quad E_{el\_other\_2} := E_E^2 \quad (19)$$

**Example:** ionized H1 in electric field of 100,000 volts/cm; assume Paschen series ( $n_1 = 3$ ,  $n_2 = 4$ ; ignore  $m_1$  and  $m_2$ )

$$n_1 := 3 \quad n_2 := 4 \quad n_3 := 0, 1, 2, 3 \quad n_4 := 0, 1, 2, 3 \quad \epsilon_r := 1.25 \quad (\text{much higher than normal due to experimental conditions; needs independent confirmation})$$

$$n_2 \cdot (n_4 - n_3) := -12, -8, -4, 0, 4, 8, 12 \quad 2 \cdot n_2 - 1 = 7 \quad \text{levels}$$

$$E_{H1\_I} = 13.61256 \quad \text{eV} \quad E_{H1\_I\_tot} := E_{H1\_I} \quad E_{H1\_I\_tot} = 13.61256 \quad \text{eV}$$

$$F_V := 100000 \cdot 10^2 \quad F_V = 1 \times 10^7 \quad \text{volts/m}$$

$$E_E := E_{H1\_I\_tot} \cdot \frac{\epsilon_r \cdot \epsilon_0 \cdot SI \cdot F_V}{Q_{uc}} \cdot \text{conv}_{\text{evtojoule}} \quad (\text{using SI for convenience and not including splitting factor yet})$$

$$\frac{1}{(s_{t\_u} \cdot 10^{-2})^2}$$

$$\nu_E := \frac{E_E}{h_{SI}} \quad \lambda_E := \frac{c_{SI}}{\nu_E} \quad \nu'_E := \frac{1}{\lambda_E} \quad \nu'_E = 643.73173 \quad \text{m}^{-1}$$

$$\nu'_{E\_cgs} := .01 \cdot \nu'_E \quad \nu'_{E\_cgs} = 6.43732 \text{ cm}^{-1} \quad (\text{back to cgs})$$

Kuhn (Ref. [10], p. 107) reports that the the experimental value is  $6.44 \text{ cm}^{-1}$  for this situation.

The frequency of this line, without the electric field, is

$$\nu := \frac{E_{H1\_l}}{h} \cdot \left( \frac{1}{n_1^2} - \frac{1}{n_2^2} \right) \quad \nu = 1.59836 \times 10^{14} \text{ Hz}$$

With the splitting factor, we have the following set of frequencies:

$$\begin{pmatrix} -12 \\ -8 \\ -4 \\ 0 \\ 4 \\ 8 \\ 12 \end{pmatrix} \cdot \frac{E_E}{h_{SI}} + \nu = \begin{pmatrix} 1.5752 \times 10^{14} \\ 1.58292 \times 10^{14} \\ 1.59064 \times 10^{14} \\ 1.59836 \times 10^{14} \\ 1.60608 \times 10^{14} \\ 1.6138 \times 10^{14} \\ 1.62152 \times 10^{14} \end{pmatrix} \text{ Hz}$$

## Conclusion



The spectroscopists and nuclear theorists have made the subject of spectroscopy and atomic structure far more difficult than it ever needed to be. The conventional equations are very complicated and mostly impractical. Complex spectra cannot be handled. In contrast, the equations derived from the principles of the Reciprocal System are very simple and very practical. All kinds of spectra, simple and complex, optical and x-ray, are all easily handled. Calculations of atomic ionization levels compare very well with those observed. Sooner or later, therefore, spectroscopists and atomic theorists will have to give up the nuclear theory of the atom and Quantum Mechanics. Going back to the classical electron theory of Lorentz, then the plum-pudding model of the atom by Thomson, then the "Old Quantum Theory" of Rutherford, Bohr, and Sommerfeld, and then the Quantum Mechanics of Schrodinger, Heisenberg, Van Vleck, Pauling, et. al, the theoretical physicists and chemists have had a fetish for explaining physical and chemical phenomena, such as atomic spectra, by means of charged electrons. That era is now ended.

## Acknowledgements

Funding for this work came from Transpower Corporation, not the government! Of course, great thanks go to Dewey B. Larson, who served as my theoretical physics mentor from 1965 until his death in 1990. He was, by far, the most intelligent and most logical of any individual I've ever known.

## References

[1] D. Larson, *The Case Against the Nuclear Atom* (Portland OR: North Pacific Publishers, 1963).

- [2] D. Larson, *Basic Properties of Matter* (Salt Lake City, UT: International Society of Unified Science, 1988).
- [3] D. Larson, *The Structure of the Physical Universe* (Portland, OR: North Pacific Publishers, 1959); *Nothing But Motion* (Portland, OR, North Pacific Publishers, 1979. The latter is Volume I of the 2nd ed. of *The Structure of the Physical Universe*).
- [4] D. Larson, *New Light on Space and Time* (Portland, OR: North Pacific Publishers, 1965).
- [5] D. Larson, *The Neglected Facts of Science* (Portland, OR: North Pacific Publishers, 1982).
- [6] D. Larson, *The Universe of Motion* (Portland, OR: North Pacific Publishers, 1984).
- [7] R. Satz, *The Unmysterious Universe* (Troy, NY: The New Science Advocates, 1971).
- [8] H. White, *Introduction to Atomic Spectra* (New York, NY: McGraw-Hill Book Company, Inc., 1934).
- [9] G. Herzberg, *Atomic Spectra and Atomic Structure* (New York, NY: Dover Publications, 1944).
- [10] H. Kuhn, *Atomic Spectra* (London, UK: Longmans, Green & Co., Ltd., 1962).
- [11] C. Candler, *Atomic Spectra and the Vector Model* (Princeton, NJ: D. Van Nostrand Company, Inc., 1964).
- [12] A. Fowler, *Report on Series in Line Spectra* (London, UK: Fleetway Press, Ltd, 1922).
- [13] S. Walker, H. Straw, *Atomic, Microwave, and Radio-Frequency Spectroscopy* (Norwich, UK: Fletcher & Son, Ltd., 1961).
- [14] S. Walker, H. Straw, *Ultra-Violet, Visible, Infra-Red, and Raman Spectroscopy* (Norwich, UK: Fletcher & Son, Ltd., 1962).
- [15] E. Condon, G. Shortley, *The Theory of Atomic Spectra* (London, UK: Cambridge University Press, 1935).
- [16] C. Moore, *Atomic Energy Levels as Derived from the Analysis of Optical Spectra*, Vol. 1 (Washington, D.C.:

[17] C. Moore, *Atomic Energy Levels as Derived from the Analysis of Optical Spectra*, Vol. 2 (Washington, D.C.: National Bureau of Standards, 1952).

[18] C. Moore, *Atomic Energy Levels as Derived from the Analysis of Optical Spectra*, Vol. 3 (Washington, D.C.: National Bureau of Standards, 1958).

[19] C. Moore, *Ionization Potentials and Ionization Limits Derived from the Analyses of Optical Spectra* (Washington, D.C.: National Bureau of Standards, 1970).

[20] NIST, <http://www.nist.gov/pml/data/asd.cfm>

[21] H. Moseley, "The High Frequency Spectra of the Elements," *Phil. Mag.*, 1913, p. 1024.

[22] K. Lang, *Astrophysical Formulae, 2nd Ed.* (Berlin, Germany: Springer-Verlag, 1980).

[23] A. Cox, ed., *Allen's Astrophysical Quantities, 4th Ed.* (New York, NY: Springer Science, 2004).

[24] I. Grigoriev, E. Meilikhov, ed., *Handbook of Physical Quantities* (Boca Raton, FL: CRC Press, 1997).

- [25] D. Lide, ed., *CRC Handbook of Chemistry and Physics*, 88th ed. (Boca Ratan, FL: CRC Press, 2008).
- [26] D. Gray, ed., *American Institute of Physics Handbook*, 3rd ed. (New York, NY: McGraw-Hill Book Company, 1972).
- [27] D. Layzer, "On a Screening Theory of Atomic Spectra," *Annals of Physics*, Vol. 8, pp. 271-296, 1959.
- [28] H. Di Rocco, "A Simple and Effective Approach to Calculate the Energy of Complex Atoms," *Revista Mexicana De Fisica*, Vol. 48, No. 1, pp. 76-87.
- [29] R. Gautreau, W. Savin, *Modern Physics* (New York, NY: McGraw-Hill Book Company, 1978).
- [30] R. Satz, "Photoionization and Photomagnetization," *Reciprocity*, Vol. XII, No. 1, Winter 1981-1982.
- [31] R. Satz, "Theory of Subatomic and Atomic Masses and Half-Lives", <http://transpower.wordpress.com>, 02/10/2012.
- [32] R. Satz, "Theory of Dielectrics, Diamagnets, Paramagnets, and Ferromagnets, including Calculation of Electric and Magnetic Susceptibilities", <http://transpower.wordpress.com>, 10/31/2011.